

Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 68 (1993)

Artikel: A fuzzy neural system for repairing bridge decks
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DOI: <https://doi.org/10.5169/seals-51864>

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A Fuzzy Neural System for Repairing Bridge Decks

Système expert pour déterminer la méthode de réparation des tabliers de pont
Expertensystem zum Bestimmen der Instandsetzungsmethode für Brückenplatten

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SUMMARY

This paper attempts to propose a method that helps maintenance engineers to choose appropriate repairing methods of reinforced concrete bridge decks by using a fuzzy neural expert system. The evaluation measures are lead on damage cause, damage degree and damage propagation speed. One of the important items in the maintenance program is that the present system can provide several appropriate repairing methods by taking account of the past experience through a neural network's learning ability.

RÉSUMÉ

Les auteurs proposent un système expert neuronal à caractère flou comme moyen d'assistance pour les ingénieurs responsables de la maintenance des ponts en béton armé, lors du choix de la méthode de réparation des tabliers. Les critères de sélection se basent sur les causes, l'importance et la vitesse de propagation des dommages. L'une des caractéristiques essentielles de ce programme de maintenance est de pouvoir proposer des méthodes de réparation appropriées qui, grâce à l'aptitude d'apprentissage du réseau neuronal, tiennent compte des expériences passées.

ZUSAMMENFASSUNG

Der Beitrag möchte ein unscharfes neuronales Ingenieurhilfsmittel bei der Wahl geeigneter Instandsetzungsmethoden für Brückenfahrbahnplatten aus Stahlbeton vorschlagen. Die Auswahlkriterien basieren auf Ursache, Umfang und Ausbreitungsgeschwindigkeit des Schadens. Eine wichtige Eigenschaft des Unterhaltungsprogrammes ist der Vorschlag mehrerer geeigneter Reparaturmethoden aufgrund früherer Erfahrungen, dank der Lernfähigkeit des neuronalen Netzwerks.



1. INTRODUCTION

In recent years, emphases in the field of civil engineering have drifted from new construction of structures to the maintenance, management and repair of existing structures, and hence maintenance and management work becomes increasingly important. In Japan, due to yearly decline and damage, functional deterioration of bridge structures, especially those built before 1965, becomes a problem. The deterioration and decline of bridges caused by cruel usage due to the changes of Japanese social environment, rapid increase of traffic volume and heavy vehicles, which was not forecasted in construction have become a serious problem. Now it attracts great concern how to plan and carry out maintenance and management of bridge structures.

For above reasons, there are many existing bridges to be repaired or altered¹⁾. However, it is impossible to rebuild all the damaged bridges because of financial limitation²⁾. Evaluation of durability is necessary for appropriate maintenance and repair of bridge structures. Future progress of the damage state of bridge structures should be estimated based on the damage cause, damage degree and damage propagation speed. Then, whether or not a bridge needs to be repaired or rebuilt, and what method should be adopted if repaired, are decided based on the estimation results.

It is difficult to maintain and repair all the bridges because the number of experts engaging in damage assessment cannot satisfy its demand. Therefore, it is very meaningful to build an expert system to help engineers without sufficient experience to make various judgments on maintenance and repair as the expert does³⁾.

This paper attempts to develop a fuzzy neural expert system for assessing the damage states of RC bridge deck and choosing its appropriate repairing method. The system consists of two subsystems: one is a fuzzy production system for making inferences and the other is a neural network system for deriving solutions. While the fuzzy production system is used to evaluate damage causes, damage degree and damage propagation speed with appropriate knowledge, the neural network system is used to select appropriate repairing methods taking into account of many factors comprehensively. It becomes possible to construct a more practical and useful system by combining these two methods with different characteristics.

2. CONSTRUCTION OF AN EXPERT SYSTEM

2.1 Utilization Environment

The present system has made improvement on the former damage assessment system⁴⁾ of RC bridge deck in the following points.

- 1) The former system was built on an engineering workstation NEWS (made by SONY) with Franz Lisp. In contrast, the present system is built on the same engineering workstation with Common Lisp and C language so that its ability of transplanted and extension is advanced, and it can be used anywhere.
- 2) In the former system, production rules have been described in Roman alphabet. In contrast, in the present system rules are described in Japanese characters so that expansion, renewal and management of rules become easier for the Japanese user.
- 3) Data are input in a dialogue form.

