New classification system for semi-rigid connections considering overall behavior of frames

New Classification System for Semi-Rigid Connections Considering Overall Behavior of Frames

Yoshiaki Goto
Professor
Nagoya Institute of Technology
Japan
Yoshiaki Goto, born 1949, got his civil engineering doctoral degree at University of Tokyo. His research interest is in the advanced analysis and design of steel structures. He has been professor at Nagoya Institute of Technology since 1991.

Satoshi Miyashita
Research Associate
Gifu National College of Technology
Japan
Satoshi Miyashita, born 1968, graduated as civil Engineer form Nagoya Institute of Technology. He is engaged in the research on advanced analysis of steel structures. He has been research associate at Gifu National College of Technology since 1993.

A new classification system of semi-rigid connections was proposed. Some typical subassemblages of multistory frames were chosen to consider the layout and member details of the structural systems. The boundary of connections between rigid and semi-rigid was established by taking into account the behaviors of the structural subassemblages at the serviceability limit state along with the ultimate limit state. The validity of the proposed classification system was examined by analyzing the overall behavior of semi-rigid frames.

1. Introduction

It is well known that real beam-to-column connections possess some stiffness that falls between the two extreme cases of fully rigid and ideally pinned. Thus, the modeling of connections as semi-rigid is more realistic. However, in engineering practice some connections can be considered pinned if their stiffness is so small that the connections are incapable of transmitting any significant moment, thus permitting almost free rotation. Similarly, some connections can be considered rigid if their rigidity is so large that no significant slope discontinuity exists between the adjoining members. The assumption of ideally pinned or rigid connections considerably simplifies the design and analysis procedures of framed structures. Thus, it is useful to estimate in advance whether the connections can be assumed rigid, semi-rigid or pinned. Proposals for the classification of connections have been presented by EC3(1992) and Bjorhovde et al(1990). The classification system by Bjorhovde et al. is intended for the case where the prior knowledge concerning the member and structural details is not available. On the other hand, EC3 proposed a classification system based on the load-carrying capacity of frames. This classification is more rational, if the layout and member details of the structural system are known in advance. However, ductility demand is not shown in EC3 classification. This is different from the proposal by Bjorhovde et al. Although EC3 considers the ultimate strength of frames in the classification of connections, it does not take in account the behavior at the serviceability limit state. Further, in order to evaluate the load-carrying capacity of frames, EC3 adopts an approximate formula, i.e. the Merchant-Rankine formula. The frame model used for this evaluation is also too simple to generally reflect the effect of layout and member details of real frames. In this way, the existing classification systems are still considerably approximate in nature. In fact, a precise elastic-plastic finite-displacement analysis showed that the EC3 boundary between rigid and semi-rigid connections is on the whole considerably restrictive in terms of the ultimate strength of frames(Goto and Miyashita 1995).

In this paper, we will propose a new classification system of connections where the behavior of frames not only at the ultimate limit state but also at the serviceability limit state is considered. The connection model used for the classification is the power model proposed by Kishi et al.(1993). The validity of the proposed classification system is examined by analyzing the elastic -plastic overall behavior of semi-rigid frames.

2. Modeling of Connections

The semi-rigid connections are represented by a discrete, inelastic, rotational spring. The connection model used herein is the three-parameter power model proposed by Kishi et al.(1993). The generalized form of this model is expressed as

$$m = \theta / (1+\theta^n)^{1/n} \qquad (1)$$

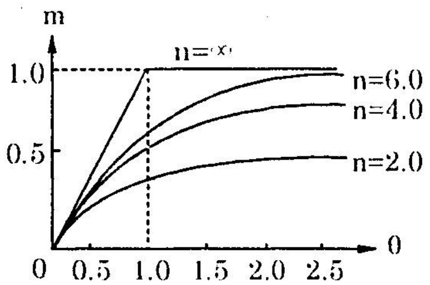


Fig. 1. Three-parameter power model

where $m = M/M_u, \theta = \theta_r/\theta_0, \theta_0 = M_u/K_I$,

M = connection moment, M_u = ultimate moment capacity of connection, θr = relative rotation between beam and column, K_I = initial connection stiffness and n = shape parameter.

