Effective length factors in precast concrete frames

Autor(en): **Elliott, Kim S. / Davies, Gwynne / Görgün, Halil**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **75 (1996)**

PDF erstellt am: **27.06.2024**

Persistenter Link: https://doi.org/10.5169/seals-56925

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.
Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.
Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Ein Dienst der *ETH-Bibliothek*
ETH Zürich, Rämistrasse 101, 8092 Zürich, Schweiz, www.library.ethz.ch

http://www.e-periodica.ch

Effective Length Factors in Precast Concrete Frames

Kim S ELLIOTT
Gwynne DAVIES
Halil GÖRGÜN
Department of Civil Engineering
University of Nottingham, UK

Drs Elliott (b 1953) and Davies (b 1935) are Senior Lecturer and Reader, respectively, in Structural Eng. Mr Görgün (b 1963) is a Research Student studying towards a PhD in this work. Drs Elliott and Davies have sup-revised some 8 projects in precast structures.

Summary

Column effective length factors β have been computed for a number of sway sub-frames in unbraced and partially braced precast concrete frames. The variable parameters were the number of semi-rigid connections in each of the sub-frames, the relative flexural stiffness α of the frame members, and the relative linear rotational stiffness K_s of the connection to that of an encastre beam. It is found that for values of $K_s < 2$ then β factors are more sensitive to changes in K_s than α. This is an important finding because experiments have shown that K_s to be less than 2 for typical sizes of beam. Parametric equations have been presented for the variations in β with K_s and α. The results enable designers to determine β factors for situations currently not catered for in codes of practice, in particular the upper storey of a partially braced frame.

1. Effective Length Factors in Column Analysis

The determination of column effective lengths in the analysis of reinforced concrete skeletal frames is well established and owes much to the work of Cranston (1), Wood (2), and Timoshenko et al (3). The notion of using effective length factors β to assess the buckling capability of a column, either individually or as part of a structural frame, has found favour with design engineers. Simple equations for β have been presented in terms of column end boundary conditions and/or relative frame stiffness functions, so that the designer may compute, not only column buckling capacities but also second order deflections and ultimate second order bending moments, often termed M_{add} (See Appendix). The British Code for concrete structures, BS 8110:1985 (4) has adopted such an approach after Cranston (1) whereby column end conditions were equated to α, the total relative stiffness of the column to that of the beam(s) (or beams and slabs) framing into the ends of the column. The approach may be used for both braced and unbraced concrete frames. The beam - column connections are assumed to be fully rigid and of equal (or greater) strength to that of the members.

Precast concrete skeletal frames, Fig. 1, are designed using pinned-jointed connections between columns, beams and floor slabs. The stability of an unbraced frame is therefore provided only by the cantilever action of the columns at the foundation because transfer of bending moments into the beams or slabs is not permitted. In determining β, BS 8110 allows a precast concrete frame to be analysed as though it were a rigid framework but with $\alpha = 10$. With the wide range of different types of beam - column connections used in precast frames, this arbitrary approach is neither rational nor representative of real behaviour, as previous and present full scale testing of connections in precast frames by the authors (5,6) and by Mahdi

(7) has shown. Although the rotational stiffness of the connections and the degree of semi-rigidity, defined by K_s = joint stiffness J / beam flexural stiffness 4EI/L, varies over a very wide range, there is clearly scope for the implementation of β factors which incorporate **both** the flexural responses of the frame and of the semi-rigid connections.

Fig.1. Typical precast concrete skeletal frame.

In the context of the present work, *stability* implies general *frame stability* in which columns will not buckle independently of one another. It is therefore necessary to investigate the stability of a framework as a whole and to take into account α effects at both ends of the column. This paper presents the results for column effective length factors in three types of sub-frames commonly occurring in precast skeletal frames. Only sway frames are considered in this work. This instability was obtained using a geometric second-order computer program analysis developed by Aksogan and Görgün (8). In all cases maximum column loads in each sub-frame, and hence β factors are obtained for given values of α and K_s.

2. Parametric Study

Precast concrete sway frames are analysed either as *fully unbraced* frames, Fig. 2(a), or as *partially braced* frames, Fig. 2(b), where shear walls (or cores) provide lateral bracing up to a certain level and the frame is unbraced above this point.

