# Heaving branch coupled flutter for long span bridge

# Heaving Branch Coupled Flutter for Long Span Bridge

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Summary

Flutter stabilization is the major subject to safely design a super long span bridge. Unsteady aerodynamic force, in other word, 8 aerodynamic derivatives, $(H_i^*, A_i^*, i=1\sim4)$ defined by R. H. Scanlan are significantly important for flutter occurrence. Especially, aerodynamic derivative, $A_2^*$, showing negative and small value, torsional flutter doesn't occur and coupled flutter stabilize. $A_2^*$ of the rectangular cylinder with vertical plate installed at the mid-chord point of rectangular cylinder with the slenderness ratio, B/D=20, showing negative and the smallest, this section doesn't have highest flutter onset velocity. Furthermore, heaving branch flutter occurs. This paper aims to clarify this mechanism of the heaving branch flutter and the relation between torsional brunch flutter and heaving brunch flutter.

## 1. Background of this study

The rectangular cylinder with vertical plate and the stable fundamental geometrical shape are shown in Fig.1[1]. Through wind tunnel test under the heaving/torsional 1DOF forced vibration, unsteady pressure around these sections is measured from pressure taps. 8 aerodynamic derivatives are shown in eq.1[2].

$$L = \frac{1}{2}\rho(2b)U^2\left\{kH_1^*\frac{\dot{\eta}}{U} + kH_2^*\frac{b\dot{\phi}}{U} + k^2H_3^*\phi + k^2H_4^*\frac{\eta}{b}\right\}$$

$$M = \frac{1}{2}\rho(2b^2)U^2\left\{kA_1^*\frac{\dot{\eta}}{U} + kA_2^*\frac{b\dot{\phi}}{U} + k^2A_3^*\phi + k^2A_4^*\frac{\eta}{b}\right\}$$

(1)

where, L,M:lift and pitching moment per unit length, $\eta, \phi$ :heaving and torsional displacement,U:wind velocity, $\rho$ :air density, b:half chord length(=B/2) k:reduced frequency (=b $\omega$/U), $\omega$ :circular frequency

These aerodynamic derivatives are calculated through the integration around sections. $A_2^*$ which is the damping term in torsional 1DOF vibration is shown in Fig.2[3]. This derivative is important factor of torsional flutter and coupled flutter. As shown in Fig.2, $A_2^*$ of rectangular cylinder with V.P. is negative and the smallest, therefore this section has the great advantage for torsional flutter and coupled flutter. Furthermore, aerodynamic unsteady force are expressed as shown in eq.1, flutter analysis is conducted. This result is shown in Fig.3[3].