Equation(1) has the shape illustrated in Fig. 1 depending on the value of n. As can be seen from Eq.(1), the connection curve is uniquely determined by three parameters, that is, ultimate moment capacity M_u, initial stiffness K_I and shape parameter n. The formulas to calculate the value of n are determined for several connection types as shown in Table 1, based on statical analysis of test data (Chen and Kishi 1989). The formulas given in Table 1 reduces the independent parameters of Eq.(1) to M_u and K_I. Thus, the classification can be made quantitatively based on these two parameters. That is, the boundary between rigid and semi-

Table 1. Empirical equation for shape parameter n

Connection Type	n	
Single web-angle connection	$0.520\log_{10}\theta_0 + 2.291$	$\log_{10}\theta_0 > -3.073$
	0.695	$\log_{10}\theta_0 \le -3.073$
Double web-angle connection	$1.322\log_{10}\theta_0 + 3.952$	$\log_{10}\theta_0 > -2.582$
	0.537	$\log_{10}\theta_0 \le -2.582$
Top-and seat-angle connection (without double web angle)	$2.003\log_{10}\theta_0 + 6.070$	$\log_{10}\theta_0 > -2.880$
	0.302	$\log_{10}\theta_0 \le -2.880$
Top-and seat-angle connection (with double web angle)	$1.398\log_{10}\theta_0 + 4.631$	$\log_{10}\theta_0 > -2.721$
	0.827	$\log_{10}\theta_0 \le -2.721$

rigid and that between semi-rigid and pinned can be estimated in terms of the values of M_u and K_i. The boundary values for K_i are decided by the behavior of frames at serviceability limit state, whilst those for M_u are determined by the behavior at ultimate limit state.

3. Frame Models

In order to take into account the behavior of frames in the classification of connections, we adopt several subassemblages which will be considered to represent the behaviors of the respective parts of the multistory multibay frames shown in Fig. 2. These subassemblages are chosen by considering the deformation patterns of the respective parts of the sway and non-sway frames illustrated in Fig. 3. The subassemblages so chosen are summarized in Fig.4. In this figure, it is denoted by the notations A_s ~F_s and A_n ~F_n how the respective subassemblages represent the parts of the frames in Fig.3. For the members of these subassemblages ,we consider the linearly distributed residual stress model, initial deflection and the uniaxial constitutive model of material which were presented by Vogel(1984).In this constitutive model, $\sigma_y = 294MPa$ and $E = 2.01 \times 10^5 MPa$ are used.

4. Classification of Initial Stiffness K_i of Connection Based on the Behavior of Frames at Serviceability Limit State

4.1 Classification Criteria

Classification of initial connection stiffness will be made by considering the behavior of frames at serviceability limit state. The following criteria defined in terms of displacements is used to classify the semi-rigid connections to be rigid.

$$\Delta_s = (\delta_s - \delta_r)/\delta_r \leq 0.05 \qquad (2)$$

where δ_s is a displacement of a frame with semi-rigid connections and δ_r is a displacement of the corresponding rigid frame. The loading conditions used to calculate the displacements are shown in Fig.4. The loads applied at the serviceability limit state are denoted by V and H. In what follows, the boundary value of the initial stiffness of connections between rigid and semi-rigid will be derived considering the behavior of sway and nonsway frames.

4.2 Sway Frame

The displacements δ_s and δ_r in Eq.(2) are represented by the horizontal displacements at the joint when a horizontal force H is applied to the subassemblages as shown in Fig.4 In the calculation of δ_s and δ_r, the small displacement theory is applied because the displacements at the serviceability limit state is small. Further, the stiffness of semi-rigid connections is assumed to be linearly elastic. The boundary of the initial connection stiffness between rigid and semi-rigid can be analytically obtained in terms of the nondimensional parameter expressed by

$$\kappa_i^b = (K_i L_b)/(EI_b) \qquad (3)$$

The boundaries so obtained for the respective subassemblages are summarized in Table 2(a) where G is a relative stiffness factor defined by