2.2 Inference Mechanism

The former system uses forward inference only in fuzzy production system. This system uses forward and backward inference and their combination. By introducing backward inference, inference time of this system becomes shorter. While the former system takes 15-20 minutes from beginning of inference to output of results, the present system reduces inference time up to 1/4.

2.3 Evaluation Method

After an appropriate goal is determined, it is necessary to establish evaluation measures and procedure in order to efficiently support a decision making on maintenance and management. Here, damage cause, damage degree and damage propagation speed are employed as the evaluation measures. It also adopted the learning ability of a neural network to automatically choose repairing methods.

3. ARCHITECTURE OF THE EXPERT SYSTEM

3.1 Fuzzy Production System

The fuzzy production system consists of inference engine including forward and backward inferences, rule base, working memory, fuzzy terms, and fuzzy predicates, as shown in Fig.1.

Both forward and backward inferences can be used in the fuzzy production system by introducing backward inference into the previous system⁴⁾.

The system uses a combination of forward and backward inferences to evaluate damage causes of RC bridge deck. Namely forward inference is implemented in the conditional part of backward inference rules. When civil engineers who have the knowledge of maintenance and management of RC bridge deck, want to inquire the results of their subjective judgment based on preliminary survey and inspection, combinational inference of forward and backward inferences can be adopted. Fig.2 shows a part of the rules in the subsystem for evaluating damage causes of RC bridge deck. Fig.2 implies that, for example, when an inquiry such as (?- ("damage cause" g-1)), namely, damage cause of RC bridge deck is [g-1: Extreme wheel load?] is implemented, the system first matches r1 result and the consequent of the backward rule, namely ("damage cause" g-1) is executed, and then moves to forward rules to execute (rules "damage cause-1" ...) using the system function *fc in

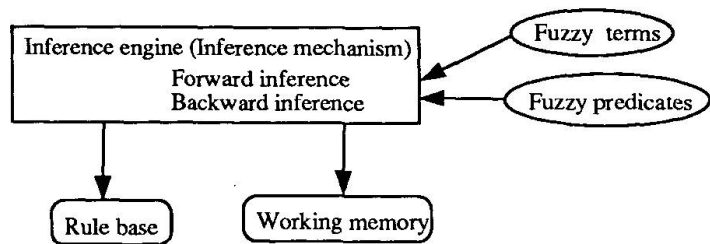


Fig.1 Architecture of Fuzzy Production System

Backward rules for damage causes:

```
(backward-rules
(r1 ("Damage cause" g-1) :- (*fc "Damage cause-1"))
(r2 ("Damage cause" g-2) :- (*fc "Damage cause-2"))
.....
(r16 ("Damage cause" g-16) :- (*fc "Damage cause-16")))
```

Forward rules for damage causes:

```
(rules "Damage cause-1"
(rule-1
if (Crack Form Width-direction)
then (change-rb "Damage cause-1-1"))
.....
(rule-dummy
if (*dummy)
then (change-rb "Result-0-1"))
.....
where, *fc is a system function executing forward inference
```

Fig.2 An Example of Rules to Damage Causes Using Forward and Backward Inferences



the conditional part of backward rules (*fc "damage cause-1"). That is, damage cause [g-1: Extreme wheel load?] is investigated in the forward-rules. Here, the forward rules are stored in the form of module. Table 1 presents the possible damage causes of RC bridge deck. Block structure of rules is considered to be an effective method when a large-scale knowledge base is constructed such as the knowledge base for the damage causes of RC bridge deck. When someone wants to know only whether a conclusion can be obtained, the rules only related to damage causes rather than the whole rules in the system need to be investigated using the combination of forward and backward inferences. For example, if the inquiry is successful, the output becomes:

g-1: Extreme wheel load (Truth value: more-or-less-large)

If the inquiry fails, the output is nothing. When an actual expert system is built, it is desirable to decide the assignment of roles of forward and backward and use their combinations according to the knowledge of object.

The advantages of using forward rules in the conditional part of backward rules are as follows^{5),6)}:

- 1) Rules can be described in the form of module because forward rules are to be executed in the conditional part of backward rules.
- 2) Knowledge can be easily added and modified by describing rules in modules.
- 3) When someone wants to know only if a conclusion can be obtained, the rules related to such a conclusion rather than the whole rules in the system need to be investigated. In other words, unnecessary search can be avoided to make the system more efficient.

