Ultimate strenght of composite structures using superposed strength method

Ultimate Strength of Composite Structures Using Superposed Strength Method

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Summary

A method to obtain the ultimate strength and the collapse mechanism of composite frames, using the generalized superposed strength method which combines the yield and bond slip conditions of the steel and concrete with the associated flow rule. The collapse load and the collapse mechanism, considering the axial and rotational deformations at the plastic hinges and the bond slip deformation at the column base, differ from those estimated by the simple plastic analysis. The diaphragm effect of the concrete slabs changes the collapse mode from overall to local side sway mechanism.

1. Introduction

In the design of composite structures in Japan, the superposed strength method has been employed. The author already presented a method to obtain the generalized superposed strength by combining the yield condition of composite members with the associated flow rule, and showed that the method can be applied in the case when the bond between a concrete and a steel is lost and slip deformation occurs.[1.2] This paper presents a method to obtain the ultimate strength of the composite frames under the vertical and horizontal forces considering the yield and slip conditions of the composite members, and to obtain the collapse mechanism considering the plastic and slip deformations derived from the associated flow rule.

2. Ultimate Strength of Composite Members

Fig.1 shows a composite member having a rectangular cross section with an ideal I-section steel. The cross section is idealized into three points model such as shown in the figure. If the stress vs. strain relationship is assumed to be rigid-perfectly plastic for both concrete and steel, such as shown in Fig.2, the yield condition of the steel is a diamond shape and the condition of the concrete is a hexagonal shape, under the axial force and bending moment, such as shown in the Fig.3 (a). The associated flow rule shows that the ratio between the axial elongation e and the rotation ω (Fig.3(c)) is constant on each ridge line of the yield surface such as shown in the Fig.3(b). By summing the strengths of the two materials so that the deformation increments of the elements coincide, the generalized superposed strength can be obtained. According to the flow rule, plastic deformation increment can be given as

$$e = -(D/3)\omega \qquad for \quad -{}_sN_o \le N \le {}_cN_o/3 \qquad (1a)$$

$$e = 0 \qquad for \quad {}_cN_o/3 \le N \le 2{}_cN_o/3 \qquad (1b)$$

$$e = (D/3)\omega \qquad for \quad 2{}_cN_o/3 \le N \le {}_cN_o + {}_sN_o \qquad (1c)$$

where ${}_sN_o = 2{}_s\sigma_o A_f$ and ${}_cN_o = {}_c\sigma_o BD$.
If the anchorage of the steel member is not perfect, slip deformation between the steel and

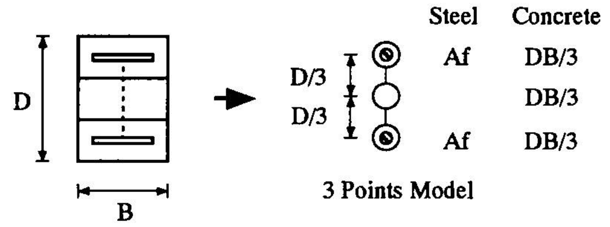
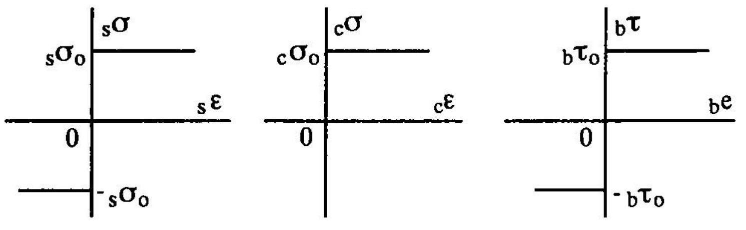
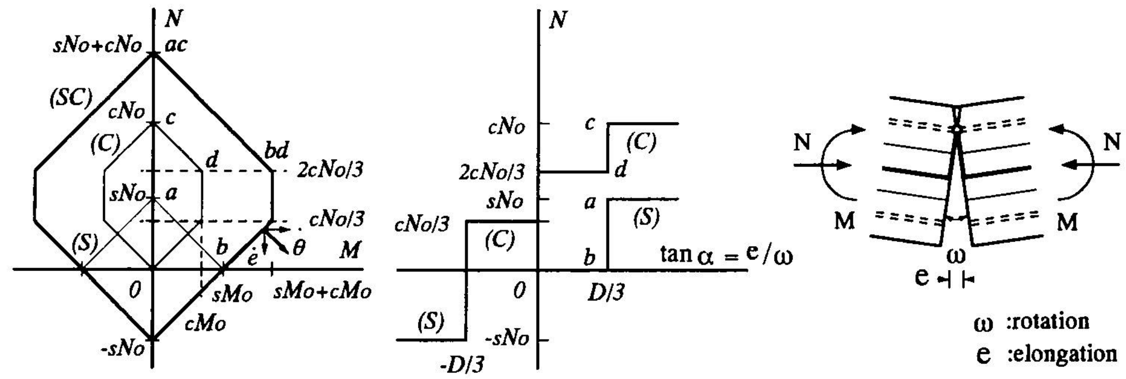


