A consistent shear design model

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Modèle cohérent de dimensionnement à l'effort tranchant

Ein konsistentes Schubbemessungsverfahren

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SUMMARY

A shear model is presented which takes into account residual tensile stresses in cracked concrete. The model treats both prestressed and non-prestressed members and accounts for the influence of amount of longitudinal reinforcement, magnitude of moment, axial force and member size.

RÉSUMÉ

Un modèle de dimensionnement à l'effort tranchant qui tient compte des contraintes de traction résiduelles dans le béton fissuré est présenté dans cet article. Il traite des éléments de structure précontraints et non-précontraints et tient compte de l'influence de la quantité d'armature longitudinale, de l'intensité du moment, de la force axiale et de la taille de l'élément-même de structure.

ZUSAMMENFASSUNG

Ein Schubbemessungsverfahren wird beschrieben, das Zugeigenspannungen in gerissenem Beton berücksichtigt. Das Verfahren behandelt sowohl vorgespannte wie auch nicht vorgespannte Elemente und berücksichtigt solche Grössen wie Längsbewehrung, Biegemomente, Axiallasten und Bauteilgrössen.

1. INTRODUCTION

In 1973 the ACI-ASCE Shear Committee [1] concluded the introduction to its state-of-the-art report with the words:

> During the next decade it is hoped that the design regulations for shear strength can be integrated, simplified and given a physical significance ...

The shear provisions of the 1984 Canadian Concrete Code [2, 3], which were based on the compression field model, introduced both strain compatibility and the stress-strain characteristics of diagonally cracked concrete, enabling some of the objectives stated above to be achieved. However, because this model neglected the residual tensile stresses in diagonally cracked concrete, it was restricted to members with shear reinforcement.

The modified compression field model [4] considers the influence of residual tensile stresses in the cracked concrete and hence provides the basis for a consistent shear design model. In this paper a design approach based on the modified compression field theory is presented.

2. RESIDUAL TENSILE STRESSES IN CRACKED CONCRETE

Tests of reinforced concrete panels subjected to pure shear [4] demonstrated that even after cracking, tensile stresses exist in the concrete between the cracks and that these stresses can significantly increase the ability of reinforced concrete to resist shear stresses.

Cracked reinforced concrete transmits load in a relatively complex manner involving opening or closing of pre-existing cracks, formation of new cracks, interface shear transfer at rough crack surfaces, and significant variation of the stresses in reinforcing bars due to bond, with the highest steel stresses occurring at crack locations. The modified compression field model attempts to capture the essence of this behaviour without considering all of the details. The crack pattern is idealized as a series of parallel cracks all occurring at angle θ to the longitudinal direction. In lieu of following the complex stress variations in the cracked concrete, only the average stress state and the stress state at a crack are considered. As these two states of stress are statically equivalent, the loss of tensile stresses in the concrete at the crack must be replaced by increased steel stresses or, after yielding of the reinforcement at the crack, by shear stresses on the crack interface. The shear stress that can be transmitted across the crack will be a function of the crack width. Note that shear stress on the crack implies that the direction of principal stresses in the concrete changes at the crack location.

The average principal tensile strain, ϵ_1, in the cracked concrete is used as a "damage indicator" which controls the average tensile stress, f_1, in the cracked concrete, the ability of the diagonally cracked concrete to carry compressive stresses, f_2, and the shear stress, v_{ci}, that can be transmitted across a crack.

3. SHEAR DESIGN OF BEAMS

In applying the modified compression field theory to the design of beams it is appropriate to make a number of simplifying assumptions. As illustrated in Fig. 1, the shear stresses are assumed to be uniform over the effective shear area, $b_w\,jd$. The highest longitudinal strain, ϵ_x, within the effective shear area is used to calculate the principal tensile strain, ϵ_1.

The longitudinal strain, ϵ_x, can be determined from a plane sections analysis (see computer program "RESPONSE" [5]) which accounts for the influence of axial load, moment, and shear. For design, ϵ_x can be approximated as the strain in the "bottom chord" of a truss as

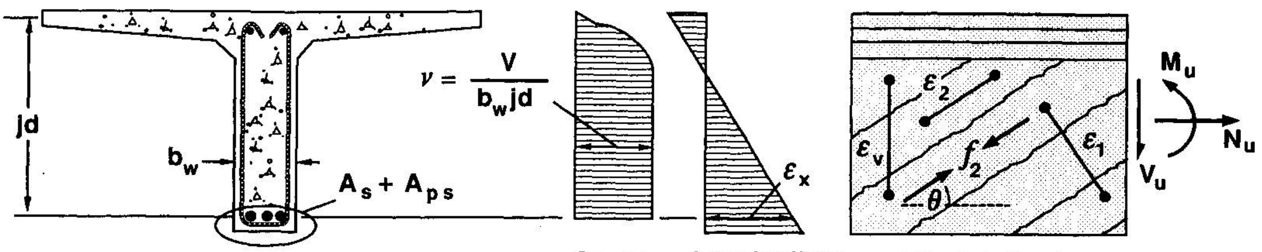

Fig. 1 Beam subjected to shear, moment, and axial load.

$$\epsilon_x = \frac{(M_u/jd) + 0.5N_u + 0.5V_u \cot\theta - A_{ps}f_{se}}{E_sA_s + E_pA_{ps}} \geq 0 \tag{1}$$

where A_s and A_{ps} are the areas of non-prestressed and prestressed longitudinal reinforcement on the flexural tension side of the member.