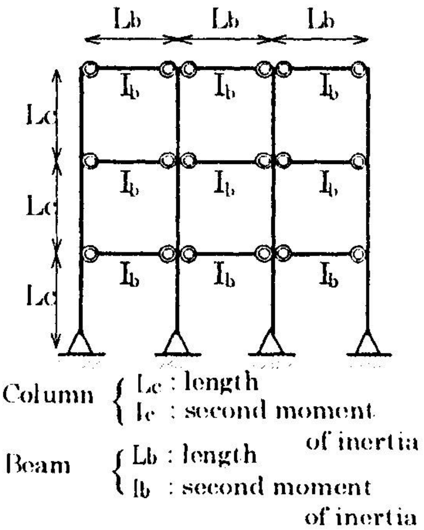
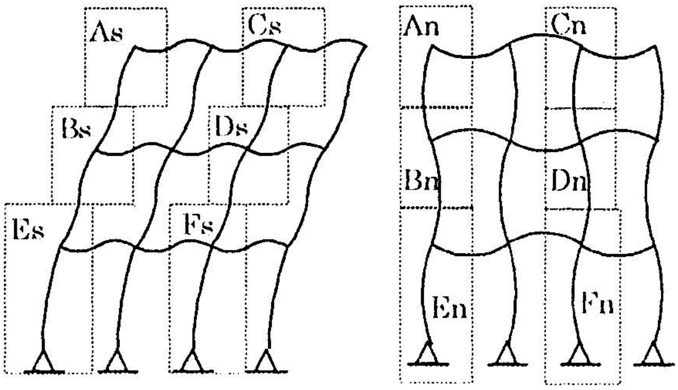
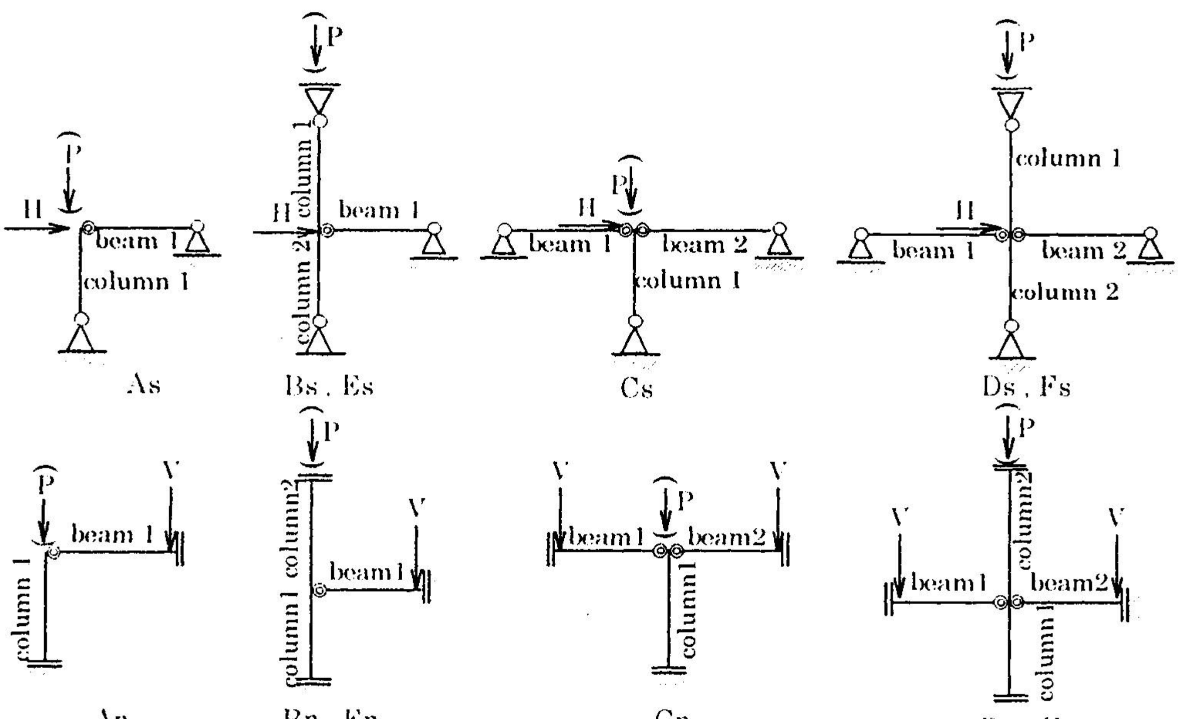


Fig. 2. Multistory and multibay frame



(a) Sway *(b) Nonsway*

Fig. 3. Deformation pattern of multistory multibay frames



	As , An	Bs , Bn	Cs , Cn	Ds , Dn	Es , En	Fs , Fn
column 1	Lc/2	Lc/2	Lc/2	Lc/2	Lc/2	Lc
column 2	—	Lc/2	—	Lc/2	Lc/2	Lc/2
beam 1	Lb/2	Lb/2	Lb/2	Lb/2	Lb/2	Lb/2
beam 2	—	—	Lb/2	Lb/2	—	Lb/2