Fig.1 Model of composite cross section



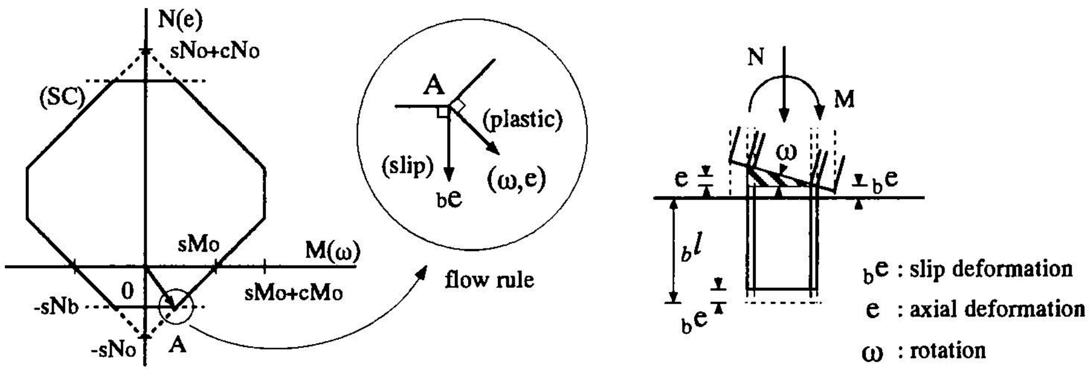
(a) Steel (b) Concrete (c) Bond

Fig.2 Stress-strain and bond stress-slip displacement relations



(a) Yield Condition (b) Flow Rule (c) Plastic Hinge

Fig.3 Yield condition and associated flow rule



(a) Yield and Slip Conditions (b) Plastic and Slip Deformations

Fig,4 Yield and slip conditions and deformation mode concrete can occur. Using the bond stress vs. slip deformation relation shown in Fig.2(c), the yield and slip conditions considering the bond slip is obtained such as shown in Fig.4. The bond strength of the steel ${}_sN_b$ can be given as

$${}_sN_b = {}_b\tau_b \, {}_bl \, {}_bS \tag{2}$$

where ${}_bl$ is a development length of the steel and ${}_bS$ he circumferential length of the steel cross section. At the stress state shown as the vector OA, both plastic and slip deformations can occur. Fig.4(b) shows a deformation mode of a column base when both plastic (ω, e) and slip deformations (${}_be$) occur.

3. Ultimate Strength and Collapse Mode of Composite Structures

3.1 Design of Composite Frames

The composite frames subjected to the vertical loads and the lateral forces are designed to collapse by forming the plastic hinges at the beam ends and the column bases (overall collapse mechanism), using the simple plastic analysis. Lateral loads are specified in conformance with the Building Standard Law of Japan, assuming that the base shear coefficient is equal to 0.3. For the sake of simplicity, we assume that the cross sections of the beams are the same ($D_b/2 x D_b$) and the cross sections of the columns are also the same ($D_c x D_c$), such as shown in Fig.5. The magnitude of the plastic moment of each member is adjusted by the cross sectional area of the formed steel.