Three sub frames, labelled F1, F2 and F3 in Fig. 2, were identified for the analysis. Sub frames F1 and F2 represent the upper floor and the ground floor levels, respectively, in an unbraced frame, whilst sub frame F3 represents the upper floor in a partially braced frame immediately above the level of the bracing. It may be seen in Fig. 2(b) that the columns adjacent to the bracing walls are fully encastre at their upper end, and may therefore be considered fully rigid at their lower end in the sub frame F3. In all cases the semi-rigid connections are positioned at the ends of the beams, reflecting the true nature of precast skeletal frames having continuous columns (Fig. 1).

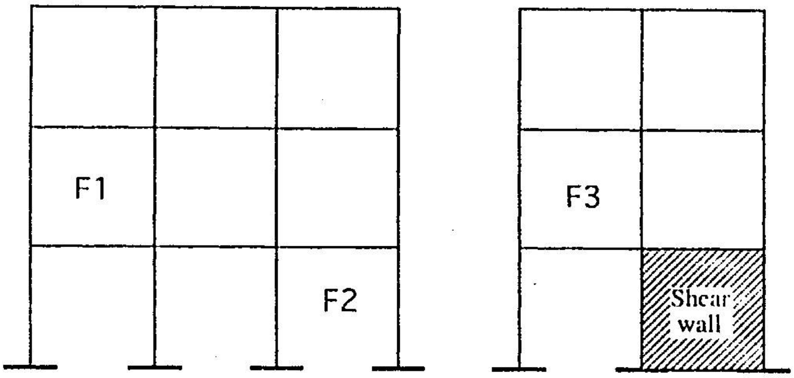

Fig. 2. Types of precast frames (a) unbraced (left) and (b) partially braced.

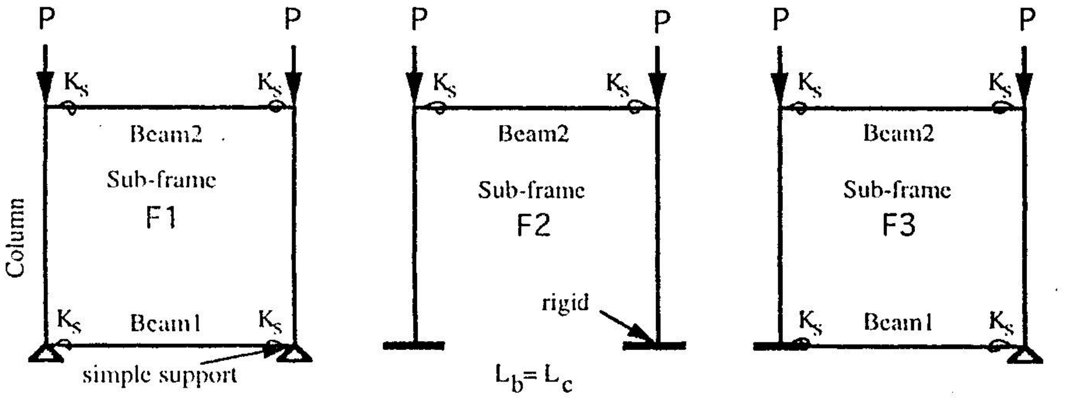

Fig. 2(c). Definitions of sub-frames used in the analysis.

The linear - elastic degree of semi-rigidity of the beam-column connections was specified as follows:

$K_s = 1 \times 10^{-9}$; to simulate a pinned-joint;
$1 \times 10^{-9} < K_s < 10$; to simulate a semi-rigid joint;
$K_s = 1 \times 10^{9}$; to simulate a fully rigid joint.

For the semi-rigid analysis, the range of values for α and K_s was obtained from realistic joint values used in typical precast concrete frames, i.e. $\alpha = 0.5$ to 2.0, and from the experimental test results for $K_s = 0$ to 10, but with a greater emphasis on the range $K_s = 0.1$ to 2.0. (In fact because the computer program requires a value for α greater than 0, $\alpha = 0.001$ was used to simulate $\alpha = 0$, and the minimum value of K_s for sub-frame F1 is 0.1 because this frame is unstable for $K_s = 0$.) For simplicity and reliability in the analysis, the length of the beams and columns in the sub-frames were made equal, and in general the cross sectional properties of the column members were varied in order to necessitate a change in α, although this is not important once the results are normalised with respect to α and β. The maximum critical load for the column converged to within an accuracy of better than 0.1 per cent of the ultimate squash load for the column, so that the error in β is approximately 0.1 per cent.