This result indicates that flutter characteristics of rectangular cylinder with V.P. is inferior to that of flat diamond shape box section and flutter of rectangular cylinder with V.P. is heaving branch flutter.
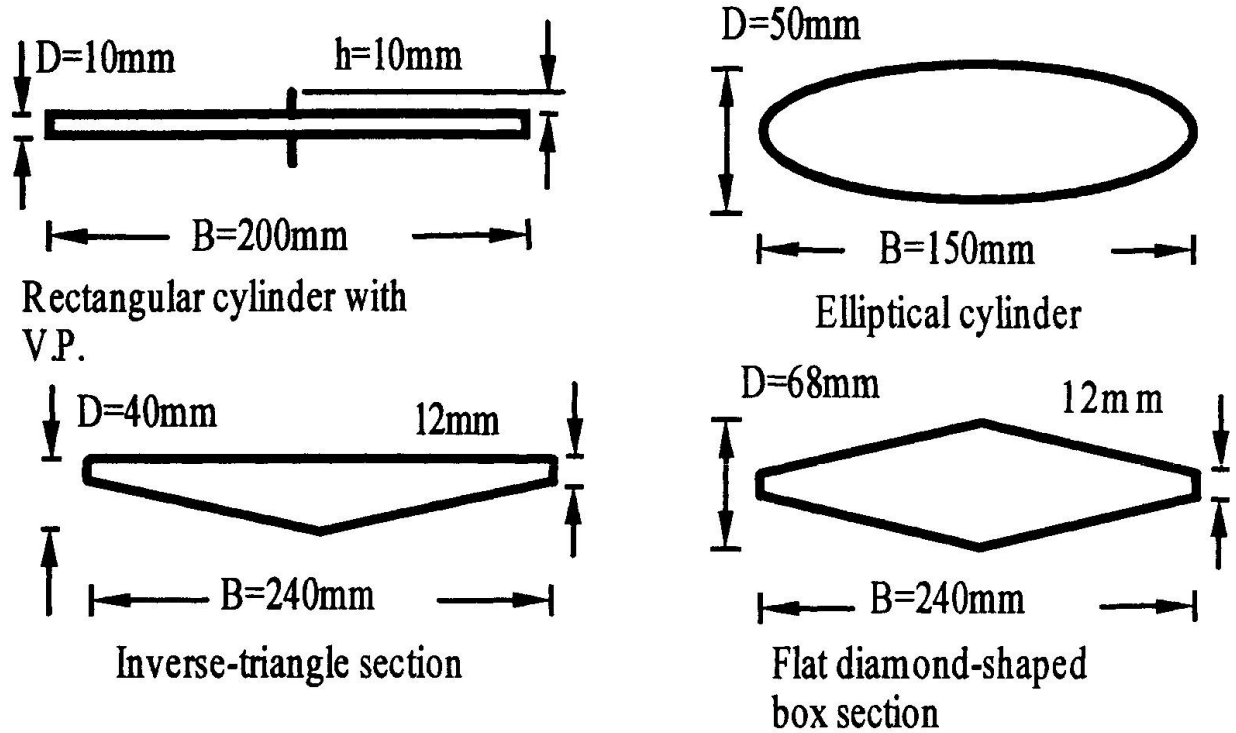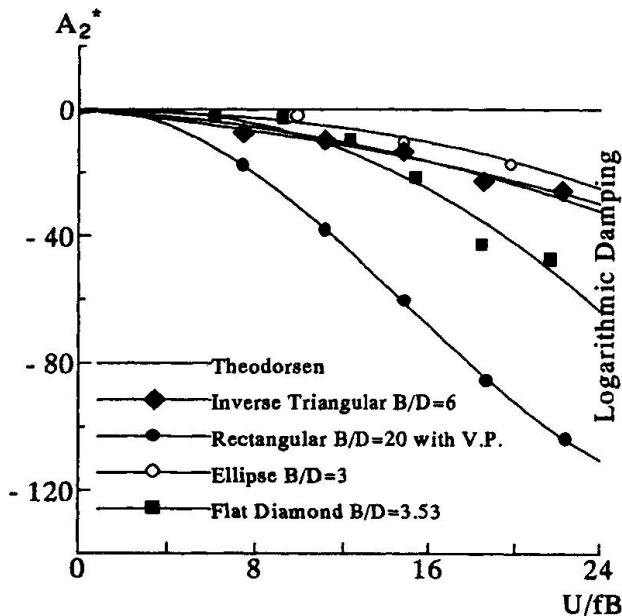
Fig.1. Some deck sections
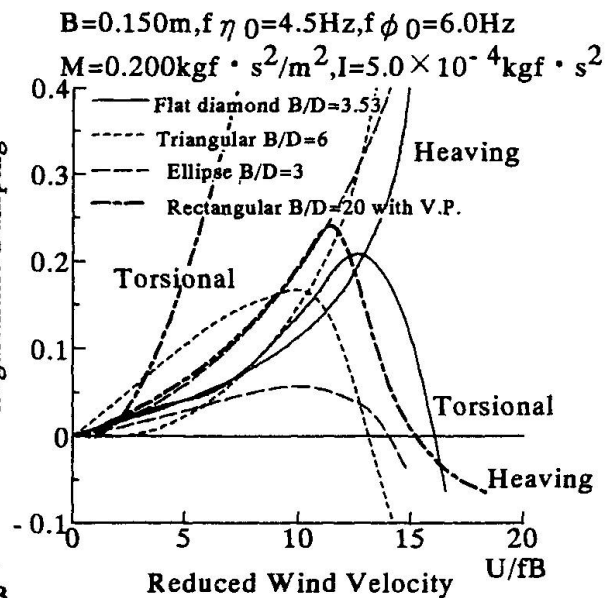
Fig.2 Aerodynamic Derivative $A_2^*$    Fig.3 V- $\delta$ diagram (Complex eigenvalue analysis)

