# Optimisation of life-cycle costs of concrete structures

# Optimisation of Life-Cycle Costs of Concrete Structures

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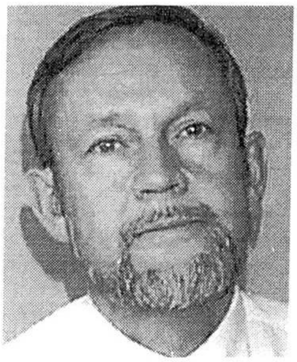

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## Summary

In the standards and design recommendations, required service life for new bridges is long, often 120 years or longer. It is also specified how the design parameters shall be chosen to reassure that the expected service life is achieved. The authors of this paper do not believe that this approach is economical for countries with a high real rate of interest. Instead, if the life-cycle costs and expected service lives for different designs are calculated, then the optimal design according to minimum annuity cost should be chosen. This approach also simplifies introduction of new materials or new designs. In this paper, an optimisation example is presented, where design parameters for an ordinary reinforced concrete bridge deck is optimised regarding to minimum annuity cost. A high concrete quality, made of Swedish high quality cement, is the most economical in this example. Other concrete mixtures would produce other results.

## 1. Introduction

Of tradition, research within structural engineering has been separated from economy. The economical consequences of improvements, innovations et cetera have been undertaken. Especially within the field structural measures to lengthen the service life of structures, in combination with the life cycle cost of the structure, very little research is done.

Nowadays it is not difficult to build and maintain bridges with respect to long service lives; but it is expensive. Nowadays, design recommendations often prescribe service lives of about 120 years or longer, and it is also specified how to choose design parameters, as for example concrete quality, concrete cover and maximum crack width, to achieve this long service life, and often with a safety marginal. These design specifications has led to higher investment costs, and if this approach really is economical for the nation has not been examined - the costs for the longer service lives have been of lower interest.

A long service life does not guarantee a good economy. Investigations show, that only about ¼ of replaced bridges are replaced because of structural deficiency /1/. To provide cost effective concrete structures, the design parameters should be chosen with respect to minimum annuity cost. It is therefore necessary to predict the expected service life for different designs of a concrete structure, already in the design phase. Then the life cycle cost, LCC, for the different alternatives can be calculated and distributed over the service life to an annuity cost.

## 2. Optimal durability

An insufficient durability leads to costs for repair, rehabilitation or replacement in a too close future. The higher durability of a structure, the higher investment costs, but also, the cost due to insufficient durability decreases. Optimal durability will be found at minimum total cost, Fig. 1.

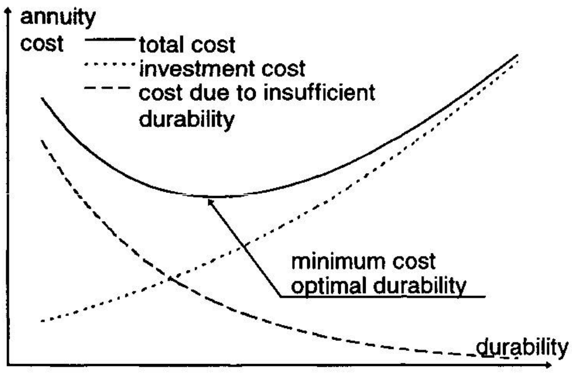


*Fig. 1 Optimal durability occurs at minimum total cost, the sum of investment cost and cost due to insufficient durability.*

A low durability, that is a short service life, causes a high annuity cost because the annuity factor increases rapidly for short service lives.

To design structures with low annuity cost, it is necessary to connect the deterioration model to the life cycle cost of the structure. The common factor in the deterioration model and the economical model is time. This enables economical design with interaction formulae. Therefore, it is important to base the design recommendations on functional demands, not specific minimum values for each single parameter. It is also important to bring useful equations for durability calculations into the design process.

## 3. Optimisation example

### 3.1 Description of the problem

In this paper, an example of how design parameters can be chosen for optimal durability is presented, some other examples will be presented in /2/. The structural design of an ordinary reinforced concrete bridge deck is optimised regarding to minimum annuity cost. Deterioration of the bridge is caused by reinforcement corrosion due to chloride diffusion. The chloride environment is aggressive, and it is supposed to correspond to de-icing salted environment. There are no beams or other bearings, the deck is carried only by itself. Span length varies between 5 and 20 m. The bridge carries an external load of 30 kN/m$^2$ but also its own weight and the weight of the pavement. Minimum tolerable concrete cover is chosen to 20 mm.