3. Results

3.1 Variations in Column Effective Length Factors for Rigid Connections

Comparing the results obtained from this work and those calculated using BS 8110 equations (see Appendix), Fig. 3 presents the results for the variation in β with α assuming fully rigid connections. Note that in the case of sub-frame F1, $\alpha_1 = \alpha_2$, where α_1 and α_2 are the relative stiffnesses of the column to the lower and upper beams, respectively. In sub-frame F2, $\alpha_1 = 0$ because the foundation is rigid. There is no equation in BS 8110 to deal with sub-frame F3.

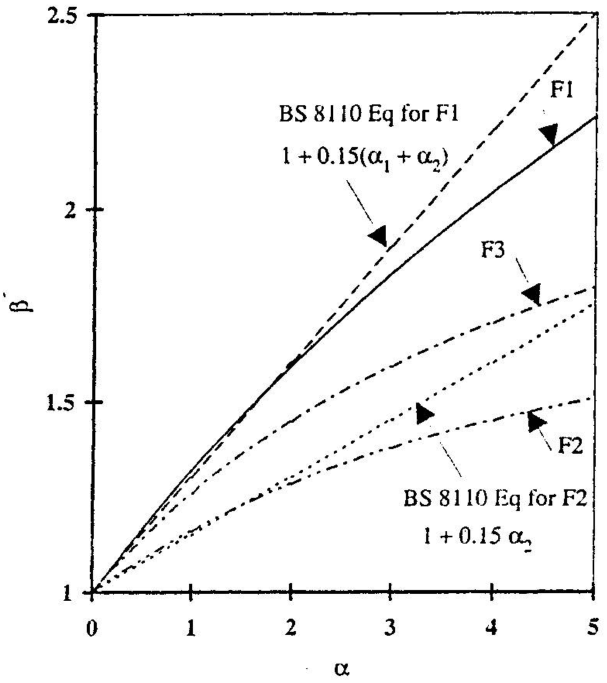

Fig. 3. Variation in column effective length factor β with frame stiffness α for rigid joints.

The results in Fig. 3 show that the code equations are in good agreement with analytical results for $0 < \alpha < 2$, and conservative thereafter. It is postulated that an equation for sub-frame F3 may be taken as the mean of the equations for F1 and F2. The results suggest that the code equations might be modified for values of $\alpha > 3$.

3.2 Variations in Column Effective Length Factors for Semi-Rigid Connections

Figs. 4 and 5 show the results for the variations in β with K_s for selected values of α in the upper storey sub-frame F1. Although a mapping function is required to demonstrate the full parametric variations, the three selected values for α show the trends clearly. The results in Fig. 4 show that for values of $K_s > 2$ or 3 the change in β is no more than about 5 per cent of its fully rigid value. For this reason Fig. 5 is an enlargement of Fig. 4 for values of $K_s < 2$. The dashed lines show the plots of the proposed parametric equations given in Section 3.3.

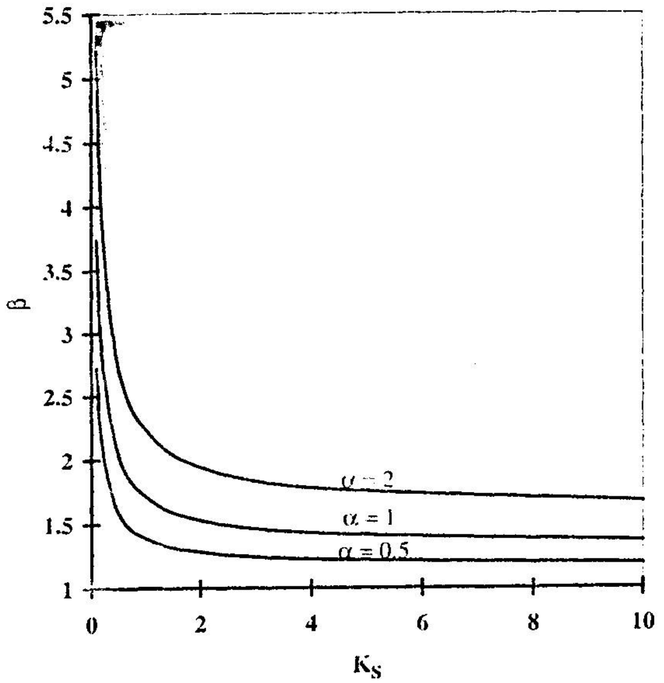

Fig. 4. β *factors vs* $K_s \leq 10$ *for selected values of* α *in sub-frame F1.*

