Plate elements under combined actions

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Combined Loading Effects in Concrete Box Girders

Effets des efforts combinés dans des poutres-caisson en béton

Kombinierte Beanspruchung von Kastenträgern aus Beton

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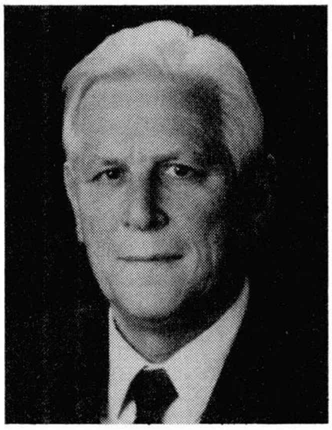

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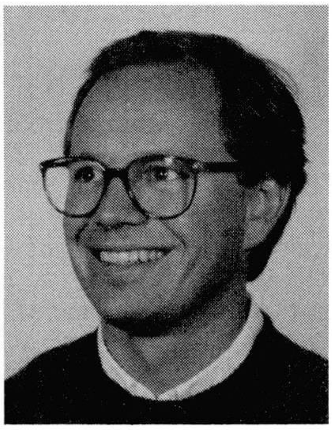

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SUMMARY
Concrete box girders are often subjected to substantial combined loading effects such as bending moments, normal forces, shear forces and torsional moments as, e.g. in the case of cable-stayed bridges suspended in the middle axis. Based on the «shear-wall-model» a consistent design method for the ultimate limit state is presented.

RÉSUMÉ
Les poutres-caisson en béton sont souvent substantiellement soumises aux actions combinées des moments de flexion, de l'efforts normal, des efforts tranchants et des moments de torsion, comme par exemple dans le cas de ponts haubanés soutenus par les câbles d'un support central. Une méthode cohérente de dimensionnement est présentée ici afin d'évaluer l'état de limite ultime.

ZUSAMMENFASSUNG
Kastenträger aus Beton unterliegen oft hohen kombinierten Beanspruchungen aus Biegung, Normalkraft, Querkraft und Torsion wie zum Beispiel bei Schrägkabelbrücken mit Mittelaufhängung. Auf der Grundlage des Schubwandmodells wird ein konsistentes Bemessungskonzept für den rechnerischen Bruchzustand vorgestellt.

1. INTRODUCTION

National codes as well as international codes dealing with concrete structures mostly cover the combined effects of actions by the use of empirical formulas or even neglect the interaction. A standardized consistent design model is missing. A more detailed consideration of the combined action-effects especially is recommended for slender box girders which are substantially subjected to combined effects of actions such as bending moments, normal forces, shear forces and torsional moments.
This paper presents a universal method for the design of concrete box girders in the ultimate limit state by the use of the "shear-wall-model" under consistent consideration of the load transfer by the webs and the chords. Combined action-effects such as slab moments and in-plane load are not treated in this paper.

2. DESIGN PROCEDURE

2.1 Design Principle

As long as bending moments acting in a horizontal plane are not considered, the action-effects such as bending and normal force as well as the longitudinal forces resulting from the diagonal compression field in the webs are allocated to the chords. The webs carry the shear forces. Torsional moments, however, affect webs and chords. Following this rule of distributing the action-effects, slender beams such as box girders can be divided into "shear-walls" being dimensioned for parts of the considered effects of actions. By this way inconsistences and overstresses are avoided. The distribution of the considered action-effects within the respective shear walls is given if a suitable inclination θ of the resulting compressive stress field in each web is derived according to the next paragraph. With regard to the following explanations it is required that the shear reinforcement is located rectangular to the axis of the beam. Furthermore all strengths, strains and action effects are supposed to be derived at "design level" according to reference [1] so that the subscript "d" is renounced within this paper. The action-effects thereby have to be derived taking account of prestressing forces.

2.2 Webs

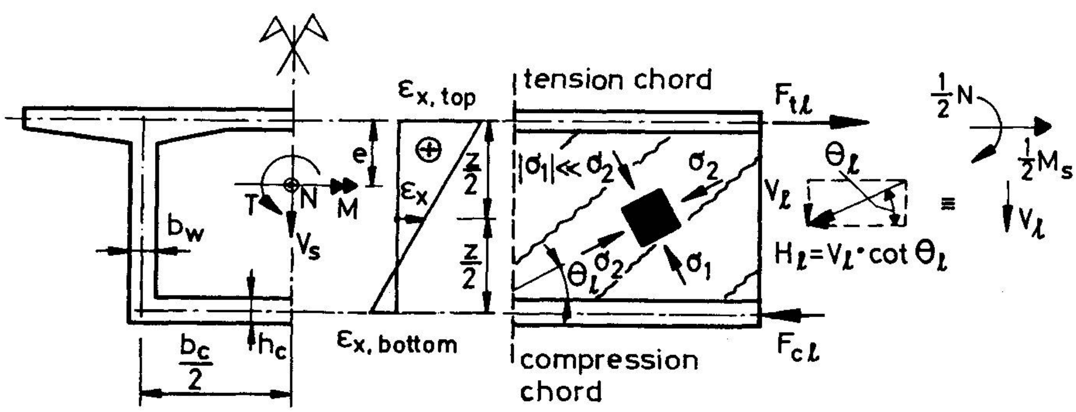

Fig. 1 Notations

Webs of continous beams mainly are subjected to shear forces V_s and the effects V_t of torsional moments T. For bearing this kind of action-effect the modified truss analogy turned out as a suitable model for calculating the magnitude of concrete stresses as well as the amount of the required shear reinforcement in the webs. In reference [2] a consistent model for calculating the resulting stress state in the webs has been developed. The model fulfills the compatibility of deformations in the web and considers the influence of the average axial strain ϵ_x of the chords upon the shear resistance of the webs. It turns out that a biaxial stress field prevails in the web concrete characterized by a predominating principal compressive stress σ_2 and a perpendicular acting subordinated principal compressive or tensile stress σ_1 (see figure 1). For practice design, however, it is confirmed that a uniaxial compressive field in the web concrete is a suitable approximation. Figure 1 illustrates the left part of a box girder (subscript "l") as well as the used notations.
Based on the above mentioned model the following applicable design formula can be derived for calculating the inclinations θ of the uniaxial compression field in the web on the left ($\rightarrow \theta_l$) and on the right hand side ($\rightarrow \theta_r$):

$$\tan\theta = 1 - [\Delta\tan\theta_{plast}] \cdot \frac{0.0015 - \epsilon_x}{0.003}$$

where $$\Delta\tan\theta_{plast} = 1 - \tan\theta_{plast} = 1 - \frac{1 - \sqrt{1-\nu^2}}{\nu}$$

so that the complete formula is:

$$\tan\theta = 1 - \left(1 - \frac{1-\sqrt{1-\nu^2}}{\nu}\right) \cdot \frac{0.0015 - \epsilon_x}{0.003} \qquad \begin{cases} \tan\theta \geq \frac{1}{3} \\ \tan\theta \geq \frac{1-\sqrt{1-\nu^2}}{\nu} \end{cases} \tag{1}$$

where $$\nu = \frac{2V}{b_w \cdot z \cdot f_{cd2}} \tag{2}$$

The vertical force V in equ. (2) acting in the web has to be determined for the web on the left hand side (V_l) as well as on the right hand side (V_r) according to the following equations:

$$V_l = \frac{V_s}{2} + \frac{T}{2b_c}; \qquad V_r = \frac{V_s}{2} - \frac{T}{2b_c} . \tag{3}$$

Thereby the average strain ϵ_x of the webs follows from the strains in the chords as:

$$\epsilon_x = \frac{\epsilon_{x,top} + \epsilon_{x,bottom}}{2} \tag{4}$$

where the chord strains (considered as positive in case of tensile strain) have to be derived taking account of the chord forces given by equ. (7) and (9) respectively. The required iterative calculation of ϵ_x and θ shows a quick convergency since the strain in the chords is influenced only slightly by the value of θ.
Thus the resulting amount of vertical shear reinforcement can be derived from:

$$\omega = \frac{2 \cdot A_{sw} \cdot f_y}{s \cdot b_w \cdot f_{cd2}} = \nu \cdot \tan\theta = \frac{2 \cdot V}{f_{cd2} \cdot b_w \cdot z} \cdot \tan\theta . \tag{5}$$

Figure 2 illustrates the resulting design curves according to the preceding equations.
The associated strut inclinations θ_l and θ_r for the web on the left as well as on the right hand side can be read from figure 2 taking account of the average axial strain in the chords according to equ. (4). Thus the resulting horizontal tensile force which has to be considered with regard to the chords in addition to the effects of M and N is:

$$\Sigma H = H_l + H_r = |V_l| \cdot \cot\theta_l + |V_r| \cdot \cot\theta_r . \tag{6}$$

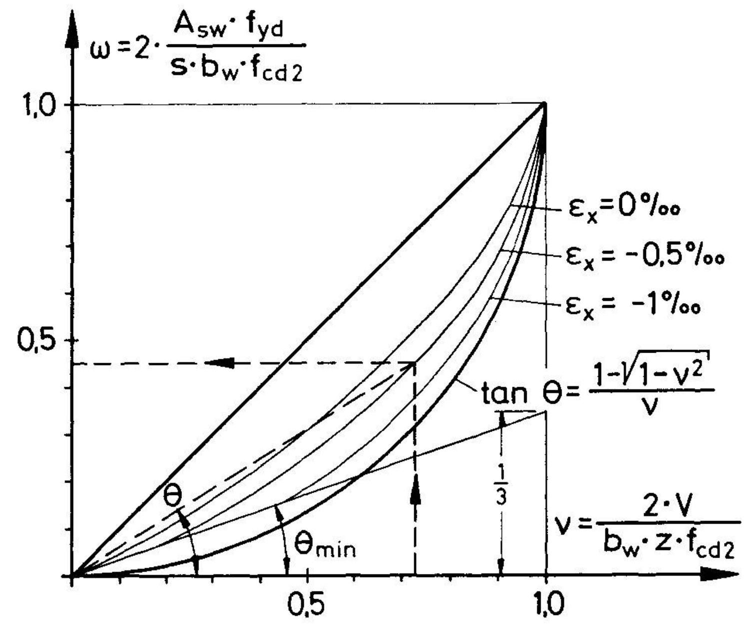

Fig. 2 Required vertical shear reinforcement in the web

2.3 Chords
2.3.1 Compression chords

The compression chords in concrete box girders are subjected to the longitudinal compressive force F_c:

$$F_c = \frac{M_s}{z} - \frac{\Sigma H}{2} \quad \text{whereby} \quad M_s = M + N \cdot e \tag{7}$$

Furthermore they are affected by a variable shear flow having a magnitude of V_l/z on the left and of V_r/z on the right hand side resulting from shear and torsion (see figure 3). With regard to the following evaluation the course of this shear flow is supposed to be approximately linear although strictly speaking a nonlinear course adjusts between the left and the right hand side due to the differential connection to the associated longitudinal stresses σ_x. In a considered element the compressive strength f_{cd1} (according to [1] applying to a uniaxial loaded concrete prism under sustained load) can not be taken as the admissible longitudinal stress σ_x if shear stresses are acting at the same time. Therefore the ultimate longitudinal stresses σ_x (see figure 3) are determined in this paper in terms of the acting shear stresses in the compression chord by the use of a suitable fracture criterion for a combined loaded concrete element as illustrated in figure 4.