V } loads at serviceability
H } limit state

P : loads at ultimate limit state

Remarks ; As ~ Fs , An ~ Fn denotes the parts of multistory multibay frames shown in Fig.3

Fig. 4. Subassemblages

$$G = (I_b / L_b)/(I_c / L_c) \tag{4}$$

EC3 determined the boundary value as $\kappa_i^b = 25$, assuming G = 1.4. Therefore, in order to compare our boundary value with that given by EC3, we also show in Table 2(a) the values of κ_i^b when 1.4 is substituted into G. The values of κ_i^b so calculated become either 50 or 31.6, depending on the types of subassemblages. These values are larger than that specified by EC3 as the boundary value between rigid and semi-rigid.

4.3 Nonsway Frame

Similar to sway subassemblages, the boundary of initial connection stiffness between rigid and semi-rigid is determined for nonsway subassemblages based on the criteria expressed by Eq. (2). The displacements δ_s and δ_r in Eq.(2) are represented by the vertical displacements of the beam at the load point when a vertical load V is applied to the beams as illustrated in Fig.4. The boundaries defined in terms of the initial stiffness are shown in Table 2(b) for respective subassemblages. To compare with EC3 classification, the values of κ_i^b with $G = 1.4$ are also shown in Table 2(b). These values which ranges from 11.2 to 29.5 are larger than 8 given by EC3.

Table 2. Boundary value for initial stiffness κ_i^b

(a) Sway Frame

	κ_i^b	κ_i^b $(G = 1.4)$
As, Bs, Cs, Ds	$\kappa_i^b = \dfrac{6}{(1+G)\Delta}$	50
Es, Fs	$\kappa_i^b = \dfrac{6(8G+1)}{(4G+3)(3G+1)\Delta} - \dfrac{6}{3G+1}$	31.6

$\Delta = 0.05$

(b) Nonsway Frame

	κ_i^b	κ_i^b $(G = 1.4)$
An, Bn	$\kappa_i^b = \dfrac{6}{(1+G)^2\Delta} - 4$	16.8
Cn, Dn, Fn	$\kappa_i^b = \dfrac{1}{2}\left(\dfrac{3}{\Delta} - 1\right)$	29.5
En	$\kappa_i^b = \dfrac{6}{(1+G)(1+2G)\Delta} - 2$	11.2

$\Delta = 0.05$

5. Classification of Ultimate Moment Capacity M_u of Connection Based on the Behavior at Ultimate Limit State of Frames

5.1 Classification Criteria

Classification of ultimate moment capacity M_u of connections is to be made by considering the ultimate behavior of subassemblages. In order to classify the connections to be rigid, EC3 used the following criteria which only considers the ultimate strength of the frames.

$$(P_{ur} - P_{us})/P_{ur} \le 0.05 \tag{5}$$

where P_{ur}, P_{us} are ultimate strengths, respectively, of rigid and semi-rigid frames.

The criteria expressed by Eq.(5), however, may not be sufficient because the displacement of frames at the ultimate limit state is not reflected. Therefore, we use herein the following classification criteria which takes into account both strength and displacement at the ultimate limit state.

$$\Delta_u = \sqrt{\{(P_{ur} - P_{us})/P_{ur}\}^2 + \{(u_{us} - u_{ur})/u_{ur}\}^2} \le \sqrt{(0.05)^2 + (0.05)^2} \cong 0.07 \tag{6}$$

where u_{ur}, u_{us} are ultimate displacements, respectively, of rigid and semi-rigid subassemblages. Based on the classification criteria given by Eq.(6), boundaries of ultimate moment capacity of connections between rigid and semi-rigid are determined. The ultimate behaviors of the subassemblages under the load conditions illustrated in Fig.4 are analyzed by the method presented by Goto and Miyashita (1995). This analysis method precisely considers the geometrical and material nonlinearities in the structural response. That is, the geometrical nonlinearity is analyzed by the co-rotational method, whilst the member plastification is taken into account by the plastic-zone method.