## 2. Heaving-Branch Flutter Mechanism

Fig.4 shows flutter characteristics of the rectangular cylinder with V.P. obtained by complex eigenvalue analysis. As shown in Fig.4, the natural frequency of heaving motion being smaller than the natural frequency of torsional one, heaving branch flutter occurs instead of torsional one. In the Velocity-Frequency diagram, it should be noted that two frequency curves, namely heaving and torsional frequencies, cross each other at certain reduced velocity, as for torsional branch flutter, these two frequency curves never cross each other. Around this velocity, the heaving branch flutter occurs. The facts suggest that some relations exist between flutter instability and this frequency crossing.
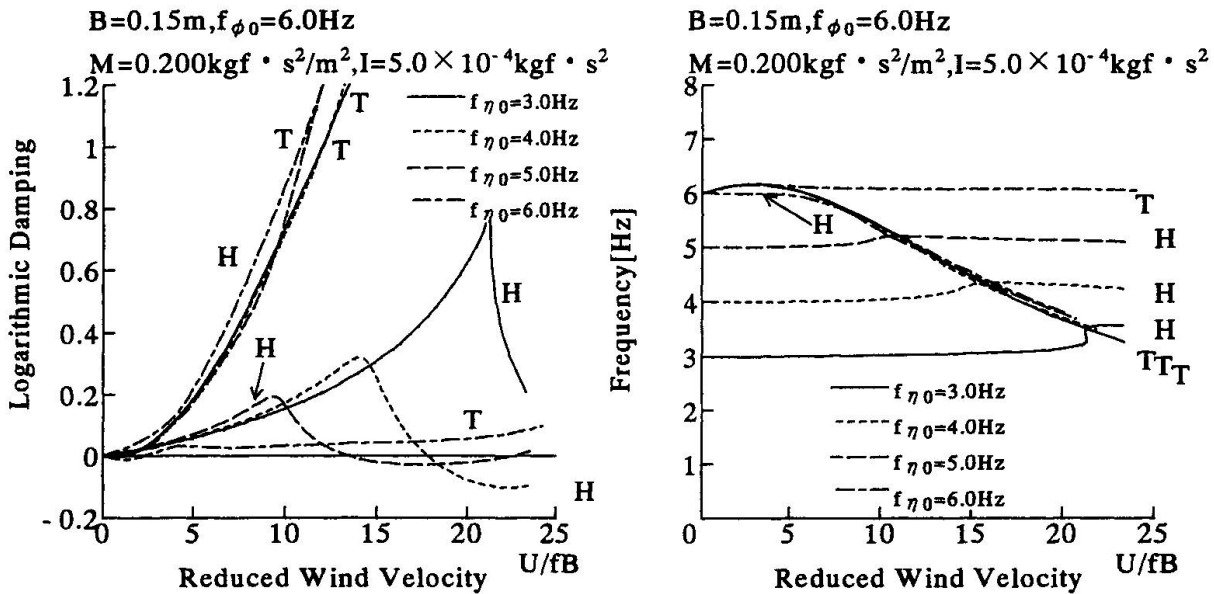


Fig.4 Flutter characteristics of the rectangular cylinder with V.P.

Based on "The step-by-step analysis" which can clarify the each role of aerodynamic derivatives on the flutter instability, the aerodynamic damping for heaving branch is given by following formula[3] :

$$\delta_\eta = -\pi\left(\frac{\rho b^2}{m}\right)H_1^* - \pi\left(\frac{\rho b^2}{m}\right)\frac{(\rho b^4/I)(\omega_f/\omega_*)^2}{\sqrt{\{1-(\omega_f/\omega_*)^2\}^2+4\zeta_*^2(\omega_f/\omega_*)^2}} \times \tag{2}$$

$$(|A_1^*|H_2^*\cos\theta_1+|A_1^*|H_2^*\cos\theta_2-|A_1^*|H_3^*\sin\theta_1-|A_1^*|H_3^*\sin\theta_2)$$

where, $\delta_\eta$ is logarithmic decrement for heaving branch and $\omega_*$ is torsional frequency.