From strain compatibility, the principal tensile strain, ϵ_1, can be related to the longitudinal compressive stress and the magnitude of the principal compressive strain, ϵ_2, in the following manner:

$$\epsilon_1 = \epsilon_x + (\epsilon_x - \epsilon_2)\cot^2\theta \tag{2}$$

Hence as the longitudinal strain, ϵ_x, becomes larger and the inclination,θ, of the principal compressive stresses becomes smaller, the "damage indicator", ϵ_1, becomes larger.

For design purposes the shear strength, V_u, of a member can be expressed as

$$\begin{aligned} V_u &= V_c + V_s + V_p \\ &= \beta\sqrt{f'_c}b_w jd + \frac{A_v f_y}{s} jd\cot\theta + V_p \end{aligned} \tag{3}$$

where V_c = shear strength provided by residual tensile stresses in the cracked concrete
V_s = shear strength provided by tensile stresses in the stirrups
V_p = vertical component of force in the prestressing tendons.

The values of θ and β, determined by the modified compression field model are given in Table 1 for members with web reinforcement and in Table 2 for members without web reinforcement.

The tabulated values of the residual tensile stress factor, β, are based on the following expressions:

$$\beta = \frac{0.18}{0.3 + \frac{24w}{a+16}} \tag{4}$$

but

$$\beta \leq \frac{0.33\cot\theta}{1 + \sqrt{500\epsilon_1}} \tag{5}$$

Equation (4) is based on the shear stress that can be transmitted across diagonal cracks and hence is a function of the crack width, w, and the maximum aggregate size, a. The crack width is assumed

v/f'_c		Longitudinal Strain $\epsilon_x \times 1000$				
		0	0.5	1.0	1.5	2.0
≤ 0.05	β	0.437	0.251	0.194	0.163	0.144
	θ	28°	34°	38°	41°	43°
0.10	β	0.226	0.193	0.174	0.144	0.116
	θ	22°	30°	36°	38°	38°
0.15	β	0.211	0.189	0.144	0.109	0.087
	θ	25°	32°	34°	34°	34°
0.20	β	0.180	0.174	0.127	0.090	0.093
	θ	27°	33°	34°	34°	37°
0.25	β	0.189	0.156	0.121	0.114	0.110
	θ	30°	34°	36°	39°	42°

Table 1 Values of β and θ for members with web reinforcement.

z mm		Longitudinal Strain $\epsilon_x \times 1000$				
		0	0.5	1.0	1.5	2.0
125	β	0.406	0.263	0.214	0.183	0.161
	θ	27°	32°	34°	36°	38°
250	β	0.384	0.235	0.183	0.156	0.138
	θ	30°	37°	41°	43°	45°
500	β	0.359	0.201	0.153	0.127	0.108
	θ	34°	43°	48°	51°	54°
1000	β	0.335	0.163	0.118	0.095	0.080
	θ	37°	51°	56°	60°	63°
2000	β	0.306	0.126	0.084	0.064	0.052
	θ	41°	59°	66°	69°	72°

Table 2 Values of β and θ for members without web reinforcement.

to equal $\epsilon_1 s_{m\theta}$ where $s_{m\theta}$ is the average spacing of the diagonal cracks. Equation (5) is based on the average residual tensile stress in cracked concrete that has a cracking stress of $0.33\sqrt{f'_c}$. See Reference 5 for more details.

In determining the values in Tables 1 and 2 it was assumed that the crack spacing, $s_{m\theta}$, equalled about 300 mm for members containing web reinforcement while, for members without web reinforcement, the spacing of diagonal cracks was assumed to be $s_{mx}/\sin\theta$ where s_{mx} is given in Fig. 2.

To avoid yielding of the longitudinal reinforcement

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M_u}{jd} + 0.5N_u + (V_u - 0.5V_s - V_p)\cot\theta \tag{6}$$

4. INFLUENCE OF MEMBER SIZE

It has been shown [6] that the modified compression field theory can predict the shear capacity of members containing web reinforcement with reasonable accuracy (coefficients of variation about 10%). The influence of axial tension on the shear capacity of members not containing web reinforcement is also predicted accurately (COV 11%) [7].