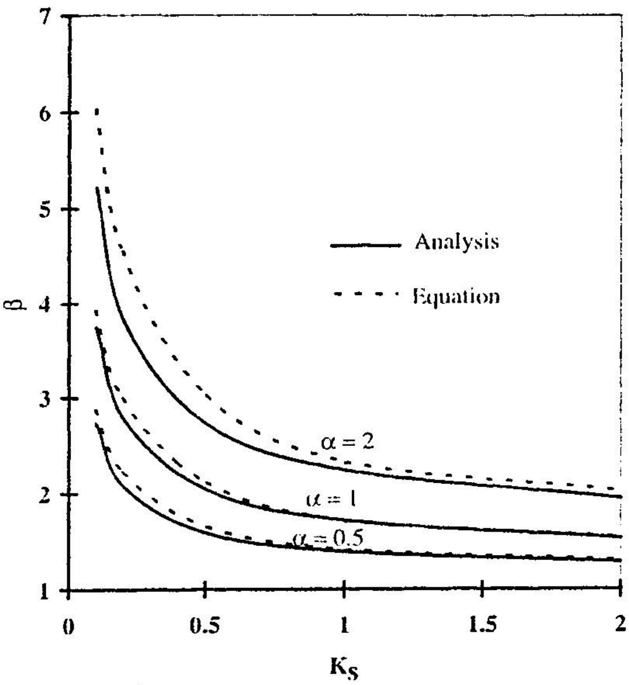

Fig. 5. β *factors vs* $K_s \leq 2$ *for selected values of* α *in sub-frame F1.*

Figs. 6 and 7 show similar sets of results for the same selected values of α for sub-frames F2 and F3, respectively. Results are presented only for values of $K_s \leq 2$ for the reasons outlined above. As expected the values of β in the ground floor sub-frame F2 converge at $\beta = 2.0$, and are independent of α. The corresponding value in sub-frame F3 is $\beta = 2.7$. A major difference between the upper floor (F3) the ground floor (F2) sub-frames is the more rapid decrease in β with K_s in the upper floor sub-frame. This is because F3 contains four semi-rigid connections,

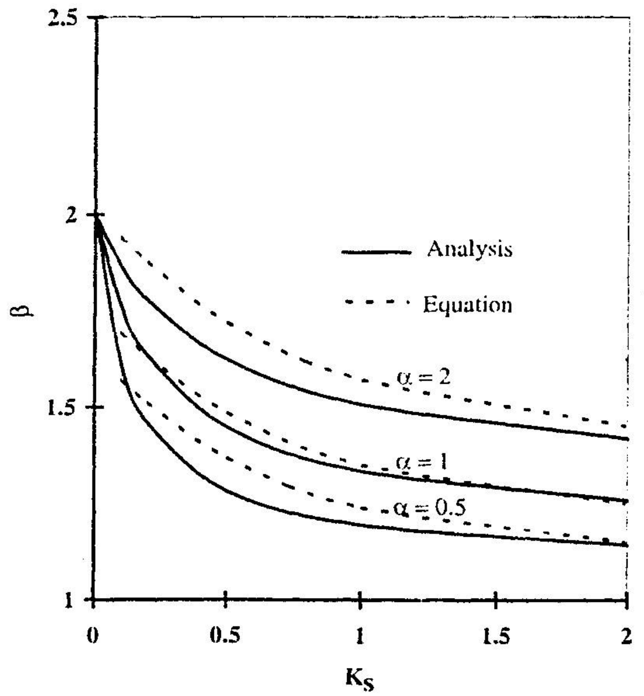

Fig. 6. β *factors vs* $K_s \leq 2$ *for selected values of* α *in sub-frame F2.*