This analysis is shown in Fig.5. Fig.5 indicates that the aerodynamic damping decreases according to the decrease of the component ◆ (-①②| $A_1^*$| $H_3^*$sin $\theta_1$ ) with reduced velocity. Therefore, this component plays a significant role of flutter excitation. Around the flutter onset velocity, | $A_1^*$| $H_3^*$ keeps negative value, and also sin $\theta_1$ changes from negative to positive (see Fig.6), then, this component ◆ (-①②| $A_1^*$| $H_3^*$sin $\theta_1$ ) turns from negative to positive.

B=0.15m,$f_{\eta 0}$=3.5Hz,$f_{\phi 0}$=6.0Hz

M=0.20kgf・$s^2/m^2$,I=5.0×$10^{-4}$kgf・$s^2$

B=0.15m,$f_{\eta 0}$=4.5Hz,$f_{\phi 0}$=6.0Hz

M=0.20kgf・$s^2/m^2$,I=5.0×$10^{-4}$kgf・$s^2$
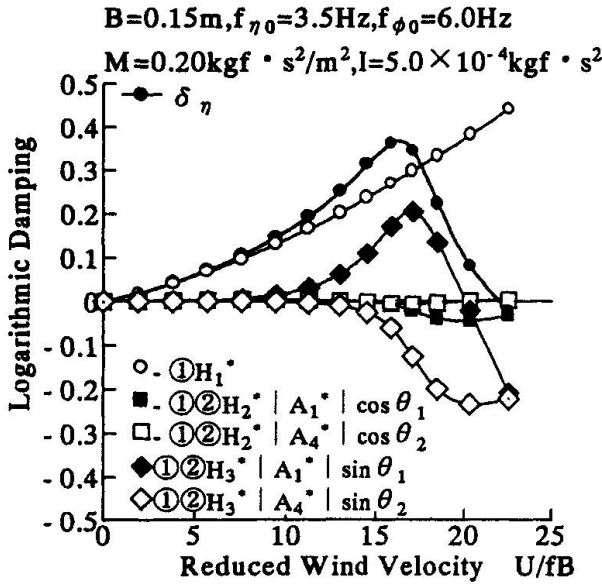
Fig.5 V-$\delta$ diagram (Step-by-step analysis)

Fig.6 V-sin $\theta$₁ diagram
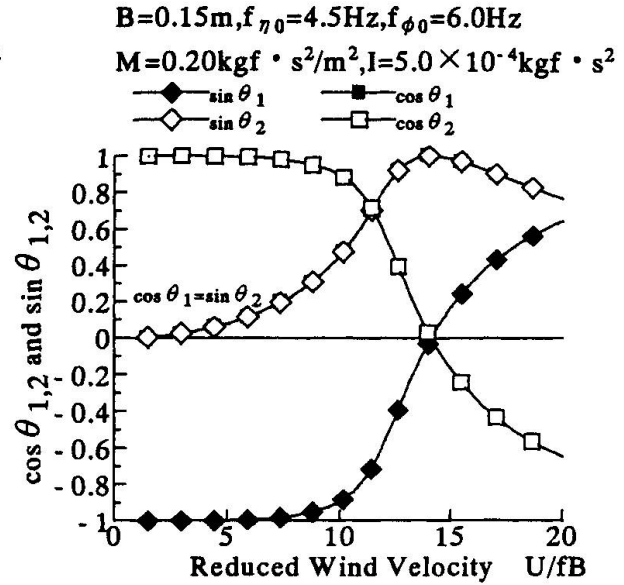
The phase lag of torsional motion to heaving motion, $\theta$₁, is expressed by folllowing equations:

$$\theta_1 = \theta - \frac{\pi}{2}, \sin \theta_1 = -\cos\theta, \theta = \tan^{-1}\frac{2\xi_\phi \omega_\eta \omega_\phi}{\omega_\phi^2 - \omega_\eta^2} \tag{3}$$