The ordinary form of an optimisation problem is:

- Given: constant parameters
- Find: design variables
- Optimise (minimise): objective function
- Satisfy: design constraints

The objective function to be minimised, is the annuity cost. The design parameters are: total height of the section, concrete cover, water to cement ratio and reinforcement area. Four constraints are given; minimum load bearing capacity in ultimate limit state, balanced section control, maximum crack width (0.4 mm) and maximum mid span deflection in the middle of the span (span length/400).

The optimisation results in a spectra of design parameters that, depending on span length and rate of interest, yield the optimal design. The partial coefficient method is used for capacity and deflection calculations. In structural design of bridges also other demands, like shear forces and accidental loads, are important. However, the optimisation problem, presented in this paper, only regards durability, loads in ultimate limit state, and deformation caused by the traffic. In an extended calculation, also other demands may be included.

The used optimisation method is the Method of Moving Asymptotes, MMA, which is a convex approximation method for structural optimisation /3/. MMA is an iterative method. In each iteration, a convex sub problem which approximates the original problem, is generated and solved. An important role in the generation of these sub problems is played by a set of parameters which influence the "curvature" of the approximations and also act as "asymptotes" for the sub problem. By moving these asymptotes between each iteration, the convergence of the overall process can be stabilised.

### 3.2 Annuity cost

For investments with equal service lives, optimal design can be chosen based on minimum life cycle cost, *LCC*. If service lives differ, a comparison between annuity costs is more suited. The annuity cost can be calculated by eq. 1 where *LCC* is the life cycle cost, discounted to present value, $F_A$ is the annuity factor, $B_n$ is the sum of all costs and benefits in year $n$, $r$ is the real discount rate, real interest rate calculated for costing purposes, and $N$ is the service life.

$$A = LCC \cdot F_A = \sum_{n=0}^{N} \frac{B_n}{(1+r)^n} \cdot \frac{r}{1-(1+r)^{-n}} \qquad (1)$$

In economical calculations, the discount rate is very important. A high discount rate favours shorter service lives. The discount rate can be chosen in different ways, where the interest rate an alternative investment can bring, or bank rate for a loan, are the most common. For cost-benefit analyses a discount rate can be calculated out from the financial situation for the society today compared to tomorrow. The discount rate may also be politically decided, as for example in Sweden, 4 % is recommended for cost benefit analyses within the transport sector. The discount rate in the following example considers real interest rate calculated for costing purposes, and varies between 2 % and 12 %.

### 3.3 Service life calculations

During the last decades, concrete durability has been a highly prioritised research area. Technical service life of a structure is usually divided into initiation time and propagation time /4/. The initiation time used to be calculated with Fick's 2$^{nd}$ law of diffusion. However, later research has showed that both diffusion coefficient and chloride surface concentration are time dependent, and the classical solution of Fick's 2$^{nd}$ law has to be modified, se for example /5/. In de-icing salted environment, chloride surface concentration increases in winter period and decreases during the summer. At a distance of 20 to 30 mm from the surface, the relative humidity is almost constant during the year and from this distance and deeper, the chloride profile often looks similar to profiles in marine environment.

Still, there is not enough knowledge of concrete deterioration, to present equations that can be used by structural designers, in purpose to predict expected service life of concrete structures in different environments, already in the design phase. Therefore, in this paper, expected service life in de-icing salted chloride aggressive environment is calculated out from eq. 2, which is an approximation of the classical solution of Fick's 2$^{nd}$ law of diffusion by /6/. The relation between diffusion coefficient and water binder ratio obtained by /7/ is used.

$$t = \frac{x_{cr}^2}{12 \cdot D_{eff} \cdot \left(1 - \sqrt{C_{cr} / C_s}\right)^2} \qquad (2)$$

In eq. 2, is $t$ the initiation time for reinforcement corrosion, $x_{cr}$ is the distance between the concrete surface and the reinforcement, $D_{eff}$ is the effective diffusion coefficient, $C_{cr}$ is the chloride threshold value, and $C_s$ is the chloride concentration at the concrete surface, extrapolated from the chloride profile inside the concrete.