Table 1 Damage Causes

Load	g-1: Extreme wheel load
	g-2: Impact effect
	g-3: Inadequacy of girder arrangement
Design and structural factors	g-4: Short of deck depth
	g-5: Lack of main steel bar
	g-6: Lack of distribution bar
	g-7: Inadequacy of distribution cross beam
	g-8: Additional moment due to differential settlement
Construction conditions	g-9: Poor quality of cement
	g-10: Poor compaction
	g-11: Inadequate curing of construction joint
	g-12: Lack of covering
Other factors	g-13: Salt
	g-14: Poor drainage
	g-15: Movement of substructure
	g-16: Action of alkali material

3.2 Neural Network⁷⁾

In the system, a subsystem for choosing appropriate repairing methods is constructed by utilizing the learning and pattern recognition abilities of neural network. Fig.3 shows the structure of a neural network.

Learning method: Back-propagation
 Combining method: Multi-layer neural network
 Input and Output form: Events
 Parameters: Synapse weights and threshold values
 Input and output function of cell: Sigmoid function

$$f(x)=1/(1+\exp(-x))$$

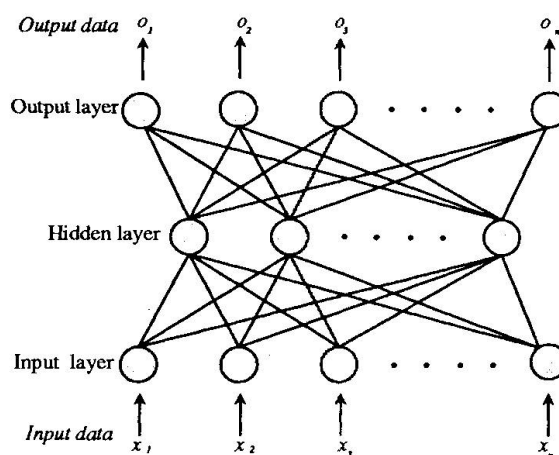


Fig.3 Multi-layer and Back-propagation Neural Network

Introducing the neural network into the expert



system, it is possible to consider not only structural factors of RC bridge deck but also economic, construction and environmental factors in choosing the repairing methods.

4. CONSTRUCTION OF THE EXPERT SYSTEM FOR DAMAGE ASSESSMENT OF RC BRIDGE DECK

4.1 Evaluation of Damage Cause, Damage Degree, Damage Propagation Speed by Fuzzy Production System

This system estimates the damage cause from the design condition, environmental condition and inspection data of a bridge, and evaluate the damage degree and damage propagation speed for each cause. The flow chart of the inference process is shown in Fig.4.

Firstly, design and environmental conditions of a bridge are surveyed in advance. Secondly, inspection is carried out at site. Inspection items include crack, pavement, concrete, steel etc. Based on the above survey and inspection results, damage causes yielding the bridge damages are estimated among the causes shown in Table 1. For example, the following rules are used.