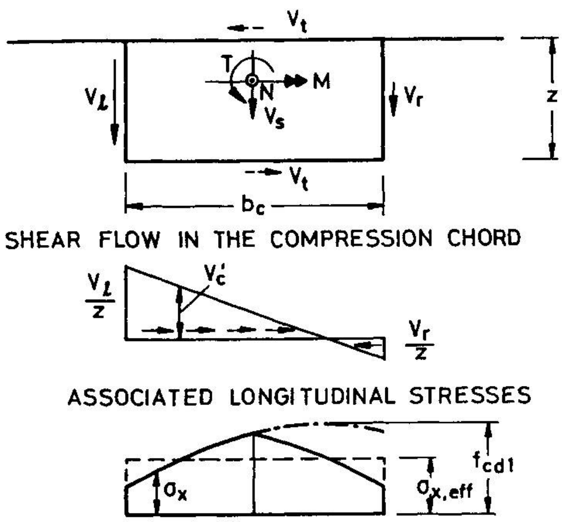

Fig. 3 Stress state in compression chords

In it the concrete compressive strength is supposed to be f_{cd1} in case of pure longitudinal compression of the uncracked concrete whereas it is reduced to 0.8 f_{cd1} in case of pure shear due to the diminishing influence of cracks. The almost elliptic course of the interaction diagram of figure 4 in the range of $0,4 \leq \sigma_x/f_{cd1} \leq 1,0$ (passed through line) is obtained by the use of reference [3] assuming a magnitude for $\sigma_y = \rho_y \cdot f_y$ (represented by the transverse reinforcement) which leads to concrete failure in the element.

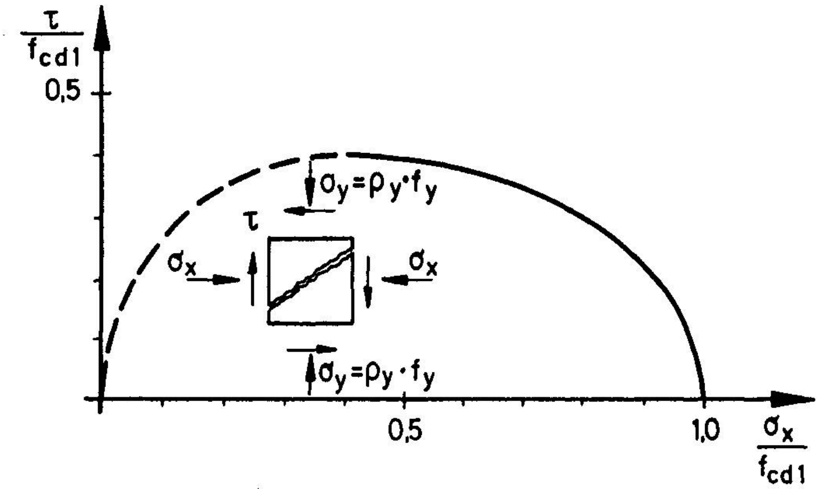

Fig. 4 Fracture criterion for a concrete element in concrete failure

As a simplification on the safe side the course of σ_x along the breadth of the compression chord is supposed to be symmetrically (More favourable values for σ_x could be obtained if the internal horizontal bending moment $(H_l - H_r) \cdot 0,5b_c$ is taken into account for both chords). Thus an average ultimate longitudinal stress $\sigma_{x,eff}$ (see figure 5) is derived applying for combined loaded compression chords by assuming an optimized distribution of load effects within the compression chord according to the theory of plasticity. As a consequence the compressive force F_c according to equ. (7) has to fulfill the following inequality:

$$F_c \leq \sigma_{x,eff} \cdot b_c \cdot h_c \qquad (8).$$

In regions of the compression chord where $V_c' > F_c/b_c$ (for V_c' see figure 3) a longitudinal reinforcement acc. to $\rho_x \cdot f_y \cdot h_c = V_c' - F_c/b_c$ is required assuming a strut inclination of 45° and a uniform stress distribution of σ_x. The required amount of transverse reinforcement in compression chords can be derived by the use of reference [1].

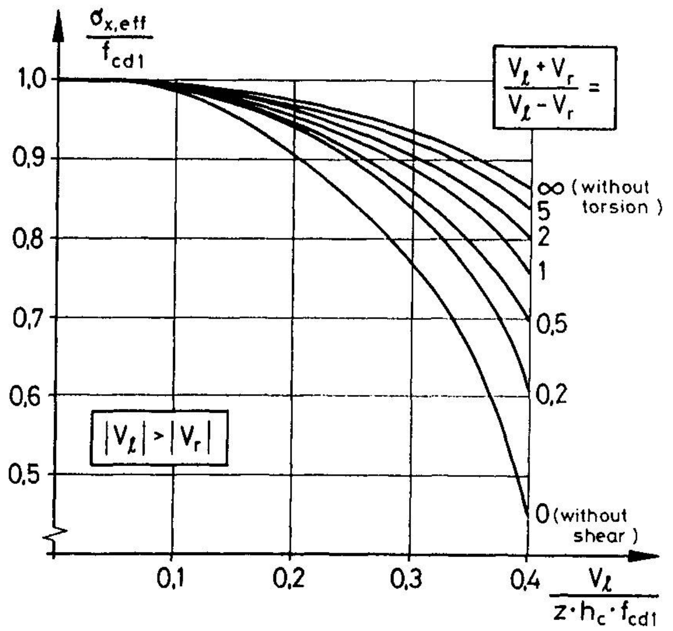

Fig. 5 Average ultimate compressive stress $\sigma_{x,eff}$ in terms of the acting shear forces in the webs

2.3.2 Tension chords

Tension chords in concrete box girders are subjected to the longitudinal tensile force F_t:

$$F_t = \frac{M_s}{z} + N + \frac{\Sigma H}{2} + \frac{T}{2z} \,. \tag{9}$$

The term $\Sigma H/2$ in equation (9) only covers the horizontal part of the diagonal forces in the webs without including the effects of the diagonal stress field in the tension chord itself. Therefore only those prestressing tendons or reinforcing bars (crossing the decisive section) can be taken into account which are anchored more far from the decisive section than from the nearest web. This is not required for reinforcing bars or prestressing tendons which are curtailed due to a varying magnitude of the torsional moment supposing that this curtailed reinforcement is steady distributed within the tension chord inbetween the webs.

3. SUMMARY

This paper presented a consistent model for the design of concrete box girders which are subjected to combined action-effects. It is based on the shear-wall-model under consistent consideration of the load transfer by the webs and the chords. A simple formula for the determination of the resulting strut inclination in the webs under taking into account the chord strains was given to the designer. The resulting distribution of the inner forces within the cross section was observed and the resulting design forces allocated to the shear walls were obtained. Furthermore an average limit value for the ultimate longitudinal compressive stress in combined loaded compression chords was derived in terms of the acting shear forces in the webs.

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Reinforced Concrete Plates under Biaxial Bending Moments

Dalles en béton armé soumises à des moments de flexion biaxiaux

Stahlbetonplatten unter zweiachsiger Biegung

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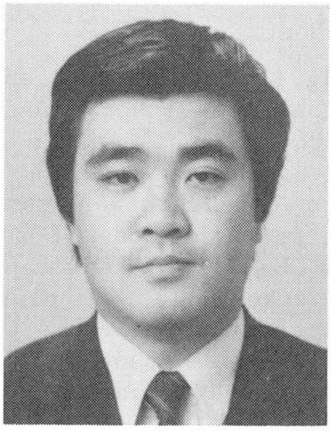

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SUMMARY

This paper proposes a method of analyzing the strength and deformation of reinforced concrete slabs for out-of-plane biaxial bending moments. In the analytical method, the tension zone and the compression zone were divided into two discrete layers, and compression field theory was employed in analyzing reinforced concrete slabs under in-plane forces. The analytical method was verified using a newly-developed apparatus for out-of-plane biaxial bending tests.

RÉSUMÉ

Cet article présente une méthode d'analyse de la résistance et de la déformation de dalles en béton armé soumises à des moments de flexion biaxiaux hors du plan. Dans la méthode analytique, la zone tendue de même que celle comprimée furent divisées en deux couches discrètes et la théorie du champ de compression fut appliquée dans l'analyse de dalles armées soumises à une force dans le plan. La méthode analytique développé fut vérifiée grâce aux tests effectués à l'aide d'un bâti spécialement développé sous effort de flexion biaxiale hors du plan.

ZUSAMMENFASSUNG

Es wird ein Berechnungsverfahren für das Trag- und Verformungsverhalten von Stahlbetonplatten unter zweiachsiger Biegung vorgeschlagen. Dabei werden die Zug- und die Druckzone als zweiachsig beanspruchte Scheiben nach der Druckfeldtheorie behandelt. Das Berechnungsverfahren wurde durch Versuche in einer neu entwickelten Prüfmaschine bestätigt.

1. INTRODUCTION

With the recent increase of reinforced concrete structures constructed underground, designers are being faced with many problems. Side walls of underground storage tanks for crude oil or LNG, for example, are subjected to out-of-plane biaxial bending or torsional moments. Design methods, however, for reinforced concrete plates (RC plates) under such moments yet remain to be established. In this paper, an analytical method for RC plates under out-of-plane biaxial bending moments is presented. This paper also describes newly developed apparatus for out-of-plane biaxial bending tests, by which the validity of the above analytical method is verified. This study assumes pure bending condition without the influence of out-of-plane shearing force and it considers the deviation of principal moment in the direction of reinforcing bars.

2. ANALYTICAL METHOD FOR OUT-OF-PLANE BIAXIAL BENDING MOMENTS

2.1 Analytical Method

As shown in Fig.1, this method analyzes RC plates, by applying compression field theory, under in-plane forces N_1 and N_2 assumed in the zone of tensile stress due to bending. The method calculates the strength and the deformation of RC plates taking into consideration compatibility conditions for deformation between the zone of tensile and compressive stresses due to bending. Given average strain ε_{ct} perpendicular to cracks in RC plates, the method can calculate the unit compressive force C_1' due to bending from Bernoulli-Eulers' assumption, the stress-strain relationship of concrete and the conditions for equilibrium shown in Fig.2 and equation (1) ~ (3). Since the unit tensile force N_1 due to bending equals C_1 when strain reaches ε_{ct}, the location x of the neutral axis can be obtained by convergence calculation. Using the location x of the neutral axis, bending moment and curvature can be calculated and thus deformational behavior can be traced. Fig. 3 shows a flowchart of the above analytical procedure. The ultimate strength is judged when one of the following conditions was met:

i. Strain at the compressive edge of concrete after the yielding of x-axis reinforcing bars (reinforcing bars at a smaller deviation angle with the maximum principal moment M_1, see Fig. 4) reaches 0.35%.

ii. Y-axis reinforcing bars (reinforcing bars at a greater deviation angle with the maximum principal moment M_1) yields after the yielding of x-axis reinforcing bars.