5.2 Determine of Boundary Value of Connection Moment Capacity

For the classification of the connection moment capacity, we introduce the nondimensional moment defined by

$$\overline{m}_u = M_u / M_{bp} \tag{7}$$

where M_{bp} denotes the full plastic moment of the connected beam. The connection curve based on the three-parameter power model is governed by the parameter $\overline{m}_u$. The moment-rotation curves for top- and seat- angle connections with double web angles are illustrated in Fig. 5 with $\overline{m}_u$ ranging from 0.4 to 1.0. The boundary value of $\overline{m}_u$ between rigid and semi-rigid is denoted here by $\overline{m}_u$

To consider the layout and member characteristics of the subassemblages, two parameters shown below are used.

$$G = \frac{(I_b / L_b)}{(I_c / L_c)} \quad , \quad \lambda = \frac{L_c}{\pi r}\sqrt{\frac{\sigma_y}{E}} \tag{8a,b}$$

where r is the radius of gyration of member cross section. G and λ respectively denote relative stiffness and normalized column slenderness ratio. The ranges of these parameters are determined by considering the layout and member details of practical semi-rigid steel frames.

Taking the sway subassemblage D_s with top- and seat-angle connections with double web angles for an example ,the boundary value $\overline{m}_u^b$ between rigid and semi-rigid is to be determined based on the criteria expressed by Eq.(6). The boundary values $\overline{m}_u^b$ obtained for the respective values of the two structural parameters G and λ are shown in Fig. 6. It can be seen from this figure that $\overline{m}_u^b$ becomes large with the increase of λ or G. It should be noted that $\overline{m}_u^b$ exceeds unity for the cases with $\lambda = 1$ or ($\lambda = 0.6, G = 0.7$). This is different either from the EC3 classification where

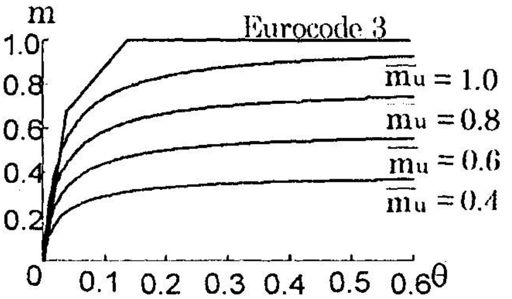


Fig.5. Moment-rotation curves for top-and seat angle connections with double web angle

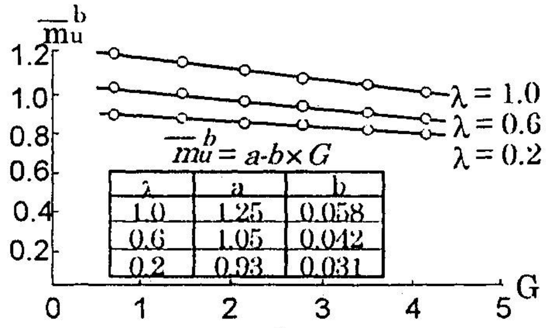


Fig.6. $\overline{mu}^b$ – G relation

$\overline{m}_u^b$ is unity or the Bjorhovde classification where $\overline{m}_u^b$ is 0.7. As can be seen from Fig.6, the relation between $\overline{m}_u^b$ and G can be well approximated by the function in the form

$$\overline{m_u^b} = a - bG \qquad (9)$$

where a and b are assumed here to be expressed by the linear functions of λ. These linear functions are determined as follows by the least square method.

$$a = 0.828 + 0.388\lambda \qquad b = 0.024 + 0.029\lambda \qquad (10a,b)$$

The formula given by Eqs.(9) and (10a,b) coincides well with the numerical results , as compared in Fig.6.

Following the same procedures as explained above, the boundaries of connection moment capacity $\overline{m}_u^b$ between rigid and semi-rigid are obtained for the rest of the sway and nonsway subassemblages shown in Fig. 4. These subassemblages are assumed to have top- and seat-angle connections with double web angles. Formulas to predict the boundaries of connection moment capacity $\overline{m}_u^b$ are shown in Table 3.