An example of measured chloride concentrations versus water binder ratio, in concrete made of ordinary Portland cement and with water reducer, is shown in Fig.2 /8/. The concentrations have been obtained from 15 to 40 years old outdoor structures. The relations between chloride concentrations and water binder ratio is likely to be exponential.

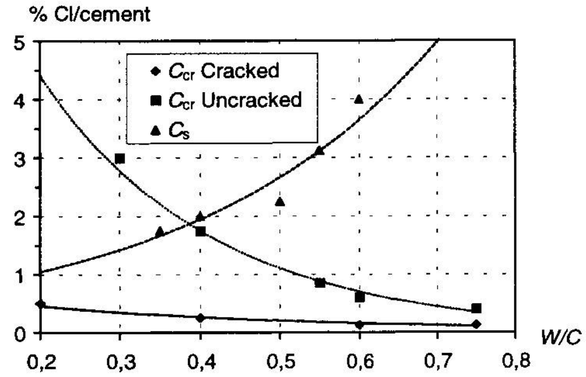


*Fig. 2 Measured chloride threshold values and maximum surface chloride concentrations by water binder ratio. Redrawn from /8/.*

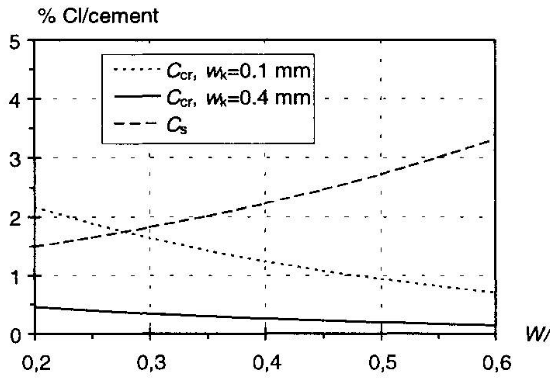


*Fig. 3 Chloride concentrations used in the calculations. Chloride aggressive environment.*

Other investigations show that the chloride threshold value is smaller than in Fig. 2, c.f. /9/. Therefore, in this paper, the chloride threshold value is supposed to be exponentially related to water binder ratio and linearly dependent on crack width as shown in Fig. 3. The chloride surface concentration is supposed to be exponentially dependent on water binder ratio, Fig. 3, and refers to de-icing salted environment is in this example.

### 3.4 Results

Optimisation of the problem results in a spectra of optimal design parameters that together yield the most economical design. It is important to remember that every single parameter is a part of the total design and can not be treated separately; it is the combination of the design parameters that is important. The parameters are chosen so that the total design is optimised.

The optimisation method MMA was easy to use, and optimal value was found already after 5-10 iterations. Optimal water to cement ratio became in all these examples as small as tolerable, here chosen to 0.35. Total height was independent on discount rates because of the deflection constraint; 255 mm for span length 5 m, 510 mm for 10 m, 764 mm for 15 m, and 1019 mm for span length 20 m.

I Fig. 4 to 7, results from optimisations for different discount rates and span lengths are presented. The objective function, Fig. 4, is the sum of concrete cost and reinforcement cost, including construction cost, and it is almost linear because of the long service lives achieved. Most important for the results were the discount rate and the interactions between chloride concentrations and water to cement ratio.