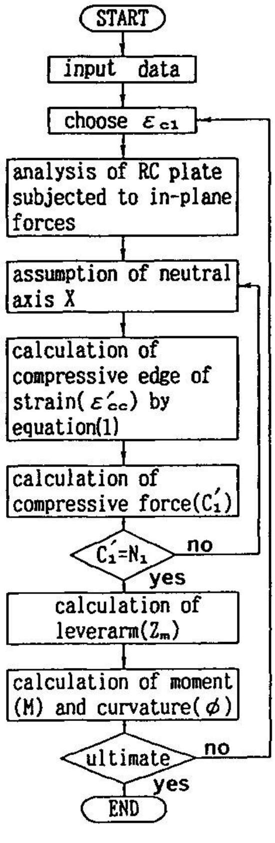

Fig.3 Flowchart for analytical method

$$\varepsilon_{cc}' = \frac{x}{d-x} \cdot \varepsilon_{ct} \quad (1) \qquad \sigma_c' = f_c' \,[2(\varepsilon_c'/\varepsilon_0) - (\varepsilon_c'/\varepsilon_0)^2] \quad (2) \qquad C_1' = \int \sigma_c' \cdot dx \quad (3)$$

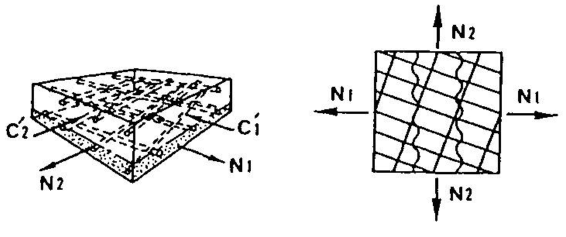

Fig.1 RC plate subjected to biaxial bending and RC plate subjected to in-plane forces assumed in tensile zone due to bending

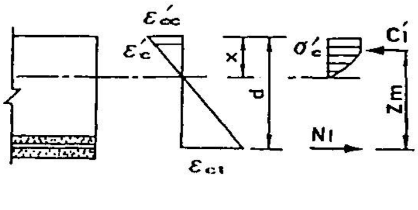

Fig.2 Assumption of strain profile and equilibrium of section

2.2 Analysis of RC Plates Under In-Plane Forces Based on the Compression Field Theory

2.2.1 Modeling of Cracks

A smeard crack model was employed, and RC plates with cracks were assumed to be continuous elements. Strain and stress dealt with in this paper, therefore, were assumed to be average strain and stress which were uniform within the element. It was assumed that the initial cracks occurred when tensile strain in the direction of the maximum principal stress reached 200μ. It was also assumed that even after the maximum principal stress reached the tensile strength, that stress was kept until tensile strain reached 200μ. Reinforcing bars in plates are usually arranged in two directions. It is likely, therefore, that reinforcing bars in the direction perpendicular to the principal moment which are virtually thought to be voids in concrete furthers the development of cracks. The cracking stress was hence determined according to equation (4) which takes this influence into consideration. Experiments conducted by the authors indicated that the direction of cracking, whether in one axis or two axes, remained throughout the cracking process. Consequently, it was assumed that the cracking direction was perpendicular to the maximum principal moment. Biaxial bending with a principal moment ratio of $Km=M_2/M_1=$ 0.5 or above causes cracking in two directions. To reflect this influence in the analysis, the authors decided to take account of the influence of bidirectional cracks in the evaluation of shear stiffness described later, while assuming that dominant cracks always occurred in a single direction.

$$f_t = 0.5\, f_c'^{\,2/3} \quad (\mathrm{kgf/cm^2}) \qquad (4)$$

2.2.2 Stress-Strain Relationship

Stress was calculated using the average stress-strain relationships shown below. In the analysis, Vecchio-Collins Model which considers softening was used as the average stress-strain relationship in a direction parallel to concrete cracks [1], while Okamura-Maekawa Model which considers tension stiffening was used as the average stress-strain relationship in a direction perpendicular to the cracks [2]. For shear stiffness of RC plates after cracking, Aoyagi-Yamada Model was used [3]. It is to be noted, however, that since the average shear stiffness was being considered, the shear stiffness of uncracked portions and the Aoyagi-Yamada Model were serially connected to determine the average shear stiffness (equation (5) with $\beta = 1$).

2.2.3 Influence of Bidirectional Cracks

The analytical method described above is intented for concrete with unidirectional cracks. Experiments showed that cracks occurred in two directions for specimens under biaxial bending with a principal moment ratio of $Km=M_2/M_1=0.5$ or above. A modeling method, however, for bidirectional open cracks yet remains to be established. The authors had presumed that bidirectional cracks reduced the bond between reinforcing bars and concrete, thus decreasing tension stiffening and shear stiffness. From the results of trail calculation for test results, the influence of bidirectional cracks was clearly reflected in the shear stiffness model. When the ratio of the principal moments in two directions was $Km = M_2/M_1 = 1.0$ (cracks fully developed in both directions), the ability of shear transfer seemed to disappear almost. Hence the authors introduced the reduction coefficient β depending on the principal moment ratio Km and gave it as $\beta = 1-Km \leq 1.0$. Then shear stiffness Ga with bidirectional cracks was evaluated using equation (5).

$$Ga = \beta \cdot \frac{Gc \cdot Gcr}{(Gc+Gcr)} \qquad (5)$$

Note here that Gc which represents the shear stiffness of the uncracked portions is given as $Ec/2(1+\nu)$; where Ec is the elastic modulus of concrete, and ν is Poisson's ratio of concrete. Gcr is the shear stiffness of the cracked portions and is given as $36/\varepsilon_{ct}$ (Aoyagi-Yamada Model).

2.2.4 Analytical Expression of RC Plates Under In-Plane Forces

This analytical method is based on the compression field theory of Vecchio-Collins. However, this method is characterized by clear distinction between the directions of the maximum principal strain (α) and the maximum principal moment (θ). If it is assumed that Mohr's circle applies to the maximum principal strain ε_{c1}, the minimum principal strain ε_{c2}, strains ε_x, ε_y and ε_{xy} in the directions of reinforcing bars, strain ε_{ct} perpendicular to cracks, strain ε_{cv} parallel to cracks, and shear strain γ_{ctv}, compatibility conditions in equations (6) and (7) should be satisfied, where θ is a deviation angle between the direction of the maximum principal moment and the x-axis reinforcing bars and α is an angle between the direction of the principal strain of RC plates and the x-axis reinforcing bars.

$$\begin{aligned} \varepsilon_x &= \varepsilon_{c1}\cos^2\alpha + \varepsilon_{c2}\sin^2\alpha \\ \varepsilon_y &= \varepsilon_{c1}\sin^2\alpha + \varepsilon_{c2}\cos^2\alpha \\ \gamma_{xy} &= (\varepsilon_{c1} - \varepsilon_{c2})\sin 2\alpha \end{aligned} \tag{6}$$

$$\begin{aligned} \varepsilon_{ct} &= \varepsilon_{c1}\cos^2(\alpha-\theta) + \varepsilon_{c2}\sin^2(\alpha-\theta) \\ \varepsilon_{cv} &= \varepsilon_{c1}\sin^2(\alpha-\theta) \quad \varepsilon_{c2}\cos^2(\alpha-\theta) \\ \gamma_{ctv} &= (\varepsilon_{c1} - \varepsilon_{c2})\sin 2(\alpha-\theta) \end{aligned} \tag{7}$$

Loads on RC plates are resisted by the stresses of reinforcing bars and concrete, and they can be added. If the principal strain ε_{c1} is given, stress in concrete in the x and y-axis directions can be calculated by equation (8) based on strain's compatibility conditions, the stress-strain relationship and the equilibrium of forces of the free body shown in Fig. 4 and Fig. 5. Equation (9) is obtained from the equilibrium of external and internal forces. If ε_{c1} is established and ε_{c2} and α are assumed, all internal forces can be calculated. The convergence calculation is carried out until the resultant n_x, n_y and n_{xy} equal the principal moment ratio km and the deviation angle θ of the principal moment.

$$\begin{aligned} \sigma_{cx} &= \sigma_{ct}\cos^2\theta + \sigma_{cv}\sin^2\theta - \tau_{ctv}\sin 2\theta \\ \sigma_{cy} &= \sigma_{ct}\sin^2\theta + \sigma_{cv}\cos^2\theta + \tau_{ctv}\sin 2\theta \\ \tau_{cxy} &= (\sigma_{ct} - \sigma_{cv})\sin\theta\cos\theta + \tau_{ctv}\cos 2\theta \end{aligned} \tag{8}$$

$$\begin{aligned} n_x &= p_x\sigma_{sx} + \sigma_{cx} \\ n_y &= p_y\sigma_{sy} + \sigma_{cy} \\ n_{xy} &= \tau_{cxy} \end{aligned} \tag{9}$$

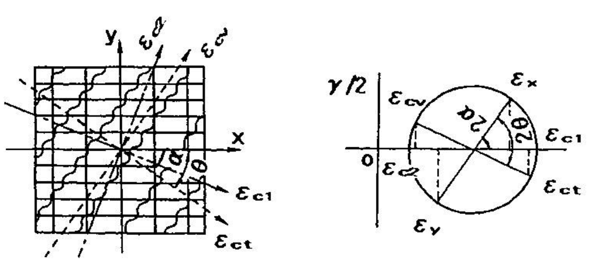

Fig.4 Average strains condition for RC plate element

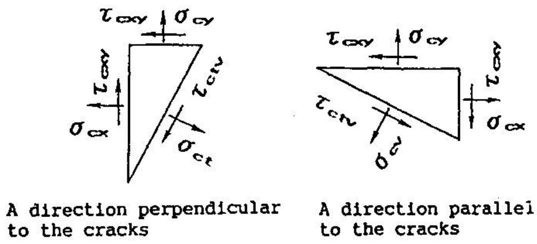

Fig.5 Average stresses acting on free-body in RC plate

3. COMPATISON OF TEST DATA AND CALCULATED VALUES

3.1 Biaxial Bending Test

An analytical model of this type requires verification using test data. However, reliable data on biaxial bending tests had not been available because of difficulty. Therefore an apparatus for biaxial bending tests of RC plates, was newly designed. It did not have the interaction of bending moments between each directions shown in Fig. 7. Using cruciform specimens shown in Fig.6, reliable test data on biaxial bending could be obtained. Table 1 summarizes the type of specimens, the compressive strength of concrete, cracking moment Mcr and the maximum moment M_{1u} .

3.2 Comparison of Test Data and Calculated Values

3.2.1 Comparison on Deformation

When the deviation on angle θ between the directions of principal moment and reinforcing bars under uniaxial bending moment increases, the RC plates show a marked tendency toward decreased flexural rigidity. Comparison of test data and calculated values for the moment-curvature relationship is shown in Fig.8. For RC plates under biaxial bending moments, comparisons on the moment-curvature relationship and the moment-reinforcing bars' average strain relationship are shown in Fig. 9 and Fig. 10, respectively. This analysis can trace deformation until the ultimate state of RC plates because it considers the compatibility conditions of deformation between the tensile and compressive stresses zone due to bending. In this analysis, the reduction coefficient β =(1-Km) is also introduced. As shown in figures, both curvature and average strain calculated accurately represent the actual behavior of deformation from the beginning of cracks through the ultimate state. The figure also shows calculated values which do not consider β. The influence of not considering β is clearly reflected in the results of the evaluation of the reinforcing bars' strain. This implies that if β is not taken into consideration, deformation could be underevaluated when the reinforcement ratio in the y-axis direction is lower than in the x-axis direction.