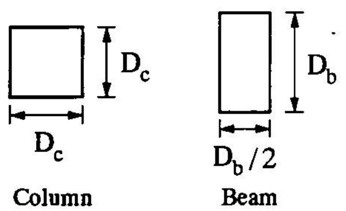


Fig.5 Beam and column cross section

3.2 Portal Frame

A portal frame under the vertical load W and the lateral load 0.3W is shown in Fig.6(a), and the location of plastic hinges obtained by the simple plastic analysis and the plastic moment of each member are shown in Fig.6(b). As the axial load levels of the columns and beams are low, the axial elongation and the rotation are supposed to occur at each plastic hinge. The collapse mechanism of the frame is shown in Fig.7(a). In the parentheses, the rotation angles of the plastic hinges are shown. The equation of virtual work is given as

$$\begin{aligned} &\alpha 0.3Wh(\omega_1 + \omega_2) - W(\omega_1 + \omega_2)(D_c/3) \\ &= 0.18Wh(\omega_1 + \omega_2) + 0.12Wh(\omega_1 + \omega_2 + 2\omega_3) \end{aligned} \tag{3}$$

where α show the collapse load factor. Assuming that the aspect ratios of the columns (D_c/h) and a beam (D_b/l) are equal to 10, the collapse load factor is calculated as

$$\alpha = 1.1120 \tag{4}$$

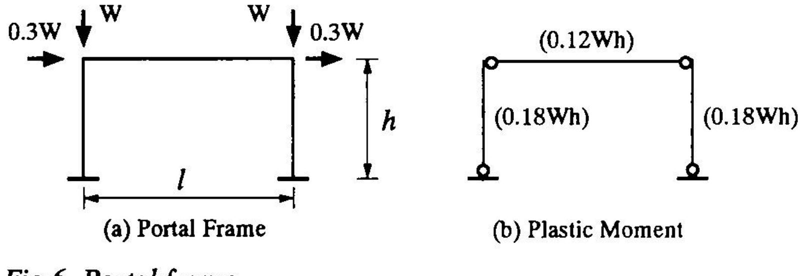


Fig.6 Portal frame

The relations between the rotation angle of the members are given as

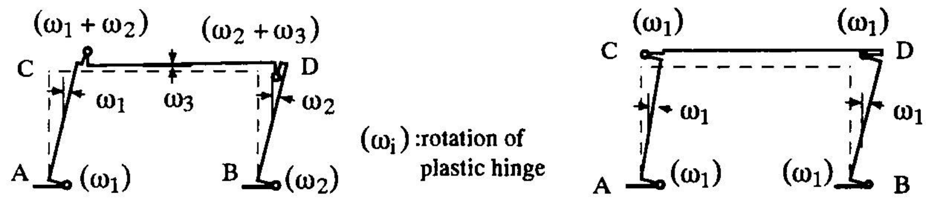
$$\omega_1 = [1 - 2(D_b/3h)/K_1]\omega_2 \quad (5a)$$

$$\omega_3 = (D_c/3l)(\omega_3 - \omega_1) \quad (5b)$$

where $K_1 = 1 + (D_b/3h)\{1 - (2D_c/3l)\}$.

Equation (5a) shows that the rotation angle ω_1 is smaller than ω_2, due to the elongation at the plastic hinges of the beam.The composite frames usually have a concrete slab. If the diaphragm effect of the concrete slab is perfect, the elongation of the beam is prevented. The collapse mechanism in this case is shown in Fig.7(b). The plastic hinges are formed at both ends of the columns. The collapse load factor is calculated as

$$\alpha = 1.4667 \quad (6)$$



(a) Collapse Mechanism without Slab Diaphragm Effect (b) Collapse Mechanism with Slab Diaphragm Effect

Fig.7 Collapse mechanisms of portal frame

3.3 Multi-story Frame

Fig.8(a) shows a three-story frame under the vertical and horizontal loads. According to the simple plastic analysis, the plastic hinges are formed at the beam ends and the column bases. Considering the elongation as well as the rotation at the plastic hinges, the collapse mechanism, such as shown in Fig.8(b), can be obtained . In this case, the additional plastic hinges are formed at the lower end of the left columns. The collapse load factor is calculated as