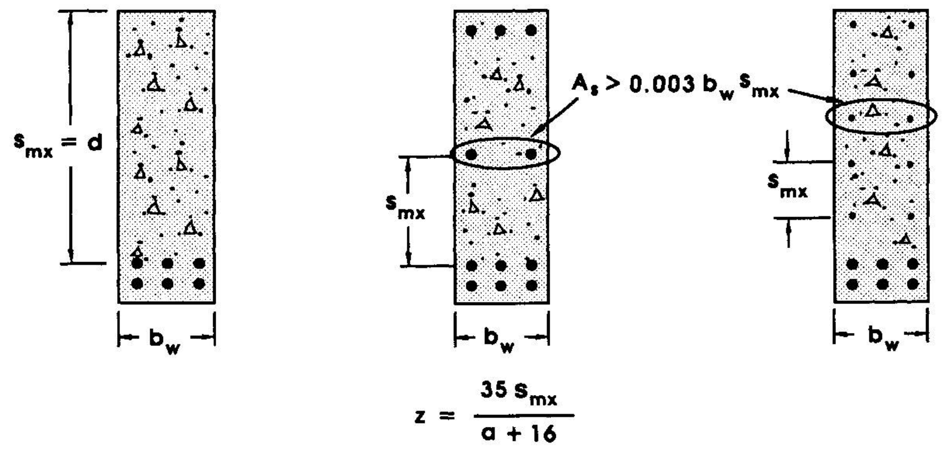

Fig. 2 Crack spacing parameter z.

For the purpose of this colloquium it is of particular interest to discuss the influence of member size upon the shear strength of members not containing web reinforcement. For members not containing crack control reinforcement (Fig. 2), as member size increases the crack spacing, $s_{m\theta}$, will increase and hence, for a given value of strain, ϵ_1, the crack width will increase. An increase in crack width reduces the shear stress that can be transmitted across the crack and hence reduces the shear strength of the member. It can be seen from Table 2 that members containing large amounts of longitudinal reinforcement or prestressed concrete members (i.e., members with low values of ϵ_x) will be less sensitive to member size than lightly reinforced members or members subjected to high moments (i.e., members with high values of ϵ_x). Thus if ϵ_x equals 0 the shear stress at failure increases by a factor of 1.33 as the size decreases by a factor of 16, while if ϵ_x equals 0.002 the shear stress increases by a factor of 3.10.

Figure 3 compares the observed shear stresses at failure for a series of lightly reinforced beams with depths ranging from 200 mm to 3000 mm [8]. Also shown are the shear stresses at failure predicted from the β values in Table 2. It can be seen that the theory predicts the strength of the larger beams very well, but is somewhat conservative for the smallest beam.

5. CONCLUDING REMARKS

The amount of stirrups required to resist a given shear, V_u, can be determined from

$$\frac{A_v f_y}{s} \cdot jd \geq \left(V_u - \beta\sqrt{f_c'}b_w jd - V_p\right) \tan\theta \qquad (7)$$

where both β and θ depend on the longitudinal strain parameter, ϵ_x, which accounts for the influence of moment, axial load, prestressing, and longitudinal reinforcement ratios. In addition, for members without web reinforcement, β and θ are strongly dependent on member size.

The sectional design model summarized above is appropriate for those regions of structures where it is reasonable to assume that plane sections remain plane. In regions where there are substantial static or geometric discontinuities, it is more appropriate to use strut-and-tie design models (see Reference 5).

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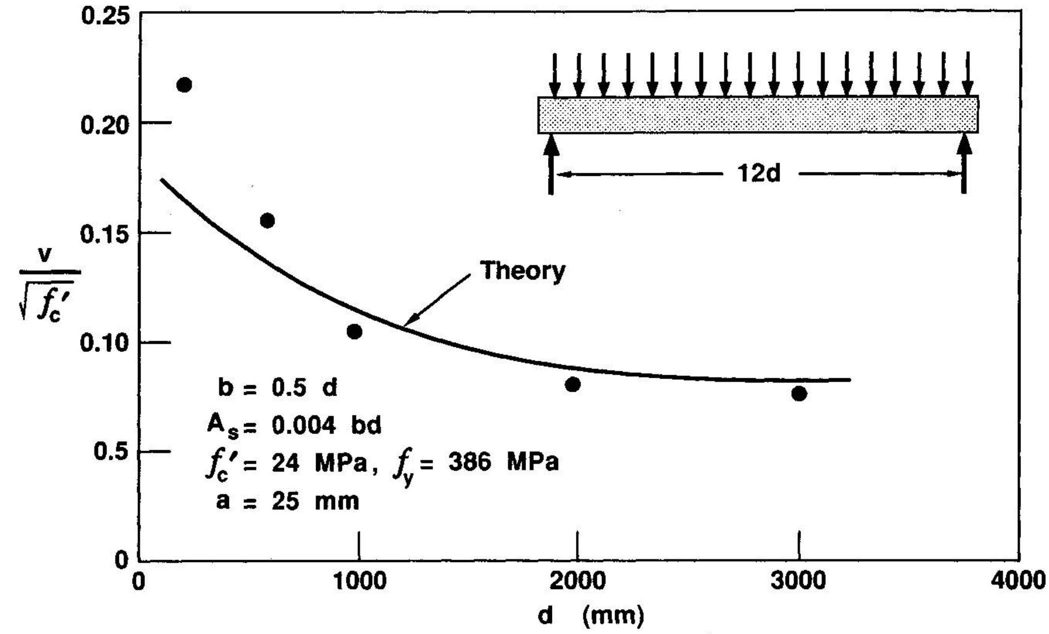

Fig. 3 Comparison of predicted and observed shear stress at failure on section distance d from the support.

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