3.2.2 Comparison on Ultimate Strength

Ultimate strength was estimated on the basis of the definition of the ultimate state described above. Comparison of the estimated values and the test values was shown in Table 1. Note that this comparison included data on not only RC plates under biaxial bending moments, but also those under uniaxial

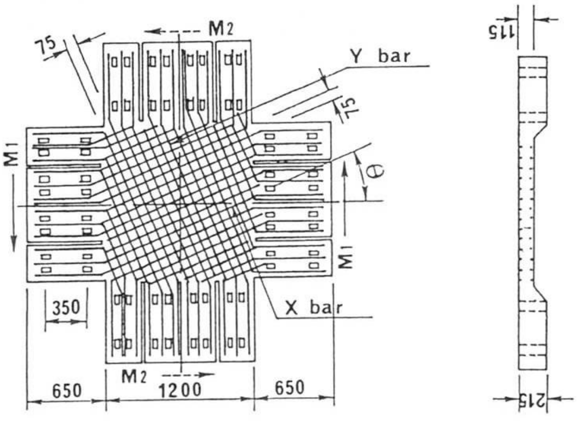

Fig.6 Shape and dimensions of typical specimen

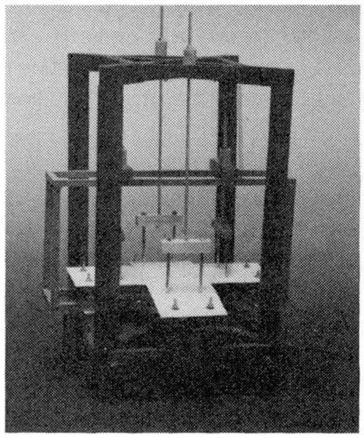

Fig.7 An apparatus for biaxial bending tests

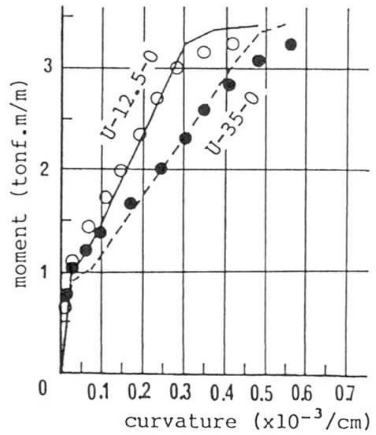

Fig.8 Comparison of test data and calculated values on moment -curvature relationship (uniaxial bending)

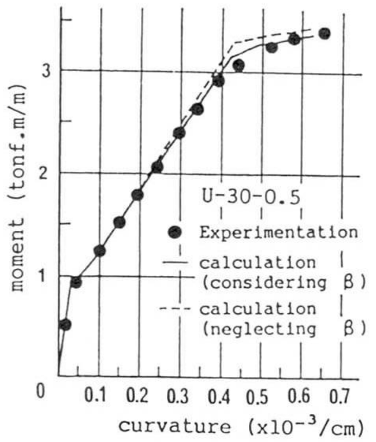

Fig.9 Comparison of test data and calculated values on moment -curvature relationship (biaxial bending)

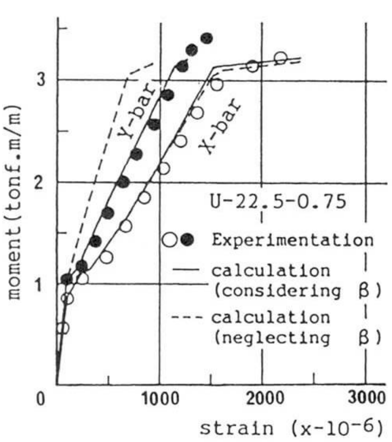

Fig.10 Comparison of test data and calculated values on moment -reinforcing bars' average strain relationship (biaxial bending)

bending moment. The analysis using ten data showed that the ratio of the test values to the calculated values averaged 1.03, giving the coefficient of variation of 4.8%. Although the average value was slightly high, the ultimate strength of RC plates under biaxial bending moment could be estimated with reasonable accuracy.

Specimen			Test Results			Calculation	M_{1utes} / M_{1ucal}
Mark	θ	K_m (M_2/M_1)	f'_c kgf/cm²	M_{cr} tonf·m	M_{1u} tonf·m	M_{1ucal} tonf·m	
U-0-0	0	0	261	0.98			
U-12.5-0	12.5	0	289	1.09	4.10	3.78 -	1.08
U-17.5-0	17.5	0	305	1.12	3.75	3.76	1.00
U-22.5-0	22.5	0	252	0.91	3.71	3.62	1.02
U-30-0	30	0	232	0.84	3.54	3.62	0.98
U-35-0	35	0	286	1.02	3.61	3.68	0.98
B-0-0.5	0	0.5	255	0.88	4.02	3.78	1.06
B-12.5-0.5	12.5	0.5	230	0.98	4.03	3.70	1.07
B-22.5-0.5	22.5	0.5	243	0.98	3.98	3.68	1.08
B-30-0.5	30	0.5	262	0.98	3.92	3.69	1.06
B-22.5-0.75	22.5	0.75	248	0.91	3.92	3.77	1.04

the averaged values = 1.03

coeffcient of variation = 4.8%

Table 1 Specimen properties, experimental and calculated value, and comparison of the both values

4. COMPARISON WITH OTHER APPROACHES

The analysis employed is a kind of sandwich method assuming a discrete layer between the compression zone and the tension zone. Some attempts have been made to analyze the behaviors of various members subjected to bending using sandwich models, and one of those was presented by Marti [4]. In his report, slabs were divided into an upper, a middle and lower layers; moment and axial force were resisted by the upper and the lower layers, while out-of-plane shearing force was resisted by the middle layer. In the model, the lever arm dv was assumed at 80% of thickness h. His proposed method of employing a truss mechanism in the middle layer is noteworthy. For the analysis of the upper and lower layers, compression field theory may be required. If the specimens used in this research were analyzed by the above methods, the ultimate strength obtained would show good agreement with test data. It is to be noted, however, that compatibility conditions in the upper and lower layers are not satisfied, those models will not be suited for the examination of serviceability limit state.

5. CONCLUSIONS

Here are the conclusions drawn from the study.

(1) Deformational behavior can be accurately traced by analyzing the lower layer of RC plates by compression field theory and by employing compatibility conditions of deformation between the upper and the lower layers.

(2) Accuracy of analysis can be enhanced by introducing the reduction coefficient β considering the influence of bidirectional cracks due to biaxial bending.

(3) Ultimate strength can be accurately represented by defining failure as a state with a strain of concrete's compressive edge of 0.35%, or as the yielding of reinforcing bars in y-axis direction.

The above results indicate that it is possible to unify the design process of RC plates under out-of-plane bending within the framework of the compression field theory as in the case of shear problems of RC plates under in-plane force.

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Computer Programme for Consistent Design of Surface Structures

Programme d'ordinateur pour dimensionnement de structures planes

Programm zur konsistenten Bemessung von Flächentragwerken

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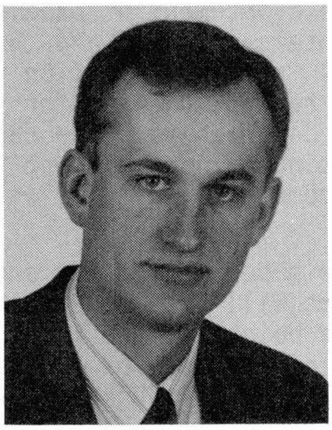

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SUMMARY

A computer program for the design of the reinforcement of surface structures has been developed. For given thickness, concrete strength, and location of the reinforcement layers, the required reinforcement areas are calculated for any desired combination of in-plane forces and bending moments. The program can be used in a stand-alone version, for example to determine the load carrying capacity of critical points in reinforced concrete panel, plate, or shell structures. It may also be used as a post-processor for finite element programs determining the required reinforcement at each integration point.

RÉSUMÉ

Un programme d'ordinateur qui calcule l'armature des structures planes a été développé. L'épaisseur lui étant donnée de même que la résistance du béton et l'emplacement des nappes d'armatures, il calcule les sections requises d'acier pour toute combinaison de forces dans le plan de la dalle, ainsi que de moments de flexion. Le programme peut aussi être utilisé par exemple pour déterminer la capacité à supporter une charge de points critiques de structures en béton armé de type panneau, dalle ou coque. Il peut aussi être employé comme post-processeur de programmes d'éléments finis capables de déterminer l'armature requise en chaque point d'intégration.

ZUSAMMENFASSUNG

Ein Computerprogramm für die Bemessung der Bewehrung in Flächentragwerken wurde entwickelt. Für vorgegebene Dicke, Betonfestigkeit und Lage der Bewehrungsscharen werden die erforderlichen Bewehrungsflächen für jede beliebige Kombination von Scheiben- und Plattenbeanspruchungen ermittelt. Das Programm kann zur Ermittlung der Tragfähigkeit von einzelnen kritischen Punkten eines Stahlbetonflächentragwerks benützt werden. Es kann aber auch als Nachlaufprogramm eines Finite Elemente Programms zur Bestimmung der Bewehrungsflächen in jedem Integrationspunkt verwendet werden.

1. INTRODUCTION

Many equations for the determination of the reinforcement in panels for deviating principal stress and reinforcement directions have been published. Baumann [1] has presented a thorough treatment of the subject and also an equation for the design of the reinforcement of panels. The subject of dimensioning the reinforcement in panels subjected to general in-plane loading and in particular Baumann's equation are also discussed in Marti's report [2].

The equations for the reinforcement of panels can be applied to plate elements if the lever arm of the internal forces and the thickness of the load carrying covers is known, as is shown in section 4.4 of [2]. This approach leads to satisfactory results if a realistic guess for the internal lever arm and the depth of the concrete compressive zone can be made. If moments and in-plane compressive forces are acting on an element a realistic estimation of the internal lever arm may become difficult, as will be shown for an example below.

In the remainder of the paper a computer program for the design of reinforced concrete surface structures will be described, which is based on rational mechanic assumptions. The height of the internal lever arm and the depth of the concrete compressive zone need not to be guessed but are the result of the design process for an element subjected to arbitrary in plane forces and moments. Although the application of the computer program requires considerably more computations than programs based on Baumann's equation, this is no serious drawback considering the computing power already available in the design offices today.

2. ALGORITHM FOR THE DESIGN OF SHELL ELEMENTS

2.1 Layered shell element

In accordance with [2] the term shell element is used for an element which may be subjected to in-plane forces and moments. A shell element with unit dimensions in the x- and y-directions and the thickness t is shown in Fig. 1. The element is divided into concrete layers and reinforcement layers. The strain and stress state within each layer is uniform. Assuming a linear variation of the strains through the thickness the strain state in each layer can be calculated for known strains and curvatures of the middle surface.

2.2 Concrete layer

Under loading three states - uncracked, cracks in one direction, cracks in two directions - are possible in each concrete layer. A layer with cracks in one direction is shown in Fig. 2, where axes 1 and 2 denote the principal tensile and principal compressive strain directions, respectively.

The uniaxial stress-strain diagram of concrete is also shown in Fig. 2. The parabola-rectangle diagram of the German code [3] has been substituted by a fourth order parabola for numeric reasons. Tensile strength of concrete is set to zero and tension stiffening is also neglected since the program is to be used as a design tool. In accordance with the code [3] an increase of the concrete strength under biaxial compression will not be considered. Concrete under biaxial compression is analyzed with an orthotropic material model using the stress-strain diagram of Fig. 2 in each principal strain direction and a shear modulus based on the tangent moduli in the principal strain directions.

Initially the cracks in a layer will form in a direction normal to the principal stress in concrete as soon as this stress becomes tensile. Upon increased loading the principal strain direction is adjusted according to the principle of the minimum of the internal energy. The crack direction and the direction of the concrete struts in a layer (Fig. 2) will always remain orthogonal to the principal tensile strain direction. Biaxial tension in a concrete layer will lead to the formation of two orthogonal sets of cracks and a complete loss of stiffness of this layer. The programing of this material model with re-orientation of the principal tensile strain direction, including the tension stiffening effect and an effective concrete strength depending on the transverse stress state has been described in [4].