Table 3. Formulas to predict $\overline{m}_u^b$

	Nonsway subassemblage	Sway subassemblage
A	$\overline{m}_u{}^b = (1.161 + 0.150\lambda) - (0.026 + 0.005\lambda)\cdot G$	$\overline{m}_u{}^b = (0.732 + 0.248\lambda) - (0.015 + 0.005\lambda)\cdot G$
B	$\overline{m}_u{}^b = (0.976 + 0.324\lambda) - (0.027 + 0.011\lambda)\cdot G$	$\overline{m}_u{}^b = (0.679 + 0.300\lambda) - (0.026 + 0.015\lambda)\cdot G$
C	$\overline{m}_u{}^b = (0.909 + 0.331\lambda) - (0.027 + 0.019\lambda)\cdot G$	$\overline{m}_u{}^b = (0.630 + 0.260\lambda) - (0.004 + 0.003\lambda)\cdot G$
D	$\overline{m}_u{}^b = (0.836 + 0.396\lambda) - (0.024 + 0.031\lambda)\cdot G$	$\overline{m}_u{}^b = (0.658 + 0.196\lambda) - (0.014 + 0.001\lambda)\cdot G$
E	$\overline{m}_u{}^b = (0.811 + 0.385\lambda) - (0.029 + 0.004\lambda)\cdot G$	$\overline{m}_u{}^b = (0.678 + 0.134\lambda) - (0.008 + 0.004\lambda)\cdot G$
F	$\overline{m}_u{}^b = (0.680 + 0.361\lambda) - (0.019 + 0.024\lambda)\cdot G$	$\overline{m}_u{}^b = (0.494 + 0.242\lambda) - (0.005)\cdot G$

5.3 Validity of the New Classification System

We shall examine the validity of the aforementioned new classification system of semi-rigid connections, when applied to the multistory and multibay semi-rigid frames. Test frames considered herein consist of two sway frames and two nonsway frames which are illustrated in