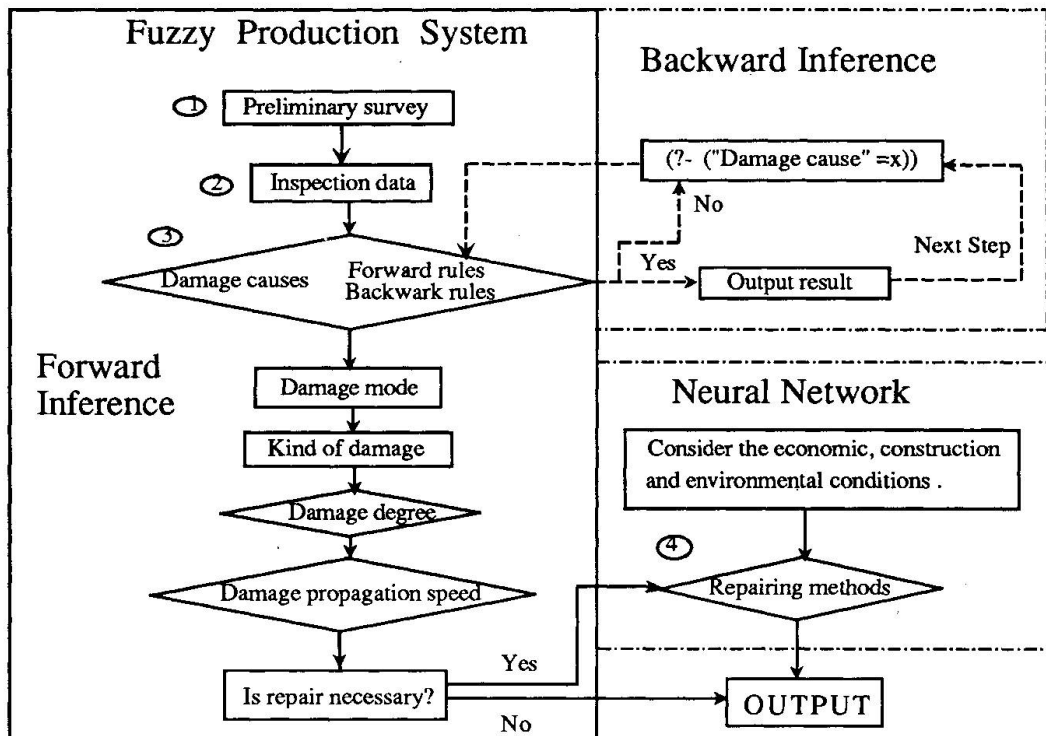


Fig.4 Inference Process of the Expert System

```
(rules "Damage causes-1-2-2"
very-true
if (structural-type plate-girder)
   (crack-configuration width-direction)
   (crack-location center-of-deck-span)
   (wheel-load-location center-of-deck-span)
then (deposit ("Damage cause" Extreme-wheel-load))
      (change-rb "Damage causes-1-3"))
```



Next, damage modes consisting of only damage types with the same damage causes are established. For example, if a damage cause is assumed to be "Extreme-wheel-load" from inspection results, then the damage mode is made up of damage types concerning crack as shown below.

```
(rules add-mode-1
(rule-1
  if ("Damage cause" Extreme-wheel-load)
  then (add (mode crack) 1.0)
        (change-rb add-mode-2))
(rule mode-1
  (rule-1-1
    if (mode crack)
    then (change-rb crack-damage-causes-1)))
```

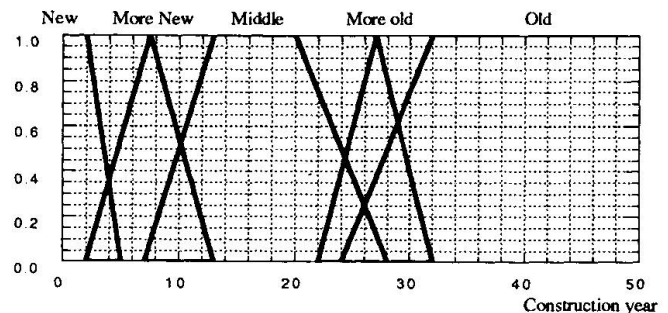


Fig.5 An example of membership function for age of RC deck

where, "add" means to add factors into the current working memory.

Next, the type of damages such as crack damage degree, steel damage degree, pavement damage degree, concrete damage degree, structural damage degree etc. are estimated. For example, following rules can be considered to determine crack damage degree:

```
(rules crack-damage-causes-1-1
(rule-1-1
  true
  if (crack-form width-direction)
      (crack-width large)
      (crack-space small)
  then (deposit (crack-damage-degree more-large)))
```

In addition to the damage degree and damage propagation speed associated with the damage mode, environmental condition and age of the bridge are considered.

While the former system uses only forward inference to judge damage causes, the present system can use both forward and backward inferences by adding backward inference. Parts of forward rules and rules of combination of forward and backward rules are shown in Fig.2. Some membership functions used in the system are shown in Fig.5.

4.2 Automatic Selection of Repairing Methods by Neural Network

As shown in Fig.4, the system first evaluates damage cause, damage degree and damage propagation speed by the fuzzy production system. Then, suitable repairing methods are chosen through the learning ability of neural network by taking into account of the effects of importance, economy and construction condition of bridges.

The architecture of the system for choosing automatically repairing methods by neural network is illustrated as follows:

- 1) As stated in previously, inference results obtained from the fuzzy production system, namely, damage cause, damage degree, damage propagation speed of RC bridge deck, are used as input data. In addition, structural property, economic, construction and social effects etc. are taken into account together. Here the results obtained from the fuzzy production system are represented in the form of fuzzy set.
- 2) A four-layered neural network is used.



First layer (input layer): In the input layer, fuzzy data are represented as a fuzzy unit group and non-fuzzy data as a crisp unit group.

Second layer: 20 unit

Third layer: 20 unit

Fourth layer (output layer): Adding stringer method, retrofitting method by steel plate deck and rebuilding method are considered as representative repairing methods, and denoted as A, B, C respectively. As an example, when the output is B, it is represented as:

(A, B, C)=(0, 1, 0)

3) The input data are learned.