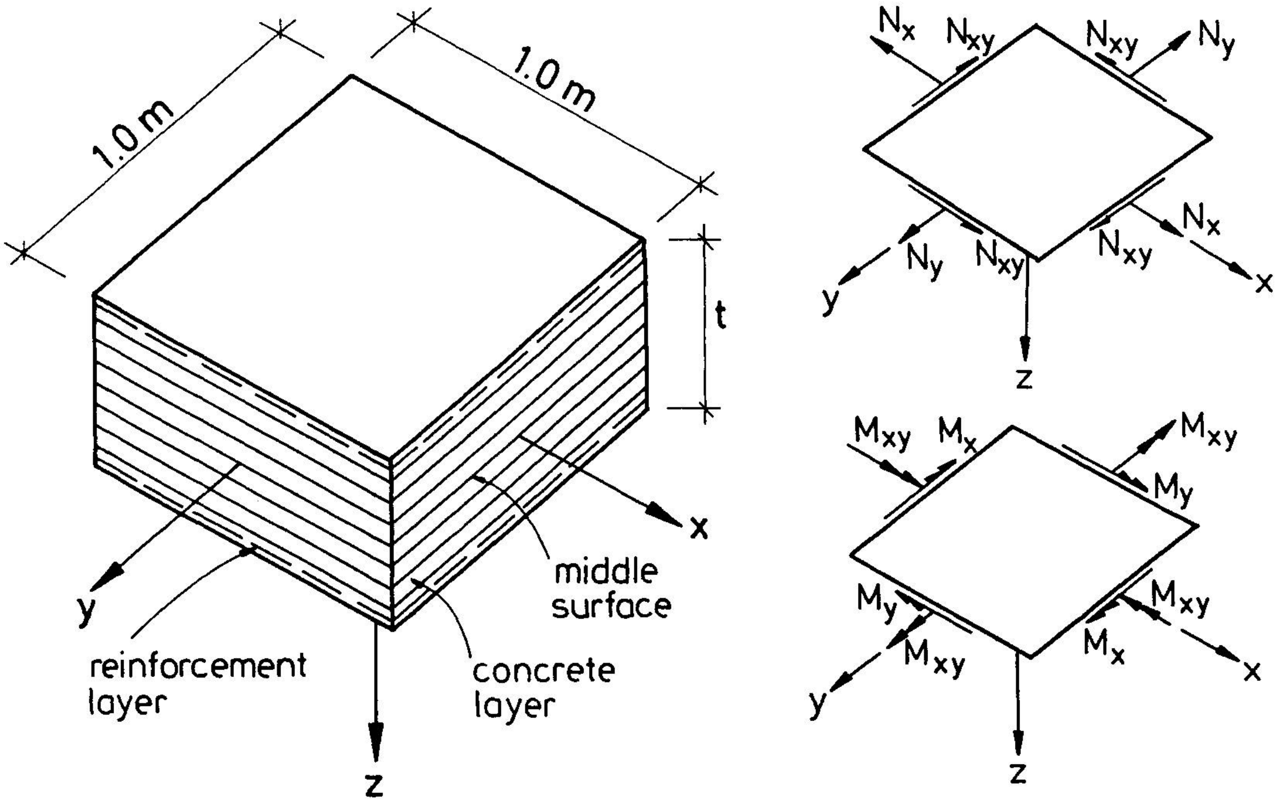

Fig. 1 Layered shell element (left), applied forces and moments (right)

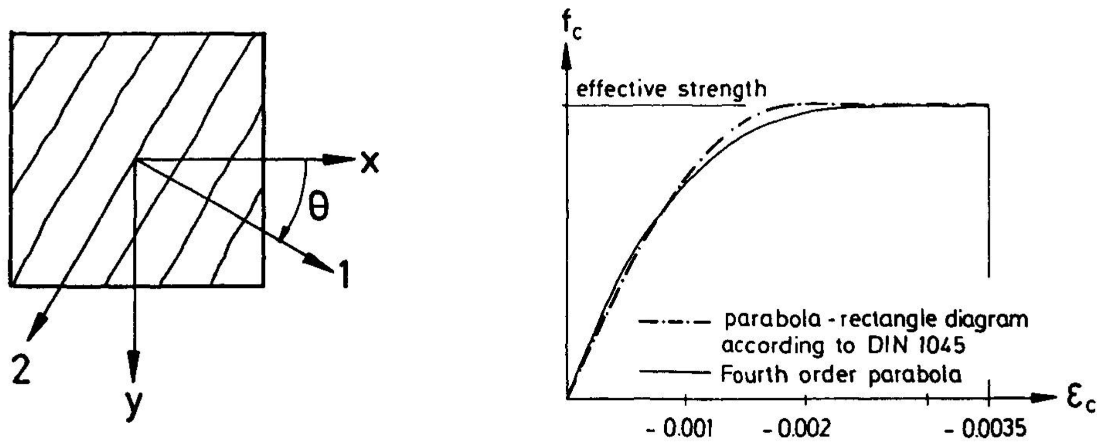

Fig. 2 Concrete layer with cracks in one direction (left) and uniaxial stress-stress strain diagram of concrete (right)

2.3 Reinforcement layer

A reinforcement layer is described by the area of reinforcement, the angle between x-axis and the reinforcing direction, and the distance of the reinforcement layer to the middle surface of the shell element (Fig. 3). An elastic-plastic stress-strain relationship with a hardening modulus is used for tensile and compressive strains as is also shown in Fig. 3. The hardening modulus is set to 1% of the elasitic modulus for numeric reasons. A strain cut-off is assumed for strains larger than 0.005 and smaller than -0.0035 in order to remain compatible with the code [3].

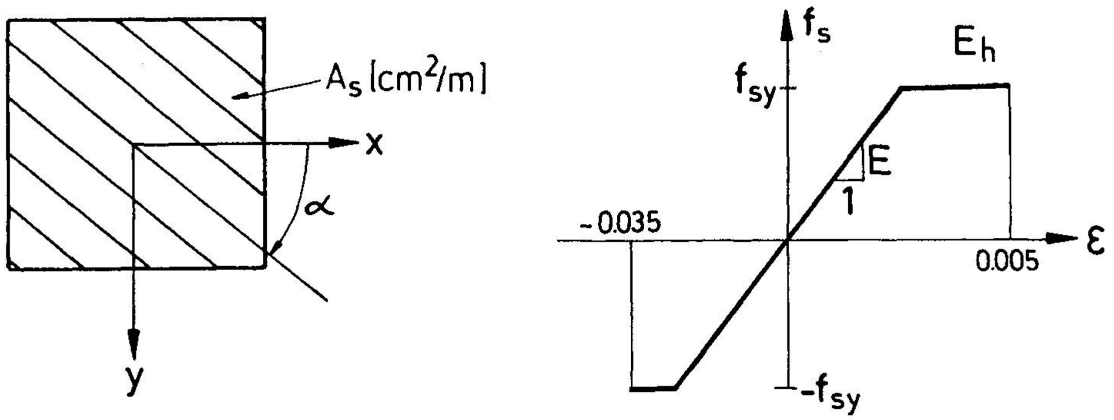

Fig. 3 Reinforcement layer and material model for the reinforcement

2.4 Numerical algorithm to determine the reinforcement of a shell element

The strains in the middle surface and the curvatures of the layered shell element are calculated for any set of applied external forces and moments in an iterative procedure. Starting with linear elastic material properties in the first iteration step, strains and curvatures are calculated. The strains in the individual layers are then determined, and using the material models for the concrete and the reinforcement described above the internal stresses of each layer are calculated. Integrating the internal stresses over the thickness of the element yields the internal forces and moments. Unbalanced forces and moments are calculated as the difference of the externally applied and the internal forces and moments of the element. The unbalanced forces and moments are applied to the layered shell element in the next iteration step. This procedure is repeated until the unbalanced member forces and the differences of the current strains and curvatures with respect to the ones of the last iteration step are smaller than required convergence limits. Then the external member forces of the next load step are applied to the layered shell element and the procedure described above is repeated starting with the element stiffness of the previous load step.

In order to automatically determine the ultimate load of a shell element subjected to proportionally increasing external member forces the following method is used: In the first step the external member forces corresponding to a load multiplier of 1.0 are applied to the shell element. Then the load multiplier is increased in a stepwise manner by 1.0 until no solution can be calculated, i.e. the ultimate load has been overestimated. In the following steps the intervalls are always divided into halves, until the difference between two consecutive load-multipliers remains under a predefined limit. For design examples a value of 0.01 is sufficient for this difference.

In using the layered shell element for the determination of the required reinforcement it is assumed that for each reinforcement layer the reinforcing direction, the distance to the middle surface, and the minimum reinforcement are known. In the first step of the automated design process the load-multiplier for a given set of external forces and moments is calculated with the minimum reinforcement. Then the reinforcement layer with the largest strain is strengthened by adding a reinforcement increment and the load-multiplier is calculated again and compared with the required safety coefficient. The incremental increase of the reinforcement areas of the individual layers and the incremental, iterative determination of the load-multipliers is continued until the required safety coefficient, e.g. 1.75 according to [3], is reached.

The incremental iterative procedure outlined above has been programmed in FORTRAN and implemented on a workstation. The validity of the program has been checked by analyzing numerous experimental tests on panel, plate and shell elements. Comparisons with design examples based on Baumann's theory showed that the same reinforcement areas as determined by Baumann's equation were obtained for panels with in-plane loading and for plates without large reinforcement ratios in the compression zone.

3. ELEMENT SUBJECTED TO AXIAL FORCE AND TWISTING MOMENT

A test program on reinforced concrete plates subjected to torsion was carried out by Marti et al. [5]. Here only plate ML5 will be considered which had a thickness of 20 cm and two reinforcement layers with 20 cm²/m in in the x-direction and two layers with 5 cm²/m in the y-direction. In the analysis an effective concrete strength of 25 MPa was used and the strain cut-off acc. to Fig. 3 was disabled. The envelope of the ultimate loads for different combinations of axial force and twisting moment is shown in Fig. 4. The calculated principal strains and the inclination of principal tensile strain direction are indicated for pure axial force, pure twisting moment, and a combination with Mxy=0.05Nx. While the ultimate load for pure axial force can easily be found by a hand calculation and the core model of [2] will predict the ultimate load for pure twisting moment, the intermediate points of the envelope are most easily determined by a nonlinear analyses with the layered shell element.