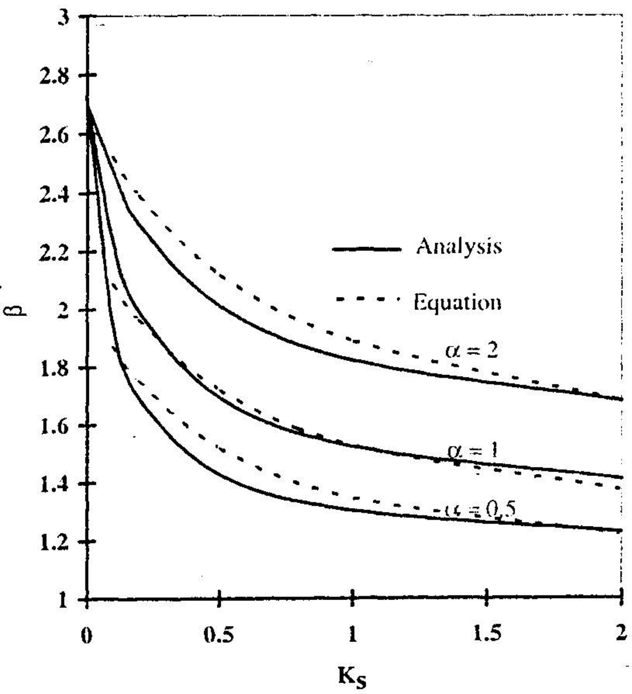

Fig. 7. β *factors vs* $K_s \leq 2$ *for selected values of* α *in sub-frame F3.*

(although one of them is located adjacent to a rigid column foundation) whereas F2 contains only two. Also, F3 has eight degrees of freedom whereas F2 has only six. This result has obvious implications for frameworks containing a small number of bays in the plane of bending, say 2 or 3, where the number of semi-rigid connections is disproportionately large to the number of columns. The variation in α does not appear to have any major influence on the behaviour of the various sub-frames once the effects of changes in K_s have been removed.

3.3 Parametric Equations

Subtracting the value of 1.0 from all the data and normalising the results with respect to α, the variation in $1/\beta$ with K_s is primarily linear and marginally quadratic. A simple analysis of a right angled knee-joint (comprising one beam and one column connected by a semi-rigid rotational spring) will show that the effect of the semi-rigid connection is to modify the relative stiffness of the members from α to α' where

$$\alpha' = \alpha\left(1 + \frac{1}{K_s}\right) \qquad [1]$$

For example if $\alpha = 0.5$ and $K_s = 0.6$ (say), then the effect of incorporating a semi-rigid connector is to increase the apparent stiffness of the column to $\alpha' = 1.33$, thus increasing β according to the results in Fig. 3. Thus the influence of the connector stiffness K_s is paramount in the present parametric equations, whilst that of α is of lesser influence over the range studied. The influence of K_s on β is greater for values of $K_s < 2$ than when $K_s > 2$, and therefore separate equations are presented to cater for the differences in behaviour at these points.

Referring to Figs. 4 and 5, the data for the upper storey sub-frame F1 may be approximated by using the following empirical relationship:

$$\beta = 1 + \frac{1}{0.2 + 10.0K_s} + \frac{\alpha}{0.3 + 1.8K_s - 0.45K_s^2} \quad \text{for } 0.1 < K_s \le 2 \qquad [2]$$

$$\beta = 1.1 + \frac{1}{7.4 + 7.4K_s - 0.4K_s^2} + \frac{\alpha}{1.6 + 0.3K_s} \quad \text{for } 2 \le K_s \le 10 \qquad [3]$$

Thus, $\alpha = 0.5$ and $K_s = 0.6$ for example, equation [2] gives $\beta = 1.50$. If the value for the equivalent stiffness from Eq 1. ($\alpha' = 1.33$) is used in the BS 8110 equation, then $\beta = 1.40$. This shows that equating a semi-rigid connection to a rigid connection in an equivalent frame under estimates β for these particular parameters.

Referring to Fig. 6, the data for the ground floor sub-frame F2 may be given as:

$$\beta = 1 + \frac{1}{2.0 + 2.0K_s + 4.0K_s^2} + \frac{\alpha}{4.0 + 0.5K_s} \quad \text{for } 0.1 < K_s \le 2 \qquad [4]$$

And for values of $K_s > 2$ not presented in the figures:

$$\beta = 1 + \frac{1}{8.6 + 8.4K_s - 0.4K_s^2} + \frac{\alpha}{3.9 + 0.9K_s} \quad \text{for } 2 \le K_s \le 10 \qquad [5]$$

Referring to Fig. 7, the data for the upper storey sub-frame F3 may be given as:

$$\beta = 1 + \frac{1}{1.25 + 2.5K_s + 2.5K_s^2} + \frac{\alpha}{2.25 + 0.5K_s} \quad \text{for } 0.1 < K_s \leq 2 \qquad [6]$$

And for values of $K_s > 2$ not presented in the figures:

$$\beta = 1 + \frac{1}{6.5 + 5.6K_s - 0.3K_s^2} + \frac{\alpha}{2.7 + 0.3K_s} \quad \text{for } 2 \leq K_s \leq 10 \qquad [7]$$

4. Discussion

It has been established that where column effective length factors β are determined within a structural framework, the nature of that framework and its boundary conditions will influence the results. All the results show an increase in β with:

i) an increasing number of degrees of freedom, and an increasing number of connections per sub-frame
ii) an increase in α
iii) a decrease in K_s.