$$\alpha = 1.0522 \quad (7)$$

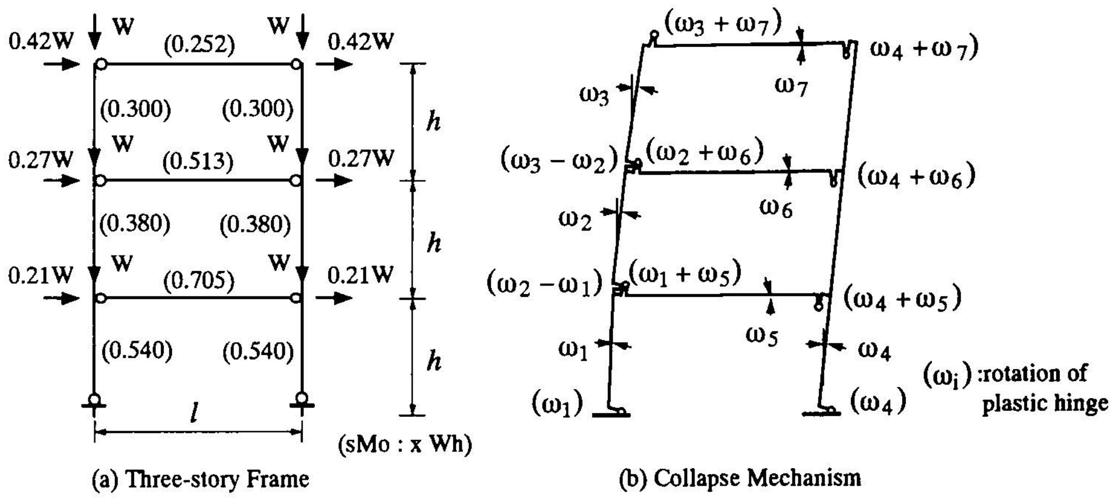
The relations between the rotation angle of the columns are given as

$$\omega_1 = [1 - 2(D_b/3h)/K_1]\omega_4 \quad (8a)$$

$$\omega_2 = [1 - 2(D_b/3h)^2\{1 - (2D_c/3l)\}/K_1^2]\omega_4 \quad (8b)$$

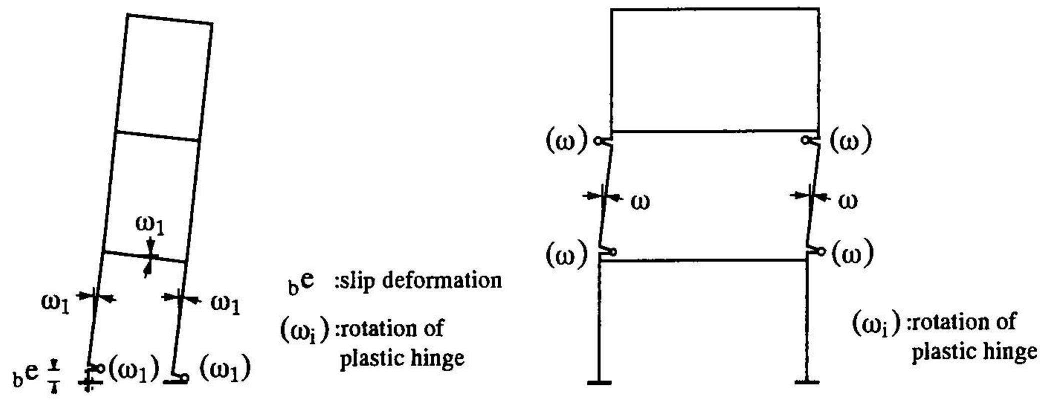
$$\omega_3 = [1 - 2(D_b/3h)^3(1 - (2D_c/3l)^2/K_1^3]\omega_4 \quad (8c)$$

where $K_1 = 1 + (D_b/3h)\{1 - (2D_c/3l)\}$.



(a) Three-story Frame (b) Collapse Mechanism

Fig.8 Three-story frame and the collapse mechanism



(a) Collapse Mechanism Considering Slip Deformation (b) Collapse Mechanism with Slab Diaphragm Effect