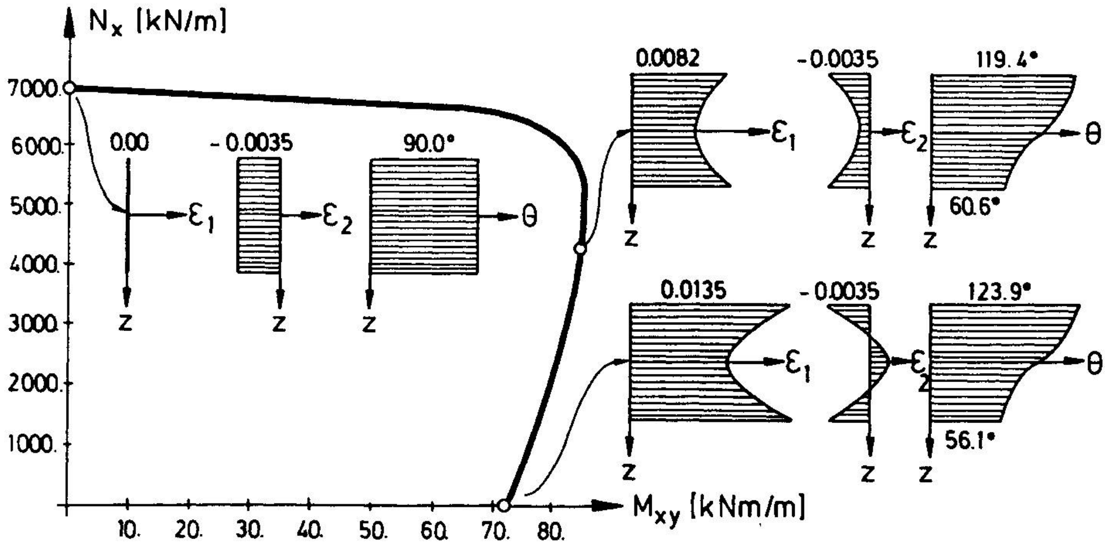

Fig. 4 Calculated ultimate load envelope and selected strain states of plate ML5 [5] subjected to axial force and twisting moment

4. ELEMENT SUBJECTED TO SHEAR FORCE AND BENDING MOMENT

The shell element SE7 was tested at the University of Toronto under a combination of shear force and bending moment [6]. In the following the reinforcement of the shell element will be determined for Nxy=1000 kN/m and Mx=113 kNm/m and a required load-multiplier of 1.75. The effective strength of the 28.5 cm thick specimen is taken as 70% of the cylinder crushing strength which was equal to 41.8 MPa. The yield strength of the reinforcement was 492 MPa and the four reinforcement layers had distances of -12.2 cm, -10.0 cm, 12.2 cm, and 10.0 cm, respectively. The reinforcement areas of the test specimen and the calculated strains at failure are shown in Tab. 1. The failure of the test specimen according to the analysis occured at a load-multiplier of 1.754, whereas a load-multilier of 1.81 is reported from the experiment [6].

For the design task the above mentioned properties remained unchanged. Only the reinforcement areas of the four layers were reduced to a minimum reinforcement of 5 cm^2/m. The load-multipliers, reinforcement areas, and calculated strains are shown in Tab. 1 for the first step with minimum reinforcement, intermediate steps with stepwise increased reinforcement, and the final step with a load multiplier larger than 1.75. The determination of the required reinforcement areas took only a few seconds on a workstation.

load-	layer 1		layer 2		layer 3		layer 4	
multiplier	area cm^2/m	strain	area cm^2/m	strain	area cm^2/m	strain	area cm^2/m	strain
analysis of ultimate load with reinforcement areas of specimen SE7 [6]								
1.754	41.8	0.0003	13.9	0.0021	41.8	0.0033	13.9	0.0049
incremental, iterative determination of reinforcement areas								
0.283	5.0	0.0004	5.0	0.0011	5.0	0.0049	5.0	0.0020
0.508	5.0	0.0007	5.0	0.0018	11.0	0.0043	5.0	0.0023
1.095	5.0	0.0009	9.0	0.0023	23.0	0.0050	11.0	0.0044
1.539	5.0	0.0015	12.0	0.0038	33.0	0.0046	15.0	0.0043
1.752	5.0	0.0015	14.1	0.0035	37.6	0.0048	16.9	0.0047

Table 1 Analysis of specimen SE7 [6] with reinforcement areas of the test and incremental, iterative determination of reinforcement areas

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A Consistent Shear Design Model for Concrete Offshore Structures

Modèle de dimensionnement à l'effort tranchant des structures en mer

Schubbemessungsverfahren für Beton-Offshore Strukturen

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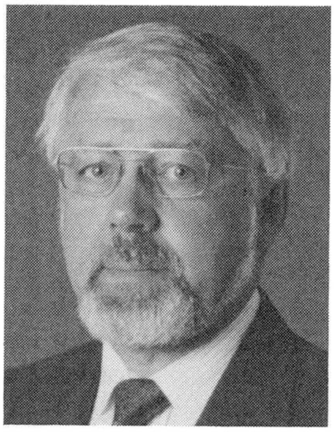

SUMMARY

A strain compatibility procedure for the sectional design of complex concrete structures is presented. The procedure, which is a generalization of the modified compression field theory for membrane elements, is capable of predicting the response of elements subjected to combined membrane forces, bending moments and transverse shear.

RÉSUMÉ

On présente ici une procédure basée sur la compatibilité des contraintes lors du dimensionnement de structures complexes tenant compte des efforts intérieurs. Cette méthode, généralisation de la théorie modifiée du champ de compression applicable aux éléments minces, est capable de prévoir la réponse d'éléments sollicités par des forces de membranes, des moments de flexion et des forces de cisaillement transversales.

ZUSAMMENFASSUNG

Diese Veröffentlichung beschreibt ein Kompatibilitätsverfahren für die Querschnittsberechnung von komplexen Betonstrukturen. Die Methode stellt eine Verallgemeinerung der modifizierten Druckfeld-Theorie für Membranelemente dar. Sie ermöglicht eine Berechnung des Elementverhaltens unter einer kombinierten Beanspruchung durch Membrankräfte, Biegemomente und Schub.

1. Introduction

Traditional sectional design procedures were developed for simple concrete structures such as buildings. The sectional forces (axial load, bending moment, and shear force) at various locations in a building frame are typically determined using a linear elastic analysis. In checking the ability of a particular section to resist the calculated stress resultants, the non-linear behaviour of concrete is taken into account. The response to axial load and bending moment is based on a strain compatibility approach, while shear design has traditionally involved empirical rules.

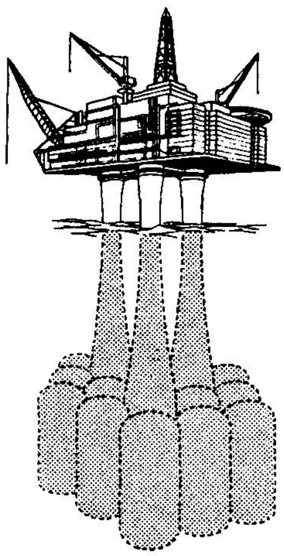

Fig. 1 Concrete Offshore Structure

The design of a more complex structure such as a concrete offshore structure (see Fig. 1) also involves determining the sectional forces at critical locations in the structure. Once again, linear elastic analysis is usually employed. However, the sectional forces for such a structure is considerably more complex. The loading demand at a particular location is expressed in terms of eight stress resultants, three membrane forces, N_x, N_y, N_{xy}, three bending moments, M_x, M_y, M_{xy}, and two trans verse shear forces, V_x, V_y, (see Fig. 2). The method currently used to design offshore structures for the three membrane forces and the three bending moments is a generalization of the strain compatibility approach used for beams, while for transverse shear the empirical beam shear design rules are used. This paper describes a procedure in which membrane forces, bending moments and transverse shear can be considered in a consistent strain compatibility approach.

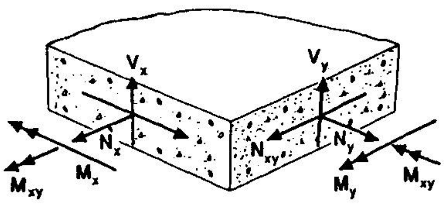

Fig. 2 Sectional Forces

2. Membrane Forces

The simplest "shear problem" is to predict the response of a reinforced concrete element subjected to only membrane forces, N_x, N_y, and N_{xy}. The problem involves relating the uniform biaxial strains ϵ_x, ϵ_y, γ_{xy} to the uniform biaxial stresses n_x, n_y, n_{xy}. A procedure for predicting the response of membrane elements was presented by Vecchio and Collins [1]. In this procedure, called the modified compression field theory, cracked concrete is treated as a new material with its own stress-strain characteristics. Rather than dealing with the variable local stresses (e.g., higher reinforcement stresses at a crack, lower away from the crack) the theory is formulated in terms of average stresses and average strains. In addition, the ability of the section to transmit the required forces across the cracks is specifically checked.

The biaxial stress-strain characteristics of cracked concrete were empirically determined from tests [1]. These tests showed that the principal compressive stress, f_{c2}, in cracked concrete is a function of not only the principal compressive strain, ϵ_2, but also of the co-existing principal tensile strain, ϵ_1. In addition, the tests showed that even severely cracked concrete stiffens the response of reinforcement. In the modified compression field theory this phenomenon is accounted for by assigning an average tensile stress to the concrete. After cracking, the average tensile stress in the concrete reduces as the strains increase (see Fig. 3).

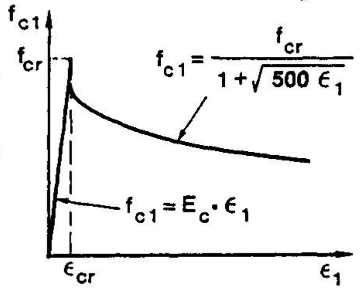

Fig. 3 Average Stress-Strain Relationship for Concrete in Tension

3. Membrane Forces and Bending Moments

A more complex problem exists when there are three bending moments, M_x, M_y, M_{xy}, in addition to the three membrane forces, because the stresses are now not uniform over the thickness of the element. The problem can be solved using a strain compatibility procedure which is a generalization of the plane sections theory. The membrane strains, ϵ_x, ϵ_y, γ_{xy}, are assumed to vary linearly over the thickness of the element, and therefore can be described by six variables. Given the six strain variables, the stresses in the concrete and the reinforcement can be determined from the biaxial stress-strain relationships. Integrating the stresses over the thickness of the element gives the six stress resultants. The concrete stresses are integrated numerically by dividing the thickness of the element into a number of membrane elements.

Finding the stress resultants which are associated with a given set of strains is a direct procedure. However, if the six stress resultants are given and it is desired to find the six strain variables, trial and error is required. Program SEP [2] was developed based on this approach. It incorporates the procedures of the modified compression field theory as biaxial stress-strain relationships.

4. Equivalent Beam Approach for Transverse Shear

Post-processing design programs based on strain compatibility procedures have been used to check sections of concrete offshore structures for the case of combined membrane forces and bending moments. However, to account for the influence of transverse shear, the empirical beam shear design rules are applied by using the concept of an "equivalent beam."

Consider the element shown in Fig. 4. An equivalent beam strip of unit width taken horizontally from this element would be subjected to the principal transverse shear, but no axial tension. However, the "beam" would be subjected to tension acting across its width. A beam strip taken vertically would be subjected to the highest axial tension, but would not be subjected to transverse shear along its length. The beam strip would be subjected to transverse shear on its "side faces." In neither case is the in-plane reinforcement parallel to the axis of the strip.

One procedure currently used to apply the beam shear design rules is as follows. For each beam strip any actions on the side faces are neglected, as is the influence of the orientation of the in-plane reinforcement. The transverse shear per unit width of beam strip is taken as

$$V = V_x \cos\alpha + V_y \sin\alpha$$

and the "axial force" per unit width of beam strip is taken as

$$N = N_x \cos^2\alpha + N_y \sin^2\alpha + 2N_{xy} \sin\alpha \cos\alpha$$

The required amount of stirrup reinforcement expressed in terms of stirrup area per unit area of concrete is determined for every possible beam strip direction using the beam shear equations. The largest amount of stirrup steel is taken as the amount which is needed.