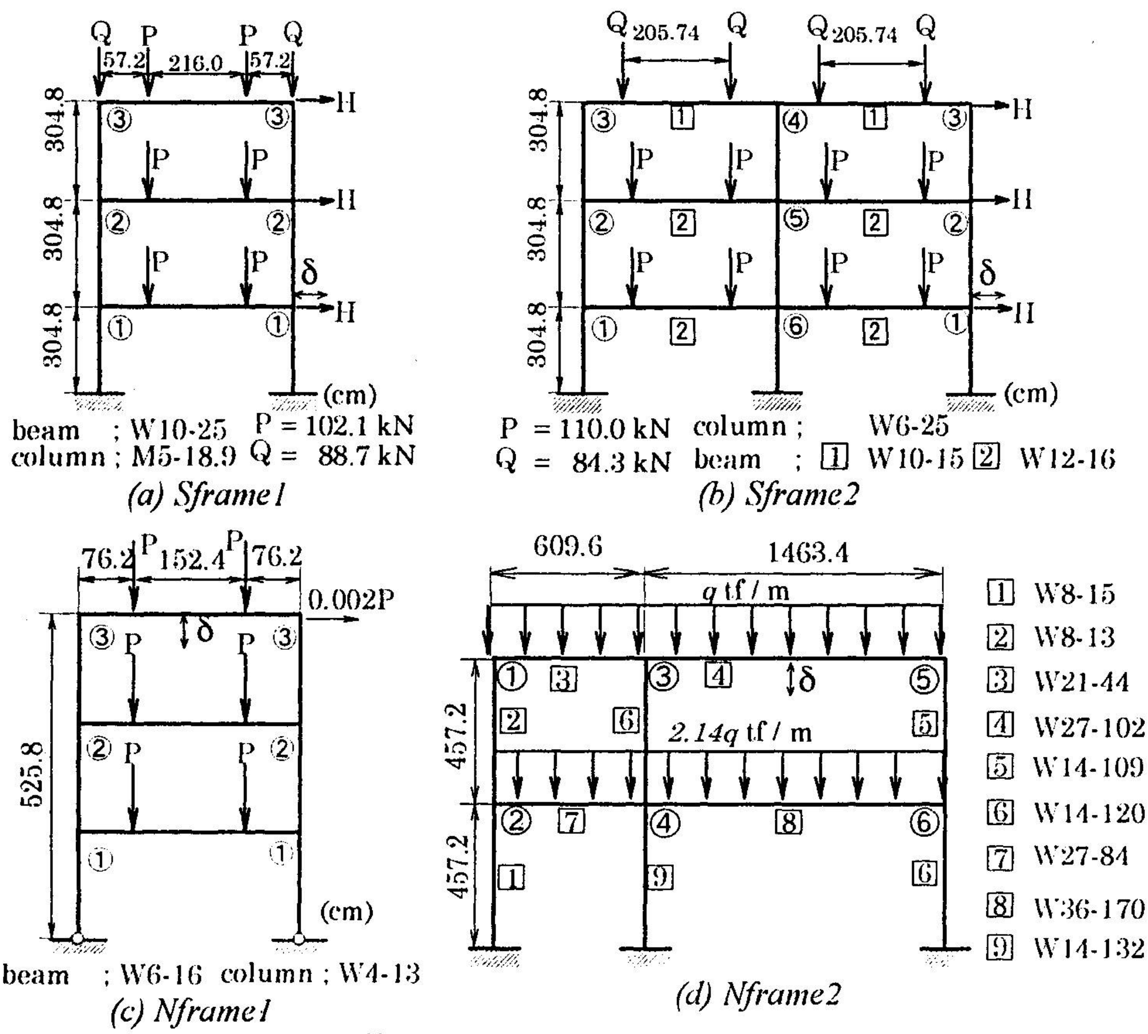


Fig. 7. Test frames

Fig. 7 along with the loading conditions. Sway frames denoted by Sframe1 and Sframe2 were shown by Yarimci (1966), while nonsway frames denoted by Nframe1 and Nframe2 were respectively designed by McNamee and Lu (1972) and Ziemian (1992). The test frames are assumed to have the top- and seat-angle connections with double web angles with the moment-rotation characteristics which coincide with the proposed boundary between rigid and semi-rigid. The governing parameters for the respective connections are determined from Tables 2 and 3, by considering the layout and details of the connected members. The connection parameters so determined are summarized in Table 4. The validity of the new classification system will be confirmed, if the behavior of the test frames satisfies the criteria expressed by Eqs.(2) and (6) within a reasonable tolerance.

The behavior of the test frames up to the ultimate states is analyzed by the elastic-plastic finite displacement analysis. For Sframe1 and Sframe2, the horizontal force H is monotonically increased with keeping the vertical loads P and Q constant, whilst the vertical load P is monotonically increased for Nframe1 and Nframe2.

As the results of numerical analysis, the load- displacement relations of Sframe1 and Nframe1 are shown in Fig. 8. In these figures, we also demonstrate the results where all the connections

Table 4. Boundary values of connection parameters between rigid and semi-rigid

	connection	κ_i^b	m_u^b	subassemblage	G	λ
Sframe1	1	10.437	1.071	As	6.477	0.758
	2	16.049	0.993	Bs	6.477	0.758
	3	16.049	1.082	Es	6.477	0.758
Sframe2	1	64.509	0.862	As	0.860	0.586
	2	52.495	0.810	Bs	1.286	0.586
	3	52.495	0.743	Es	1.286	0.586
	4	64.509	0.777	Cs	0.860	0.586
	5	52.495	0.751	Ds	1.286	0.586
	6	52.495	0.629	Fs	1.286	0.586
Nframe1	1	8.638	0.964	En	1.633	0.529
	2	13.309	1.094	Bn	1.633	0.529
	3	13.309	1.194	An	1.633	0.529
Nframe2	1	83.065	1.267	An	0.174	0.739
	2	40.956	1.076	En	0.458	0.725
	3	29.500	1.006	Cn	0.820	0.379
	4	29.500	0.781	Fn	1.280	0.379
	5	28.822	1.193	An	0.912	0.381
	6	4.203	0.885	En	2.370	0.381

are assumed to be rigid. The values of Δ_s and Δ_u defined by Eqs.(2) and (6) for respective test frames are summarized in Table 5. The criteria at serviceability limit state is checked by the load level which is 1/1.4 of the maximum load of the corresponding rigid frame. The value of 1.4 is considered here as a load factor. The value of Δ_s ranges from 0.002 to 0.004, while that of Δ_u ranges from 0.019 to 0.041. All these values of Δ_s and Δ_u satisfy the criteria given by Eqs.(2) and (6). Although all the values of Δ_s are rather small compared with the specified value of 0.05, those of Δ_u are almost comparable to 0.07 specified by the criteria. In order to further examine the validity of the boundary of connection parameters between rigid and semi-rigid, we analyze the behavior of the test frames by decreasing the value of the connection parameter $\overline{m}_u$ from $\overline{m}_u^b$. In this analysis, the value of κ_i^b is kept constant. We show in Fig .9 the