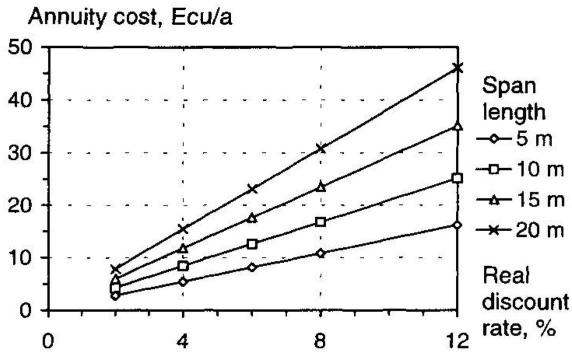


*Fig. 4 Objective function, annuity cost.*

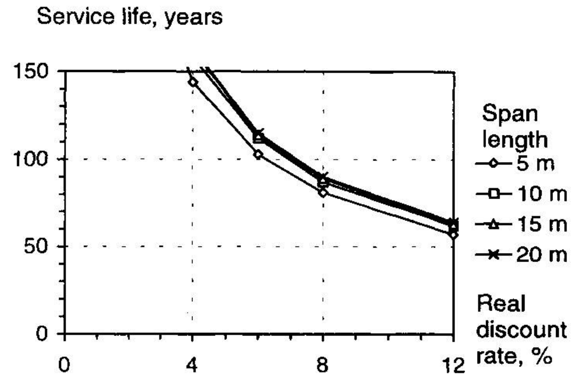


*Fig. 5 Optimal service life.*

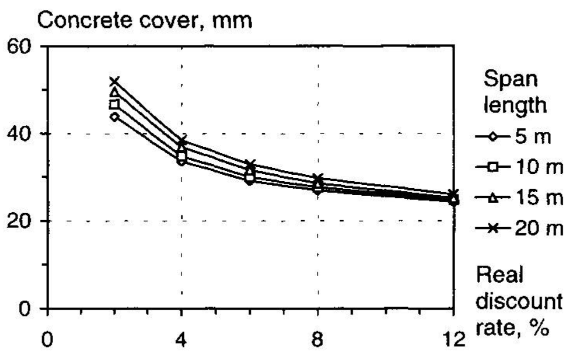


*Fig. 6 Optimal concrete cover.*

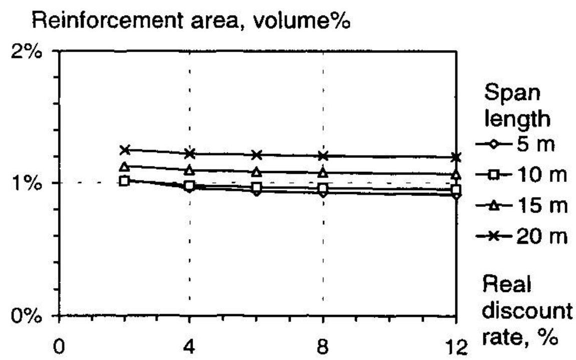


*Fig. 7 Optimal reinforcement area in % of concrete volume.*

In Fig. 5, optimal service life, related to discount rate and for various span lengths, is shown. The service lives for the optimised structures are long, about 60 years at discount rate 12 %, 80 years at discount rate 8 %, 110 years at discount rate 6 % and still longer at lower discount rates. This is mainly caused by the used relations between chloride threshold values respective chloride concentrations and water to cement ratio. Typical Swedish bridge concrete made of Swedish high quality cement withstands deterioration due to chloride ingress quite well. The cost for better concrete quality is rather low and the benefit exceeds the cost.

At small discount rates, concrete cover is about 50 mm in optimal design, but optimal concrete cover decreases with increasing discount rate, Fig. 6. Optimal concrete cover increases with increasing span length. The reinforcement amount is rather high, cf. Fig. 7, crack widths are limited, and with the assumed crack width approach, a high chloride threshold value can then be used.

In all optimisations, constraint number one, load bearing capacity in ultimate limit state, and constraint number four, maximum mid span deflection, was dimensioning. Constraint number two and three, balanced section control respectively maximum crack width, were not dimensioning in any case.

## 4. Conclusions

In this paper, the calculations show that economical optimisation of concrete structures in the design phase is possible. However, the result of an optimisation calculation is strongly dependent on input. The calculations are only intending to be examples of how to optimise structures with respect to economy, durability and structural design. Different concrete mixtures, environments and other conditions would of course produce other results. These examples are made for bridge concrete made of Swedish high quality cement with water reducer and air content adhesives. More research is needed to find better relations between concrete recipes respectively structural design and service life of concrete structures. However, the inputs to the calculations in this report are intended to be as close to reality as possible.

The example shows, that high concrete quality, normal concrete covers and small crack widths are economical if a higher threshold value then can be used. Optimum service life is long. However, the calculated service life is the mean service life, not calculated with safety marginal. In the calculations, the benefit of the bridge expects to be unchanged during the service life. Functional deficiency may decrease the benefit.

To build cost effective structures, the structural designer has to choose optimal design with respect to long term economy. If the expected service life for different durability alternatives can be calculated, and the maintenance and repair cost can be estimated, then the design corresponding to minimum annuity cost can be chosen. In this way, different designs can be economically compared, and introduction of new designs, new materials, or new construction methods will become easier.

## 5. References

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