$\theta$ moving from first quadrant to second one, the sign of sin $\theta$₁ changes (see Fig.7). The Velocity-Frequency diagram obtained by the step-by-step analysis is shown in Fig.8. As shown in Fig.8, the torsional frequency decreases with reduced velocity, on the other hand, the heaving frequency keeps almost constant value, and then, the crossing point between these two frequencies exists. Because of this crossing, the sign of $\theta$ changes as shown in eq.3. As the result that, the component ◆ (-①② | A₁ | H₃sin $\theta$₁) decreases and the total aerodynamic damping of heaving branch becomes negative value, which means the heaving branch becomes aerodynamically unstable. Thus, it is concluded that the heaving branch flutter becomes unstable because of the crossing of frequency curves.
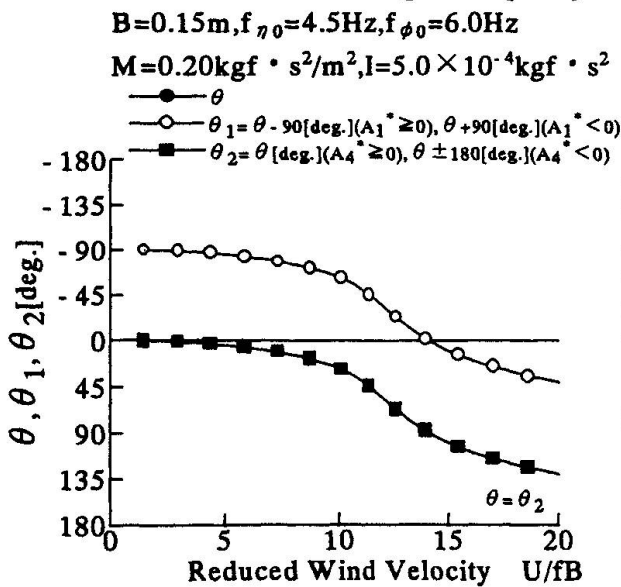
B=0.15m,$f_{\eta 0}$=4.5Hz,$f_{\phi 0}$=6.0Hz

M=0.20kgf・$s^2/m^2$,I=5.0×$10^{-4}$kgf・$s^2$

B=0.15m,$f_{\eta 0}$=4.5Hz,$f_{\phi 0}$=6.0Hz

M=0.20kgf・$s^2/m^2$,I=5.0×$10^{-4}$kgf・$s^2$

Fig.7 V-$\theta$ diagram

Fig.8 V-$\omega$ diagram
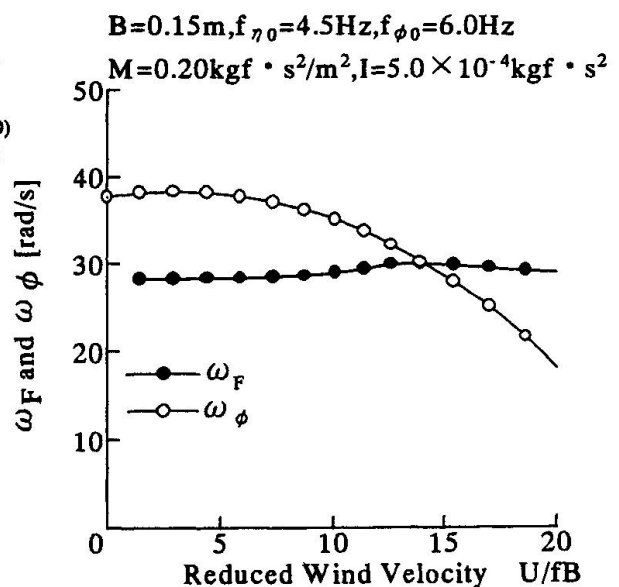
For Fig.4, it becomes clear that the flutter onset velocity of heaving branch is controlled by the reduced velocity of the crossing point. Therefore the lower natural frequency of heaving motion leads the higher flutter onset velocity of heaving branch.