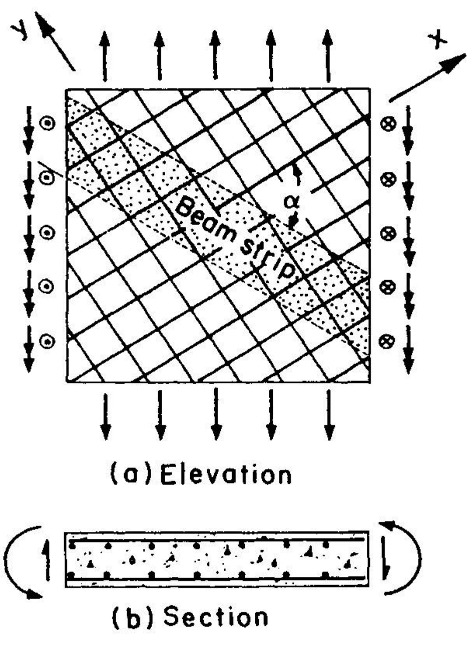

Fig. 4 Equivalent Beam Approach

5. Membrane Forces, Bending Moments and Transverse Shear

The case of combined membrane forces, bending moments and transverse shear is complicated by the need to deal with triaxial strains and triaxial stresses. A simple practical solution to the problem is possible by considering, the variation of biaxial strains over the thickness of the element, but triaxial strains at only the mid-plane of the element. The influence of the triaxial strains on the biaxial stresses is assumed to be uniform over the thickness of the element. That is, the transverse strains are assumed to influence only the membrane forces, not the bending moments.

The procedure is done as follows. From the applied bending moments, M_x, M_y, M_{xy}, and the applied membrane forces plus a "first guess" correction to account for the influence of the triaxial strains, $N_x+\Delta N_x^I$, $N_y+\Delta N_y^I$, $N_{xy}+\Delta N_{xy}^I$, the in-plane strains of the middle surface are calculated using the procedure described in the previous section. The in-plane concrete stresses f_{cx}, f_{cy}, v_{cxy} at the middle surface of the shell are first calculated from these biaxial strains neglecting any transverse strains, and then are recalculated considering the influence of the transverse strains (ie. considering triaxial strains). The differences between these two sets of stresses (ie. the additional concrete stresses due to transverse shear) are assumed to be uniform over the effective shear depth. The resultants of these additional concrete stresses, ΔN_x^{II}, ΔN_y^{II}, ΔN_{xy}^{II}, must equilibrate the membrane force corrections. For example, $N_x+\Delta N_x^I+\Delta N_x^{II} = N_x$ which means that $\Delta N_x^I+\Delta N_x^{II} = 0$. Several iterations are generally needed to determine the three membrane force corrections to account for transverse shear.

The triaxial stresses at the mid-depth of the shell are calculated by a trial and error procedure which adjusts the three additional strains ϵ_z, γ_{xz} and γ_{yz} until the required values of the transverse shear stresses v_{cxz} and v_{cyz} are obtained and the resultant normal stress on the z plane is zero (ie. the concrete compressive stress in the transverse direction equilibrates the tensile stress in the transverse reinforcement). The six concrete stresses, f_{cx}, f_{cy}, f_{cz}, v_{cxy}, v_{cxz}, and v_{cyz}, are determined from the six concrete strains, ϵ_x, ϵ_y, ϵ_z, γ_{xy}, γ_{xz} and γ_{yz}, using a three dimensional generalization of the modified compression field theory.

The procedure involves first finding the principal strains, ϵ_1, ϵ_2, ϵ_3, and their directions. The principal concrete stresses, f_{c1}, f_{c2} and f_{c3}, are then found from triaxial concrete stress-strain relationships which are generalizations of the Vecchio and Collins biaxial relationships. In addition, a "crack check" is made to ensure that the loads resisted by the average triaxial stresses can be transmitted across the cracks. When significant transverse shear is present this check is often critical in determining the failure load.

6. Transmitting Forces Across Cracks

The forces resisted by the average stresses are transferred across cracks by a combination of increased reinforcement stresses at the crack and shear stresses on the crack interface. The magnitude of the shear stress depends on the relative increases in reinforcement stress in the various directions as well as the direction of the crack.

The ability of a crack to resist shear by aggregate interlock depends primarily on the crack width. It has been suggested [1] that if there are no compressive stresses on the crack interface the limiting value of the crack interface shear stress, v_{ci}, (in MPa) be taken as

$$v_{ci} \leq \frac{0.18}{0.3+\dfrac{24w}{a+16}}$$

where w is the crack width, a is the aggregate size, and f'_c is the cylinder compressive strength.

The width and direction of a crack can be estimated from the average stresses and average strains. The crack direction is assumed to be normal to the principal average tension direction defined by the three angles θ_x, θ_y, θ_z, while the crack width can be estimated as the product of the principal average strain ϵ_1 and a crack spacing parameter $s_{m\theta}$. That is, $w = \epsilon_1 s_{m\theta}$ where

$$\frac{1}{s_{m\theta}} = \left(\frac{\cos\theta_x}{s_{mx}} + \frac{\cos\theta_y}{s_{my}} + \frac{\cos\theta_z}{s_{mz}}\right)$$

s_{mx}, s_{my} and s_{mz} are indicators of crack control provided by the x, y and z reinforcement directions.

Once the crack direction is known and an assumption is made about the increases in reinforcement stress at a crack, the shear stress on the crack surface can be calculated from

$$v_{ci}=\sqrt{\xi_{xy}^2+\xi_{xz}^2+\xi_{yz}^2}$$

$$\xi_{xy} = (\rho_x\,\Delta f_{sx} - \rho_y\,\Delta f_{sy})\cos\theta_x\cos\theta_y$$
$$\xi_{xz} = (\rho_x\,\Delta f_{sx} - \rho_z\,\Delta f_{sz})\cos\theta_x\cos\theta_z$$
$$\xi_{yz} = (\rho_y\,\Delta f_{sy} - \rho_z\,\Delta f_{sz})\cos\theta_y\cos\theta_z$$

ρ_x, ρ_y and ρ_z are the reinforcement ratios in the x, y and z direction, and Δf_{sx}, Δf_{sy}, Δf_{sz} represent the differences between the average reinforcement stresses and the reinforcement stresses at a crack. Rather than using a strain compatibility approach to determine the reinforcement stresses at a crack, a lower bound approach is used in the modified compression field theory. The relative increases in reinforcement stress are chosen to minimize the required shear stress on the crack surface.

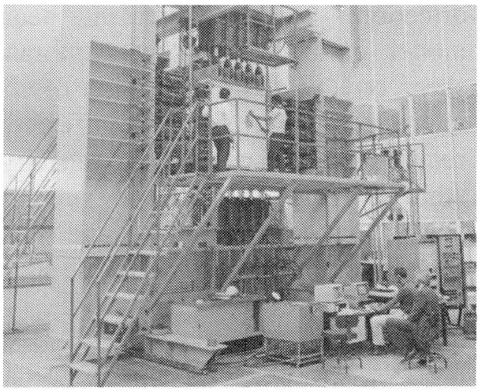

Fig. 5 University of Toronto Shell Element Tester

The increase in reinforcement stress at a crack may be limited by either yielding of the reinforcement or the inability of the crack to transmit the required shear. The average tensile stresses in cracked concrete is in turn limited by the increase in the reinforcement stress at a crack. That is,

$$f_{c1} = \rho_x\,\Delta f_{sx}\cos^2\theta_x + \rho_y\,\Delta f_{sy}\cos^2\theta_y + \rho_z\,\Delta f_{sz}\cos^2\theta_z$$

7. Computer Program SHELL474

The procedures described in this paper have been incorporated into a computer program called SHELL474 [3]. The program operates in two modes. In SLS mode the program calculates the strains (e.g., steel strains, crack widths, etc.) associated with a specified set of eight sectional forces, while in ULS mode the program calculates the complete load-deformation response of a section including the maximum loads which the section can resist.

8. Comparison with Experiments

A large testing machine capable of applying combined membrane forces, bending moments and transverse shear to reinforced concrete elements was constructed in 1984 at the University of Toronto. The machine uses sixty double acting hydraulic actuators to load 1.5 m high by 1.5 m wide elements of varying thickness (see Fig. 5).

Nine specimens were subjected to combined membrane forces, bending moments and transverse shear using the tester [4]. Seven of the specimens were 310 mm thick and had large amounts of in-plane reinforcement ($\rho_x=\rho_y=3.6\%$) but only a small amount of transverse shear reinforcement ($\rho_z=0.08\%$). The concrete (cylinder) strengths were approximately 52 MPa.

The experimentally observed interaction of transverse shear and membrane shear is shown in Fig. 6. Predictions are given from both program SHELL474 (strain compatibility model) and the equivalent beam model. The equivalent beam model gives a reasonable prediction for the pure transverse shear case, but is excessively conservative regarding the influence of membrane shear. This is because the equivalent beam model makes use of the traditional beam shear design rules, which are excessively conservative regarding the influence of membrane tension. Also, the equivalent beam model does not properly account for the in-plane reinforcement direction or the membrane shear direction. The equivalent beam model predicts that the membrane shear on the right hand side of the interaction (Fig. 6) significantly reduces the transverse shear capacity, while SHELL474 correctly predicts that in this case the membrane shear actually improves the transverse shear response.

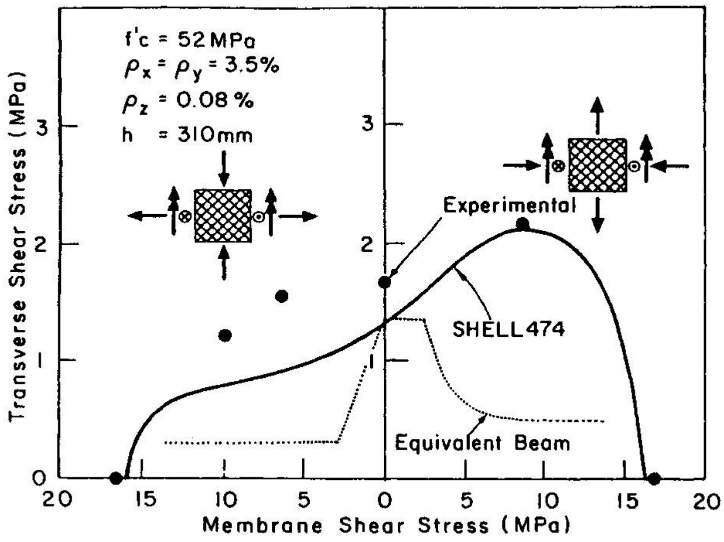

Fig. 6 Membrane Shear - Transverse Shear Interaction

9. Concluding Remarks

Traditional empirical shear design rules developed for simple building components are not appropriate for use in the design of complex concrete structures. If a consistent design approach is to be used for all types of structures then "*a more general approach is required for the* (*shear*) *design of structural concrete*" [Breen; Bruggeling].

A general approach to sectional design is possible by considering the following: (1) the compatibility of uniaxial, biaxial or triaxial strains; (2) realistic stress-strain relationships for the materials especially "cracked concrete," and; (3) equilibrium. As was demonstrated in this paper, such a general approach can be consistently applied to structural concrete.

Acknowledgements

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Offshore Structural Concrete

Béton armé des plateformes pétrolières

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SUMMARY

This report deals with design and dimensioning methods related to North Sea offshore platforms. Post-tensioning, shear and fatigue in connection with high strength concrete in particular, are assessed. Results from in-situ tests taken from slip are discussed and compared with results regarding bond behaviour of bars in slipform casted concrete.

RÉSUMÉ

Ce document traite les méthodes de dimensionnement et de conception des plateformes pétrolières de la Mer du Nord. Les modes de précontrainte, le mode de cisaillement et la fatigue relatifs au béton à haute résistance sont tout particulièrement traités. Des résultats d'essais in situ réalisés sur les structures érigées suivant la méthode du coffrage glissant sont discutés et comparés avec ceux concernant le comportement d'adhérence de barres d'armatures contenues dans des structures coffrées selon la même méthode.

ZUSAMMENFASSUNG

Dieser Artikel befasst sich mit Berechnungs- und Bemessungsverfahren für Offshore-Plattformen in der Nordsee. Insbesondere werden die Vorspannung, Schub und Dauerfestigkeit von hochfestem Beton behandelt. Es werden Ergebnisse von Baustellenversuchen zum Verbund von Bewehrungsstäben im mit Gleitverfahren hergestellten Beton diskutiert.