4) In order to evaluate the learning accuracy, it is necessary to achieve some criteria for the correctness of identification. In this study, it is thought that a highly precise diagnosis has been made if the result obtained from the neural network is nearly consistent with actual records.

5. NUMERICAL EXAMPLES

The applicability and usefulness of the present system is demonstrated by using actual data in the durability assessment of RC bridge deck. Numerical experiments on the selection of repairing methods are carried out using both the fuzzy production system and neural network. The data are collected from the actual repairing records made past for 13 bridge in Osaka City, Japan.

5.1 Reasoning by Fuzzy Production System

The input of data such as structural type, inspection results, environmental condition, etc. are implemented interactively as shown in Fig.6. Here, the following rules (about 700 rules) are used:

1. Rules for damage reasoning	31
2. Rules for estimating damage causes	223
3. Rules for damage mode	5
4. Rules for evaluation of crack damage degree	55
5. Rules for evaluation of road pavement damage degree	37
6. Rules for evaluation of reinforcing steel damage degree	4
7. Rules for evaluation of concrete damage degree	20
8. Rules for evaluation of structural damage degree	20
9. Rules for judgment of comprehensive damage degree	187
10. Rules for judgment of damage propagation speed	28
11. Rules for selection of repairing method	31
12. Rules for output of results	51

First, the data obtained from preliminary survey and inspection are input. For example, Table 2 and Table 3 present the design conditions, environmental conditions, and inspection data of a RC bridge deck. Using these data, working memory is rewritten as shown in Fig.7. For instance, the working memory concerning the age of the RC bridge deck is rewritten as follows:

```
> <Input of Data>
1. Construction of data file
2. Load existing data file
=> 1

Curve bridge?
y: yes
n: no
=> y

Degree of truth value?
1. truth
2. absolute-true
3. more-true
4. true
5. more-or-less-true
6. fairly-true
x. other
=>1

a. Add input data?
re: renew the input data from first
sp: stop
except keyboard: next input
=>3

Structural type of bridge?
1. plate girder
2. box girder
3. arch (below road)
4. arch (upper road)
x. other
=>3

.....
Write above data into file?
y: yes
n: no
=> y
Input file name => file.1

Make another file?
1. Make another file
2. Implement the inference
=> 2

Inference results:
.....
```

Fig.6 Interactive input



Table 2 Design and Environmental Conditions

Kind	Factor	Data	Truth value
Design conditions	Structural type	Plate girder(straight)	1
	Design specification	Before 1967	1
	Construction year	Old	very-true
	Deck thick	20cm	1
	Bridge length	69.00m	1
	Bridge width	12.95m	1
	Lanes	3 lanes	1
	Foot way	One side	1
Environmental conditions	Erection location	Near city	1
	Road rank	Main road	1
	Ratio of heavy vehicle	Many	very-true
	Traffic flow	Many	very-true
	Wheel load location	Center of deck span	absolute-true

Table 3 Inspection Data

Kind	Factor	State	Truth value
Crack	Configuration	Width direction	absolute-true
	Location	Center of deck span	very-true
	Density	6.63	1
	Space	Large	very-true
	Width	Large	very-true
Road surface	Pavement	Large uneven	very-true
		Small log	true
Concrete	Color change	Medium	true
	Oldness	Medium	true
	Alienation lime	Center of deck span	absolute-true
Steel	State of rust	Rusting	absolute-true
Other	water around	Medium	true
	Leaking water	Center of deck span	very-true

very-true/(deck age medium)

Next, forward inference is commenced using inference rules and the rewritten working memory. The results shown in Table 4 can be obtained after about two minutes inference. Here, truth value is expressed in linguistic forms.

