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Geometrical Shape of Stone Arches

Forme géometrique des arcs en maçonnerie

Geometrische Form von Mauerbögen

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Despite it could appear obvious, the effective structure of an arch made by stone voussoirs seldom fits its geometrical shape. In this paper it is shown that, in particular cases such as the round arches, structural stability almost always requires static collaboration of the backfill. As it is known, successful design of a masonry arch depends more on its geometrical shape than on the characteristics of the materials. Indeed, in order to state the stability of this kind of structures, a thrust line of acting loads lying wholly within the arch profile needs to be found (if it exists); in particular, the closer the thrust line fits the arch centreline, the better is the shape designed.
This kind of circumstance can never be verified dealing with round arches under gravitational loads, considering that the affinity between the thrust line of the loads and the respective bending moment diagram of a simply supported beam having the same span excludes vertical tangents at the ends. With reference to figure 1, let us consider the subsequent arches, featuring cross-section of uniform dimensions, obtained by increasing the angle α and leaving unchanged the bending radius R and the depth s. At the beginning, being α small, it is always possible to find a thrust line of the applied loads (self-weight, backfill and overload) lying wholly the arch outline; this is true, in general, up to a certain value α^*. The problem can be faced defining the minimum thickness needed to contain the thrust line, once fixed R and per each value of α.
The minimum depth can be defined as follows: let us suppose that depth is exactly equal to the minimum one, then just one line of thrust lying wholly within the masonry can be found. This line touches the arch profile in a finite number of cross-sections. Being α small, one can observe that the thrust line touching the arch profile intrados at the crown and at the springing is wholly above the intrados. This consideration suggests the way to find the minimum thickness: if the depth is equal to the minimum one, the thrust line touches the arch profile intrados at the crown and at the springing, as well as the extrados at the haunches. If the thrust line touches the arch outline in these points, the structure turns into a mechanism. Being α greater than a certain value α_1, the collapse mechanism concerns only a part of the structure, defined just by α_1, as, in general, the thrust line is included in the arch profile up to an angle value $\alpha_2 < \pi$; therefore the minimum thickness found for $\alpha=\alpha_1$ keeps unchanged up to this limit (figure 2). In order to determine the minimum thickness beyond this threshold (α_2), one should refer to the condition that the thrust line be within the arch outline near the springing, as there is no more question of mechanism type failure.
To the purposes of the present paper, a numerical code for the automatic research of the minimum thickness has been written, solving the problem both in the case of mechanism type criterion and in the case of thrust line exceeding the arch profile near the springing [3,4]. A comprehensive numerical investigation has been carried out to show the influence of each parameter concerning the problem, whose major results are shown in figures 3 and 4. In the first one (fig. 3) the lines corresponding to the values of s_{min}/R (minimum thickness over bending radius) obtained varying α are plotted for

different values of the ratio γ between the stone self-weight and the backfill weight; in this figure the overload height h is equal to 0. Figure 4 shows the same lines plotted for different values of h, representing an uniform load over the arch, being $\gamma=0.5$. As estimated, the curves show three ranges of α. In the first one s_{min}/R increases with α up to a certain value, say s_1, corresponding to α_1. Being α in the range defined by α_1 and α_2, s_{min}/R keeps unchanged to s_1. When α increases beyond α_2, and up to the maximum value π, s_{min}/R fast climbs to exaggerated values. As in general stone arches do not feature such large values of the thickness, one can state that, in these circumstances, just the static collaboration of the backfill assures the structural stability.
The threshold values α_1 and α_2 can be considered not very variable; they can be assumed to be equal respectively to $2\pi/3$ and to $\pi/1.1$.
The diagram showed can be used to assess the stability of an arch, once known its bending radius and the loads. The ratio between the actual depth and the minimum one provides a measure of the safety degree of the structure. Should the actual thickness be smaller than the minimum one, the structural stability can be thought entrusted to the static collaboration of the backfill near the springing. One has to stress the role played by the backfill in defining the acting loads as well as in participating to the effective structure of stone arches.

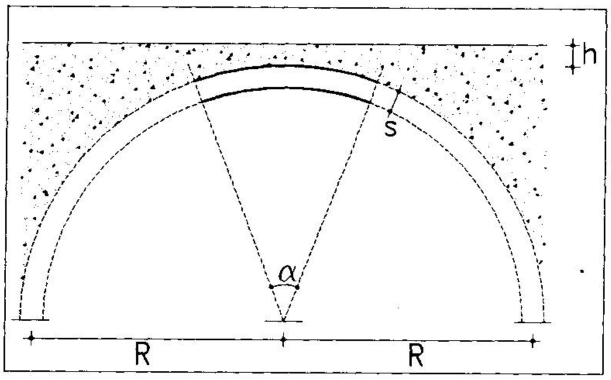

Fig. 1

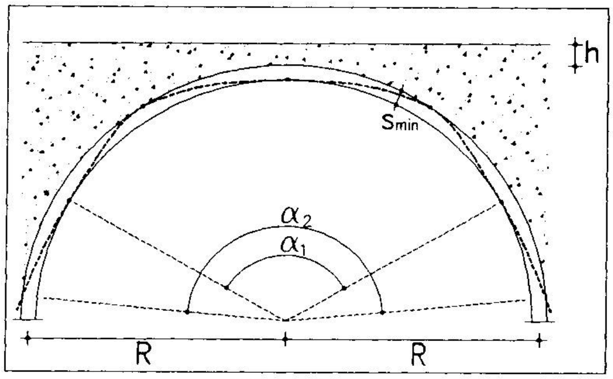

Fig. 2

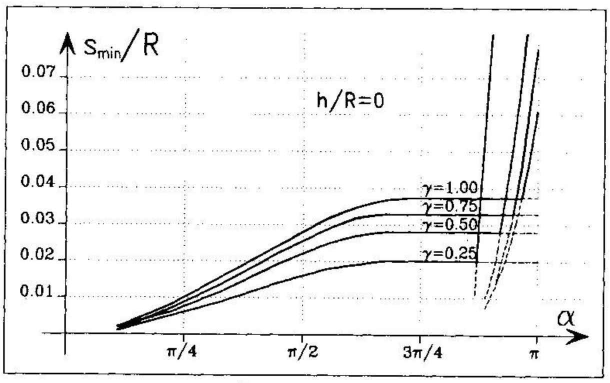

Fig. 3

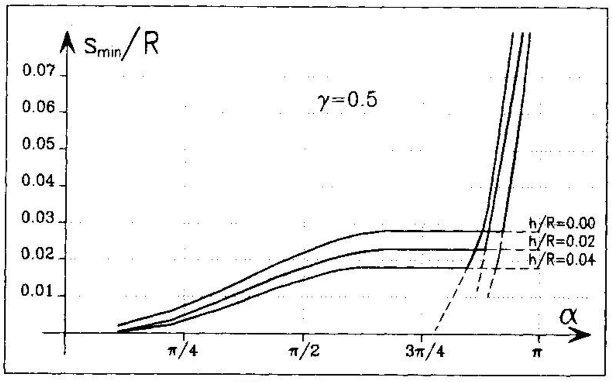

Fig. 4

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