1. INTRODUCTION

Design of concrete structures for use in offshore petroleum production involves detailed analytical investigation of complex shell structures subject to dynamic loading from waves and also seismic action. A detailed knowledge of the dynamic response, the local distribution of forces and stresses and resistance to failure in fatigue as well as the ultimate load carrying capacity is therefore of paramount importance.In order to cover these aspects refined global finite element analyses using shell or solid element models are generally required, the results from which must be scaled to cover the structural dynamic response which is determined by use of stocastic methods.

The Sleipner A platform /4/ for 82 m water depth in the North Sea incorporates new features in concrete design and analysis. This platform with a volume of 74000 m^3 of concrete is presently being constructed in Stavanger Norway. Slipform construction is used for the cells and the shafts. The top of the shafts has a near rectangular form to suite the supports of the Modul Support Frame (MSF). This is well covered in the analyses of the structure. The near rectangular top of shafts will be slipformed through application of new slipform technology.The conventional steel transition rings between the MSF and the shafts are eliminated as the steel MSF is placed directly on the concrete shaft. Furthermore the deck is supported on only two, respectively three, supports on each shaft resulting in savings in steel for the MSF. To absorb the high compression forces incurred by few supports and to optimize the design of the walls and the shafts, high strength concrete with quality C65 is necessary.The entire cells up to a level of 48 m are constructed in dry dock. To get sufficient buoyancy out of dock, the four foundation areas are cast in light weight aggregate concrete LC65.

In the following chapters bond strength for slipformed concrete structures will be discussed as well as dimensioning of post-tensioning and application of T-headed bars in cell walls. Shear and fatigue aspects are furthermore reviewed.

2 CONCRETE STRENGTH

2.1 Design strength of concrete

The design strength of concrete should correspond to the concrete strength as obtained in the structure (structural strength) reduced by an appropriate factor of safety to account for undertainties in material characteristics, quality of construction and uncertainties related to prediction of capacity and mode of failure /5/.

The format for the design strength used in the Norwegian concrete design code NS3473 /10/ refers to "structural strength" and differs in this way from the CEB-FIP format. However, the design value is close to that of CEB-FIP 1990.

The grade of concrete - eg C60 - refers in some codes to the cube strength (eg. NS3473) and in others to the cylinder strength. (eg. CEP-FIP, ACI318 and CAN3-A23.3). Uniformity on this simple point is welcome.

2.2 Effects on strength from slipforming

Slipform construction has long tradition. For tower type structures slipforming has been common. Since 1971 slipforming has been used for construction of large offshore structures in the North Sea. The Sleipner A platform described in /4/ which has been in construction since 1989, is shown in Figures 1 and 2.

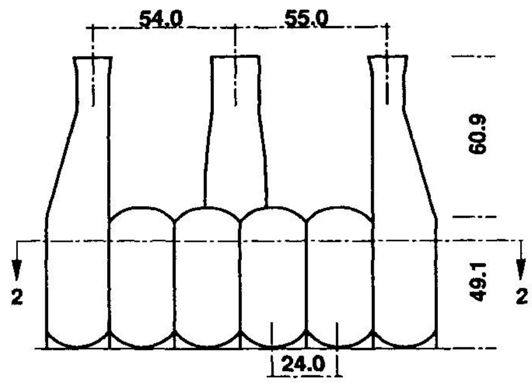

Fig.1 SECTION 1-1

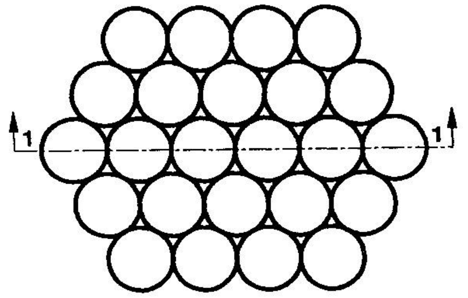

Fig.2 SECTION 2-2

The slipform construction employed for offshore platforms has given high quality well compacted concrete. In /5/ the result of in-situ tests of 1042 cores and 808 cubes from 3 platforms are described. The concrete compressive strength was 5 to 10% higher than for none slipformed parts. There were no indications that the tensile and bond strength should not follow the compression strength. Tests of bond strength of concrete from slipformed structures in Germany, indicate the opposite results. /6/ . The conclusion of the German tests is that the ratio of the computed bond lengths (slipform/ordinary formwork) was found to be 2 for vertical bars and 4 for horizontal bars.

The different results in Germany and Norway indicate that the quality of concrete may be sensitive to the slipform technique. The effective bond length will depend on workmanship and slipform technique as well as wall thickness and specific weight of the concrete.

Statoil is, however confident in using slipform construction with the high quality control of the work maintained for construction of the North Sea structures.

3. PRESTRESSING

Prestressing has been used in the construction of all North Sea concrete structures. The advantages of using prestressing are listed in /2/. Of special importance for offshore platforms is the possibility to reduce structural dimensions and to limit crack widths. According to Statoil requirements, cracks are limited to 0,15mm in the splash zone and 0,30 mm in the submerged zone. Prestressing will also improve the fatigue resistance, due to the better fatigue resistance of the concrete when only exposed to compression.
Global forces (membrane tension) are covered by prestressing imposed by post-tensioned tendons and also by hydrostatic loading. Local moments are taken up by normal reinforcement. Cables are typically located at top and base of shafts and in the ringbeam in the upper and lower domes. Normally, the shafts are also prestressed vertically.

NS 3473, does not prescribe different computational methodology for ordinary reinforced and prestressed concrete. The problems with different design methods for ordinary reinforced and prestressed concrete are thus eliminated, see /1/ and /2/. In determination of capacity in flexure and combination of flexure and membrane action the residual strength of the prestressing steel (the top part of the stress-strain curve not utilized for pretensioning) may be used. This may in many cases, especially for heavy shell structures lead do considerable savings in ordinary reinforcement and avoide congestion of reinforcement. Thus, ensuring better concreting and a better structure.

4. SHEAR

Regarding the Sleipner A platform, several locations are exposed to high shear forces requiring shear reinforcement; the transition areas between the domes and the cell wall as well as in the so called star walls, see Figure 2.

The determination of shear capacity varies between most codes. Also the value of load and material factors are treated differently in the most codes. A comparative study of the shear capacity from the concrete is reported in /8/. According to NS3473 the shear capacity can be calculated by a simplified method, by the truss model method or by a general method for inplane shear. The truss model method is less used in the design of offshore concrete structures. It was first included in NS 3473 in the 1989 edition. The practical use of it in computerized design is more complicated than the simplified method due to the necessity of having in principal one truss model for each load case and the general complexity of the method.The simplified design method given in NS 3473 is partly modifications of the CEB/FIP Model code 1978. The tensile strength is by NS3473 lower than by CIB/FIP Model code 1978 for high strength concrete, /7/.

Shear tests reported in /7/ show that NS3473 predicts well the variation of the diagonal cracking strength for beams with different types of high strength concrete.The determination of ultimate shear strength for high strength concrete above a cylinder strength of 80 MPa is however less well defined. Results reported in /7/ show that shear capacity of high strength concrete beams is more influenced by the scale than beams of normal strength. The more brittle behavior of high strength concrete and the lesser aggregate interlock due to fracture of the aggregate may be part of the reason for reduced shear strength. Further work appears necessary on this subject.

Results reported by M.P.Collins /9/ from a series of thirty-one tests involving concrete panels reinforced in only one direction and loaded in various combinations of tension and shear, provide most valuable information.Use of Collins modified compression field theory on available test results show that the existing reinforced concrete design codes typically are very conservative in their estimate of the shear strengt of elements subjected to combined tension and shear.The modified compression field theory represent the state of the art. The theory takes into account the contribution of the concrete aggregate interlock mechanism in a rational manner. However, as discussed in /3/ the method appears complicated since there is no simple procedure for obtaining transverse strain.
A rational and simple method to predict shear compression and tension based on M.P. Collins theory should be developed in order to get a simple, accurate and economical dimensioning method for shear.

5. FATIGUE

The subject of resistance to fatigue of structural concrete to random loads was first codified in DnV Rules for Offshore Structures 1977. Through joint industrial R and D projects, S-N curves were established for reinforced concrete, normal weight (NW) and light weight (LW) in compression - compression and tension - compression, including the effects of water. During the last 10 years considerable advancement has been made on the subject in terms of refinements of the S-N curves as the data base has been expanded.

At present fatigue of structural concrete is covered in two design codes - i.e. CEB-FIP Model Code 1990 and in NS3473 - for the following modes of loading:

- compression - compression
- compression - tension
- tension - tension

Procedure for estimating the fatigue life for transverse shear and bond based on extrapolations of the unaxial test data are also given. The S-N curves given in CEB-FIP 1990 and in NS3473 differs in some respect. A main difference is the way the stress range or rather the minimum stress is introduced. The NS3473 has also introduced an endurance limit, while the S-N formulation of CEB-FIP assumes a straight S-log N relation.The calculation procedures given in NS3473 is basically a further development of the DnV-77 method, and it is in some respect more detailed than the procedure of CEB-FIP 1990.Recent Norwegian tests on air dry and wet high strength concrete specimens - normal density and light weight tested in cyclic compression are reported in /13/. It is noted that the natural moisture contained in sealed concrete is enough to reduce the fatigue strength to that of wet concrete. Recommended S-N curves including endurance limits for various levels of minimum stress are given in /13/ for concrete in compression. The state of the art is reported in /14/.

Test data and improved computational models should be developed for the following areas:

- out of plane shear action
- 2D cyclic action alternating in tension-compression
- local 3D effects in joints including reinforcement detailing.

6. T-HEADED STIRRUP BARS

Structural elements subject to high out-of- plan shear and those needing confinement for ductility in the post-elastic range require concentrated transverse reinforcement. The traditional hoops of bent stirrups are difficult to place, especially if the tails must be bent back into the core. These conventional stirrups are also relatively inefficient.A T-headed bar with plate anchor at the ends has therefore been developed for use as transverse shear reinforcement. The use of T-headed bars is cost efficient compared to stirrups.Tests described in /11,12/ show these bars to have excellent anchorage under ultimate loads, providing exceptional confinement and ductility to the member, with test specimens yielding displacement ductility factors of over 40. Also tests have indicated that confining effects from T-headed bars have yielded enhancement of concrete strength in the order of 10% as compared to the use of 90° hooked bars. As to the use of 90° hooks these will open up during ultimate strength situations and loose its capacity. Use of 90°- hooks should be limited to ø 12 mm bars.

Structures to be used in the arctic must withstand intense punching shears from the impact from sea ice and iceberg. Offshore structures must withstand the impact from boats and barges, and the base slab and other connecting elements of offshore platforms may have to resist high shear from concentrated reactions. Other extreme load effects that may have to be considered are large earthquakes, explosions, or impact from falling objects. To provide the required strength and ductility the percentages of transverse steel may reach 1.5 to 2.5 % (15000 to 25000 mm^2 /m^2). Such reinforcement percentage can only be provided by use of heavy bars fully anchored.

Use of T-headed bar is an effective and structurally reliable method of providing transverse reinforcement to resist shears and to provide confinment of the concrete core. T-headed bars have been used in the North Sea offshore structures, the Gullfaks A and Sleipner A and the Ekofisk Protective Wall.

7 CONCLUSIONS

Slipformed high strength concrete is an excellent material for constructing of offshore platform support structures.

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