The reason why heaving branch becomes unstable earlier than torsional one can be explained as follows. In general, the smaller aerodynamic derivative $A_2^*$ makes the higher flutter onset velocity for torsional branch. $A_2^*$ of the rectangular cylinder with V.P. being the smallest in Fig.2, this section is most stable for conventional coupled flutter. In this section, however, the heaving branch flutter occurs instead of torsional one. It is likely that this section is stable for torsional branch, in return, the heaving branch becomes unstable. To clarify the effect of $A_2^*$, this derivative being three times as large as the value of the thin airfoil theory by T.Theodorsen, the coupled flutter is controlled by the heaving branch (see Fig.9,10). Therefore, one of the conditions for the heaving branch flutter is considered that $A_2^*$ has large negative value.
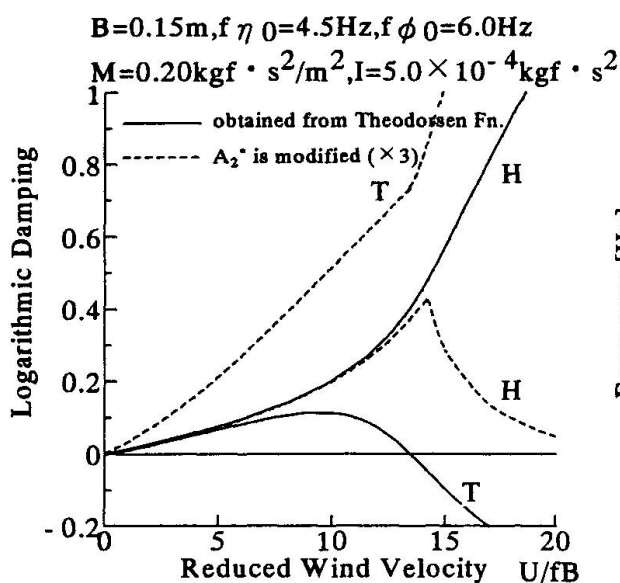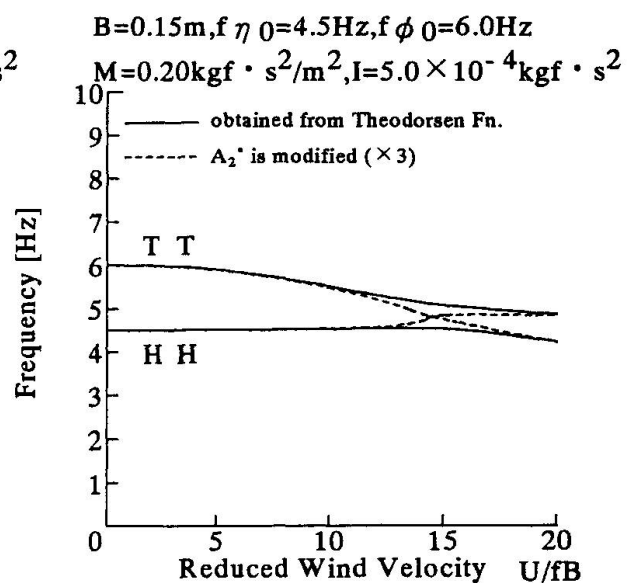
B=0.15m,f $\eta$ 0=4.5Hz,f $\phi$ 0=6.0Hz
M=0.20kgf · $s^2/m^2$,I=5.0×$10^{-4}$kgf · $s^2$

B=0.15m,f $\eta$ 0=4.5Hz,f $\phi$ 0=6.0Hz
M=0.20kgf · $s^2/m^2$,I=5.0×$10^{-4}$kgf · $s^2$



Fig.9 V- δ diagram (Complex eigenvalue analysis)          Fig.10 V-f diagram

## 3. Conclusion

The conclusions obtained in this study are summarized as follows:

(1) It is revealed that some relations between the flutter instability for heaving branch and the characteristic of frequency exist in case of the rectangular cylinder with V.P..

(2) It is possible that $A_2^*$ having large negative value, the heaving branch flutter occurs lower velocity than the torsional one.

## Reference

[1] M.Matsumoto, Hamasaki: Flutter stabilization of super long bridges, the 5th East Asia-Pacific Conference on Structural Engineering and Construction (WASECV), (Australia),pp1141-1146, 1995

[2] R.H. Scanlan and J.J Tomko, "Airfoil and bridge deck flutter derivatives", Journal of ASCE,EM6, 1971

[3] M.Matsumoto,Y.Kobayashi, Y.NIihara, H.Shirato and H.Hamasaki: Flutter mechanism and its stabilization of bluff bodies, Proc. of the 9th ICWE( Wind Engineering),New Delhi. 1995