If one wants to know whether some damage cause exists, forward inference can be used in the antecedent part of backward rules when judging damage causes of RC bridge deck in the system. In this case, matching is implemented between inquiry content and consequent part of backward rules to investigate if the antecedent condition of rules is satisfied. As an example, provided that an inquiry whether or not

(working-memory

```
{
    (structural-type girder-plate),
    (design-specification 1967),
    very-true/(construction-year old),
    (deck thick 20),
    (bridge-length 69),
    (bridge-width 12.95),
    (lanes 3),
    (foot-way one-side),
    (erection-location near-city),
    (road-rank main-road),
    absolute-true/(location-of-wheel-load center-of-deck-span),
    very-true/(rate-of-heavy-vehicle many),
    very-true/(traffic-flow many),
    absolute-true/(crack-configuration width-direction),
    very-true/(crack-location center-of-deck-span),
    (crack-density 6.63),
    very-true/(crack-space large),
    true/(crack-width w-large),
    very-true/(road-pavement uneven),
    true/(road-pavement small-log),
    true/(concrete-color-change medium),
    true/(concrete-oldness medium),
    absolute-true/(concrete-alienation-lime center-of-deck-span),
    absolute-true/(steel rusting),
    true/(water-around medium),
    very-true/(leaking-water center-of-deck-span),
}
```

Fig.7 An Example of Working Memory



the damage cause is [g-1: Extreme wheel load], the following inference result is obtained:

>(?- ("Damage causes" g-1))

[Inference Result]

g-1: Extreme wheel load (Truth value: more-large)

Table 4 Inference Results

Damage causes (Truth value)	g-1: Extreme wheel load (more-large)	
	g-6: Lack of distribution bar (small)	
	g-15: Poor drainage (large)	
Damage degree (Truth value)	g-1: more-large (small)	
	g-6: more-large (small)	
	g-15: large (more-large)	
Damage propagation speed (Truth value)	g-1: more-large (more-large)	
	g-6: medium (small)	
	g-15: more-large (large)	

5.2 Selection of Repairing Methods by Neural Network

Because the above inference results are given by fuzzy sets to deal with the inference results and take into account of the effects of importance, economy and construction condition of bridges in a unified manner, the data containing fuzziness are expressed by fuzzy unit groups and the data not containing fuzziness are expressed by crisp unit groups. For example, it is defined that [crack damage is small] = {1/0, 0.66/1, 0.33/2, 0/3, 0/4, 0/5, 0/6, 0/7, 0/8, 0/9, 0/10}. As a fuzzy unit group, [crack damage is small] = {1, 0.66, 0.33, 0, 0, 0, 0, 0, 0, 0, 0} are input to 11 units of the input layer in the neural network, whereas the data not containing fuzziness are represented by binary values as a crisp group. For example, it is 1 when steel reinforcing bars are exposed and 0 otherwise. The output of results is A or B or C, where A denotes the repairing method by additional stringers, B the retrofitting method by steel plate deck and C rebuilding method. For example, when the output is B, it can be expressed as (A, B, C) = (0, 1, 0). Then, through the neural computation, it is possible to judge which repairing method is suitable for this case. The output values for bridge No.1 are A=0.94, B=0.05, C=0.04. Thus A should be chosen as repairing method. It can be seen that the outputs agree completely with the past record, and hence the learning has been carried out precisely.

6. CONCLUSIONS

Appropriate damage assessment is important in maintenance program of bridge structures. This research has developed a practical expert system for damage assessment of bridge structures, based on the knowledge of experts who make various judgments efficiently in daily maintenance and management work. The durability of RC bridge deck is considered as a main evaluation object in the system. Based on the damage cause, damage degree and damage propagation speed, repairing method can be chosen automatically. By introducing the fuzzy logic into the system, it becomes possible to deal with linguistic data given by the subjective judgments of engineers⁸⁾. Practical and useful solutions can also be obtained even from incomplete data. Furthermore, repairing methods can be chosen automatically, while considering a lot of factors by introducing a neural network into the expert system.

The characteristics of the present expert system for damage assessment of RC bridge deck can be summarized as follows:



- 1) Since the system was built on a 32 bit engineering workstation NEWS (made by SONY) and written in Common Lisp and C language, anyone can use it at any time and place without difficulty.
- 2) Using this system, it is possible to evaluate a lot of bridges with relatively short time.
- 3) Inference time can be reduced by module of rules. While the former system takes 15-20 minutes from beginning of inference to output of results, the present system reduces up to 1/4.
- 4) Because data can be input in a dialogue form, anyone can use it without difficulty.
- 5) Because backward inference can be implemented in the system, both forward and backward inference can be used in the fuzzy production system. Implementing forward rules in the antecedent part of backward rules, it is possible to reduce the inference time.
- 6) It becomes clear from this study that, in order to solve efficiently a practical problem, appropriate results can be obtained by using fuzzy production system when knowledge is easily attainable, or relationships between events are clear. However, when it is difficult to make out rules, meaningful results can be obtained by utilizing the learning ability of neural network⁹⁾⁻¹²⁾.

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