In the context of precast concrete frame connections, where full scale experimental results indicate values of K_s between 0.2 and 2.0 [5,6,7] it is significant that for values of $K_s < 2$ the influence of connection stiffness on β is much greater than that of the relative stiffness of the frame members, particularly in sub-frame F1 where all connections are semi-rigid (see Fig. 4 and eq. 2 and eq. 3). In the sub-frames comprising at least one rigid foundation (i.e. F2 and F3) the variation in β with K_s and α is about equal for $K_s < 1$, and more dependent on α for $K_s > 1$. It is therefore concluded that maximum benefit in obtaining reductions in β with greater connection stiffness will accrue in upper storey sub-frames where $K_s < 1$, and in the ground floor sub-frame where $K_s < 0.5$.

The results obtained for the upper storey in the partially braced sub-frame F3 are of particular interest to designers because the boundary conditions for the column which is not adjacent to a shear wall is unspecified in codes of practice. Treating the column alone would lead to very high β factors and an impossible design situation (which can be appreciated from the design rules given in the Appendix). A pinned jointed frame can be idealised as shown in Fig. 8. In Fig. 8[a] the deflected profile of a column held in position but not in direction at level N, and a free cantilever above this level will have a β factor of at least 3.0 (assuming equal storey heights). However the true manner of slenderness induced deflections would be as shown in Fig. 8[b] where the effective length of *all* columns is 2.7. The restoring force in the beam is small but very significant in terms of frame stability. Bending moments resulting from sway in the unbraced part are carried over into the braced part of the frame, diminishing to zero with distance to the level of the floor below, such that β for the columns in the lower braced regions may be taken as 1.0.

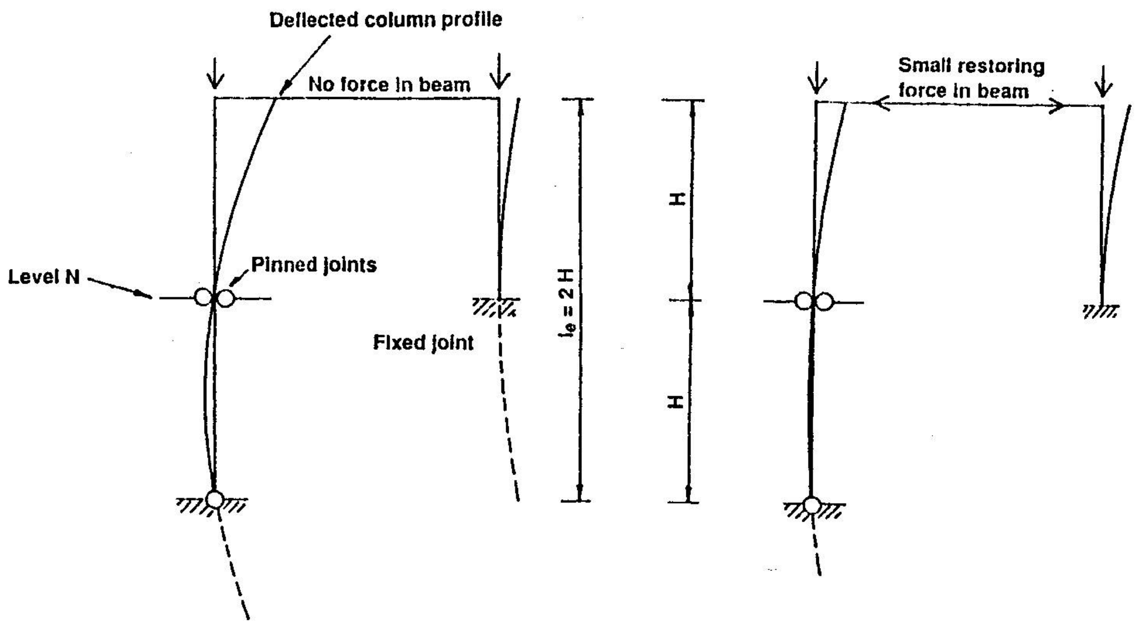

Fig. 8. Behaviour of a partially braced frame, (a) discrete column deflection profiles (left), (b) column deflection profiles in a frame environment (right).