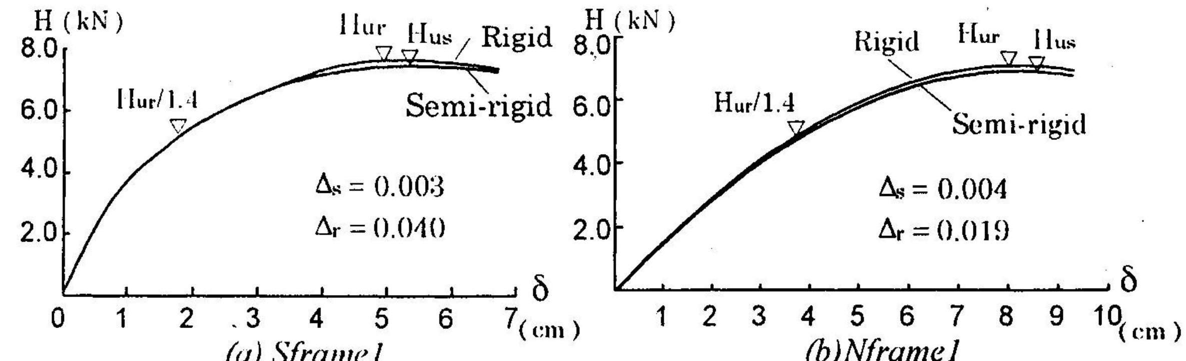


Fig. 8. Load-displacement relations

relation between $\overline{m}_u$ and Δ_u that is calculated based on the ultimate behavior of the respective test frames. It can be seen from this figure that Δ_u approaches the boundary value of 0.07 specified by Eq.(6), when $\overline{m}_u$ is reduced to $0.94\overline{m}_u^b \sim 0.96\overline{m}_u^b$. This implies that the proposed boundary values of connection parameters are relatively accurate to classify the connections into rigid and semi-rigid specifically in terms of the ultimate behavior of frames.

Table 5. Values of Δ_s and Δ_r

	Sframe1	Sframe2	Nframe1	Nframe2
Δ_s	0.003	0.004	0.002	0.003
Δ_r	0.040	0.019	0.041	0.039

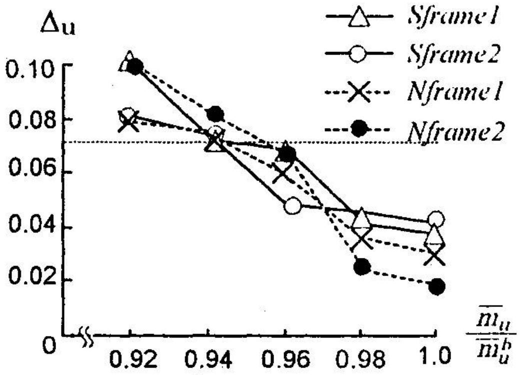


Fig.9. $\Delta_u - \overline{m}_u / \overline{m}_u^b$ relations

6. Summary and Concluding Remarks

A new classification system for semi-rigid connections was proposed. In the new classification system we considered the behavior of semi-rigid frames at the serviceability limit state along with the ultimate limit state. Taking the top- and seat-angle connections with double web angles for an example, we showed a procedure to determine the boundary of connection curves between rigid and semi-rigid. The validity of the new classification system was confirmed by analyzing the elastic-plastic overall behavior of semi-rigid frames. This new classification procedure is also applicable to the other types of semi-rigid connections

7. References

Bjorhovde ,R., Colson ,A. and Brozzetti , J. (1995) A classification system for beam to column connections , J .Struct. Engng, 116(11) , pp3059-3076, ASCE

Chen ,W.F and Kishi ,N.(1989) Semi-Rigid Steel Beam-to-Column Connections: Date Base and Modeling , J . Struct. Engng , 115(ST7) , pp.105-119 , ASCE

Eurocode3 (1990) Design of steel structures: part 1 - General rules and rules for buildings, Vol.1 Issue 3

Goto ,Y. and Miyashita,S. (1995) Validity of classification systems of semirigid connections Engineering Structures ,Vol.17 , No.8 , pp544-553

Kishi ,N. et al.(1993) Design aid of semi-rigid connections for frame analysis , Engineering Journal .3rd quarter , pp90-107 , AISC

McNamee, B. , M. and Lu ,L.W. (1972) Inelastic Multistory Frame Buckling , Journal of the Structural Division, Vol.98, No.ST7, pp.1613-1631, ASCE

Vogel , U. (1994) Ultimate limit state calculation of sway frames with rigid joints, ECCS publication 33(first edn) , Rotterdam , ECCS

Yarimci ,E.(1966) Incremental Inelastic Analysis of Framed Structures and Some Experimental Verifications, Ph.D. Dissertation Dept.of Civil Engineering ,Lehigh University ,Bethlehem ,PA

Ziemian, R. ,D. (1992) A Verification Study for Methods of Second-Order Inelastic Analysis , Annual Technical Session , Structural Stability Research Council , Pittsburgh , PA