Fig.9 Collapse mechanisms of three-story frame

The difference between the rotation angle of the left and right columns decreases in geometric progression as the number of storys increases, showing that the deformation of the plastic hinges at the lower end of the left columns decreases as the story number increases.
In the case of a slender frame whose beam span (l) to column height (h) ratio is equal to 0.5, tension force acts at the left column base. If the anchorage of the steel is not perfect, the slip deformation as well as the plastic deformation may occurs at the column base. The collapse mechanism in this case is shown in Fig.9(a). The plastic hinges are formed only at the column bases, and the sip deformation is observed at the left column base. The collapse load factor in this case is calculated as

$$\alpha = 0.9938 \tag{9}$$

when the bond strength of the steel at the column base is given as ${}_sN_b = 0.15\,{}_sN_o$. The result shows that collapse load is smaller than the load obtained using the simple plastic analysis.
Consider again the frame shown in Fig.8(a). If the diaphragm effect of the concrete slab is perfect, the elongation of the beams is prevented. The flow rule shows that the bending strength of the beams increases up to ${}_sM_o + {}_cM_o$, when only the rotational deformation is permitted, as shown in Fig.3. The collapse mechanism in this case is shown in Fig.9(b). Only the side sway mechanism of the second story occurs. The collapse load factor is calculated as

$$\alpha = 1.1981 \tag{10}$$

3.4 Multi-bay Frame

Fig.10 shows a one story multi-bay frame. According to the simple plastic analysis, plastic hinges are formed at the beam ends and the lower end of the columns. Considering the elongation and rotation of the plastic hinges, the collapse mechanism such as shown in Fig.11 can be obtained. The rotation angle of the m-th column can be obtained as

$$\omega_m = \{1 + (2D_b / 3h)/K_3\}^{m-1}\omega_1 \tag{11}$$

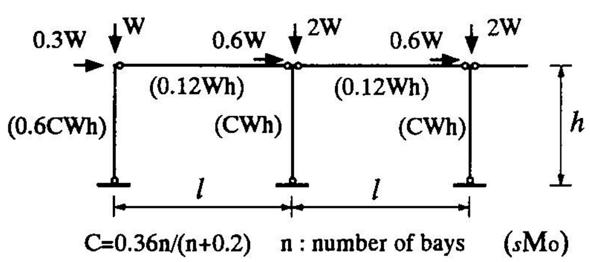


Fig.10 One-story n-bay frame

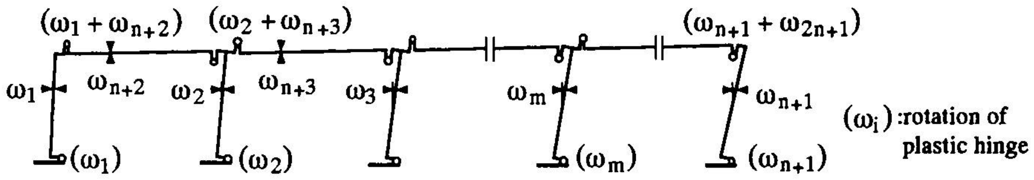


Fig.11 Collapse mechanism of n-bay frame

where $K_3 = 1-(D_b/3h)\{1+(2D_c/3l)\}$.
The rotation angle of the column increases in geometric progression as the number of bays increases, due to the elongation of the beams. Assuming that $l/D_b = 10$ and $h/D_c = 10$, the relation between the rotation angle ω_1 and ω_n is obtained as

$$\omega_n = 1.14354^{n-1}\omega_1 \qquad (12)$$

When the number of bays is n=6, Equation(12) gives $\omega_n = 1.9955\omega_1$, showing that the rotation angle of the right side column is twice of the left side column.

4. Conclusions

This paper presented a study for obtaining the strength and collapse mechanism of the composite frames, considering the axial and rotational deformations at the plastic hinges. Major findings obtained from this study are as follows:

(1) The collapse load and collapse mechanism of the composite frames differs from that estimated by the simple plastic analysis, due to the axial and rotational deformations at the plastic hinges. Except for one story frames, additional plastic hinges should be formed at some column ends.

(2) The axial and rotational deformations at the plastic hinges differ from one hinge to another. The relation between the rotational deformations of the plastic hinges at the column ends can be given in geometric progression form, as the number of bays or number of storys increases.

(3) When the diaphragm effect of the concrete slab is perfect, the collapse mode of the composite frames changes from overall side sway mechanism to local (one story) side sway mechanism.

References

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[2] Tsuji,B. and Wakabayashi,M.: Ultimate strength of composite beam-columns and beam -column joints using superposed strength method, Proc. of plasticity'95, 1995, pp.467-470.