5. Conclusions

Frame stability analyses on three types of one-storey x one-bay sub-frames in *unbraced* and *partially braced* precast concrete skeletal frames have shown that column effective length factors β increase due to:

i) an increasing number of total degrees of freedom at the connections in the ends of the beams. This is a direct measure of the number of semi-rigid connections per column, and is influenced by the location of the connections in the sub-frame;

ii) an increase in α, the relative stiffness of the columns to the beam members;

iii) a decrease in K_s, the relative stiffness of the connection to a fully encastre beam member. The influence of K_s on β is considerably greater for values of $K_s < 2$ than when $K_s > 2$.

Parametric equations have been presented for the variations in β with K_s and α. There are significant differences in the equations for values of K_s less than or greater than 2. For $K_s > 2$ the change in β is no more than about 5 per cent of its fully rigid value.

The results enable designers to determine β factors for situations currently not catered for in codes of practice, in particular the upper storey in a partially braced frame.

References

1. Cranston, W. Analysis and Design of Reinforced Concrete Columns, Research Report 20, .Cement & Concrete Association, Wexham Springs, Aug. 1972.

2. Wood, R. H. Effective lengths of columns in multi-storey buildings, Parts 1, 2 and 3, The Structural Engineer, Vol. 52, Nos. 7, 8 and 9, July, Aug., Sept. 1974.

3. Timoshenko, S. and Gere, J. M. Theory of elastic stability, McGraw-Hill, 2nd. ed., 1961.

4. British Standards Institute, Structural Use of Concrete, BS 8110, London, 1985.

5. Elliott, K. S., Davies, G. and Mahdi, A. A. Semi-rigid joint behaviour on columns in precast concrete buildings, COST C1 Proceedings of the 1st Workshop, Semi-rigid Behaviour of Civil Engineering Structural Connections, E.N.S.A.I.S., Strasborg, Oct. 1992, p 282 - 295.

6. Görgün, H. Semi-rigid behaviour of connections in precast concrete structures, Dept. of Civil Engineering Report No. SR 96-001, University of Nottingham, Feb. 1996, 344 pp.

7. Mahdi, A. A. Moment rotation effects on the stability of columns in precast concrete structures, PhD thesis, University of Nottingham, 1992.

8. Aksogan, O. and Görgün, H. The nonlinear analysis of planar frames composed of flexurally connected members, Cukurova University Journal, Faculty of Engineering and Architecture, Vol. 8, No. 2, Dec. 1993, p117-129.

Appendix. Design Rules in BS 8110 for Columns in Unbraced Frames

β factors in Part 2, clause 2.5 for columns in frames may be taken as the lesser of:

$$\beta = 1.0 + 0.15\,(\alpha_{c1} + \alpha_{c2})$$
$$\beta = 2.0 + 0.3\,\alpha_{cmin}$$

where α_{c1} = *ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the lower end of a column,*

α_{c2} = *ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the upper end of a column,*

α_{cmin} = *lesser of* α_{c1} *and* α_{c2}

In the calculation of α_{cmin}, α_{c1} and α_{c2}, only members properly framed into the end of the column in the appropriate plane of bending should be considered. In specific cases of relative stiffness the following simplifying assumptions may be used:

(b) simply-supported beams framing in to a column: α_c to be taken as 10;

(c) connection between column and base designed to resist only nominal moment: α_c to be taken as 10;

Deflection induced moments in solid slender columns, BS 8110, Part 1, clause 3.8.3.

... account has to be taken of the additional moment induced in the column by its deflection.

This may be taken as $M_{add} = N(\beta l_o/b)^2 K h/2000$, *where* N = *ultimate axial load*, l_o = *clear height*, K = *a reduction factor that corrects the deflection to allow for the influence of axial load*, h = *overall depth of a column in the plane considered, and* b = *smaller dimension of a column.*

Acknowledgement

The authors wish to thank EPSRC and The Turkish Government for financial support. The computer program was developed with the assistance of Professor O Aksogan at Cukurova University, Turkey.