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On the Limit Span of Cable-Stayed Structures

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Summary

The self-weight of the cable plays a fundamental role in the behaviour of very-long span cable-stayed structures. In fact, as span becomes longer, cable stays become longer and heavier. So a large percentage of their capacity is required to carry their own self-weight. In the present paper the behaviour of a cable-stay under a fixed load is studied in an "exact" theory by evaluating the actual stress and axial stiffness. The limit span of a steel cable-stay is determined by means of a numerical procedure. The results can be easily extended to cables of new composite materials, which will allow to cover very-long spans in the next future.

1. Introduction

Even thought the first cable-stayed structures were built in the seventeenth century, only in the last forty years the growth of such structures has been phenomenal. Since 1955, when the Strömsund Bridge in Sweden was built, cable-stayed structure span growth has first been gradual and steadily. A terrific increase has occurred in the last decade: first the 602 *m* span Yang Pu Bridge in Shanghai, then the 856 *m* span Normandie Bridge and finally the 890 *m* Tatara Bridge in Japan have been completed.

Leonhardt suggested that cable-stayed spans of 1200 to 1500 *m* were feasible (Billington and Nazmy 1990) and proposed a span of 1472 *m* for the Messina Strait Bridge. A hybrid variety with spans of 5000 *m* was designed for the Gibraltar Strait Crossing (Lin and Chow 1991).

The reasons for increased spans were individualised by Podolny (1995) very well. Among these were the increase of the horizontal navigation clearances, in order to accommodate the increasing size and volume of marine traffic; the economic trade off of span length cost of deep water foundations, as opposed to shallow water foundations; the risk of ship collision with piers.

The feasibility of longer spans is related to the implementation of new high-strength light-weight materials. As spans become longer, cable stays become longer and heavier and therefore their installation becomes very difficult. The structure will show a low stiffness because of the low stiffness of the cables due to their sag. A long and heavy stay is also difficult to put in tension and a high percentage if its stress is related to its self-weight. Stays with large diameter also determine the wind drag forces to be higher.

In this paper the feasibility of very long-span cable-stayed structures is investigated. The behaviour of a single cable-stay, subject to vertical loads only, is analysed in order to find out the theoretical limit span. The slope of the cable is fixed to 0.4, which represents the optimum ratio between the height of the pylon and the half span of the girder from an economical point of view (Clemente and D'Apuzzo 1995).

2. The cable in the actual configuration

The behaviour of the stays under dead loads depends on the erection procedure. The girder is usually cantilever erected and forces in the stays are controlled, so that, in the configuration under dead loads, there are no bending moments in the structure. The bending moment in the girder has to be considered as a local stress related to the distance between the cables. Therefore the analysis of the structure under dead load can be carried out by referring to the statically determinate truss scheme, in which hinges are placed at nodes. To analyse the structural behaviour under live loads, the bending stiffness of the girder must be taken into account (Clemente and D'Apuzzo 1990). Its influence on the structural behaviour becomes negligible for very long-span structures. On the other hand the ratio p/w between live and dead loads becomes very low when the span length approaches to its limit value. So stresses due to live loads are very low with respect to those due to dead loads.

For all these reasons a suitable model for the stay is that of Figs. 1 and 2. The cable is fixed at left end and its right end can move in the vertical direction only. The pylon bending stiffness is supposed to be infinite. Actually, a the displacement of the pier top due to its deformability, can be neglected when evaluating the slope angle α. The girder is supposed to have an infinite axial stiffness.

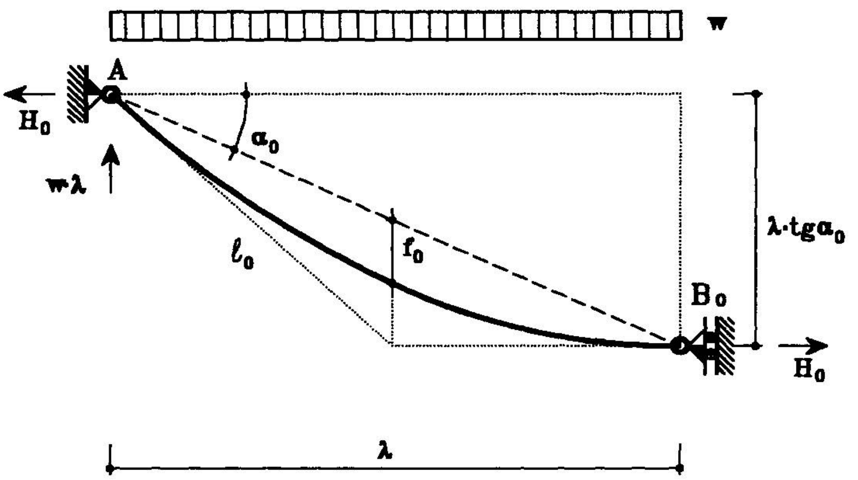

Fig. 1 Cable subject to the self weight only

Consider first the cable subject to the self-weight only, whose length in the actual configuration is ℓ_0 (Fig. 1). It assumes a configuration with horizontal tangent at B_0. The shape of the cable is supposed to be parabolic, and its self weight w uniformly distributed:

$$w = \gamma_c \cdot A_c \cdot \ell_0 / \lambda \tag{1}$$

γ_c and A_c being the weight per unit volume and the cross-section area, respectively. As will be shown later, these hypotheses cause negligible errors in the determination of the cable geometry. Tension at B_0 is:

$$H_0 = \frac{w \cdot (2\lambda)^2}{8 \cdot \lambda \tan\alpha_0} = \frac{W}{2 \tan\alpha_0} \tag{2}$$

where $W = w \cdot \lambda$ is the total weight of the stay. The vertical component of the reaction at A is $(w \cdot \lambda)$. If the force P acts at the lower end, the cable assumes a new equilibrium configuration (Fig. 2). This being closer to the straight line connecting A and B, the assumed hypothesis about w is better satisfied than in the case of self-weight only. The vertical component of the reaction at A is

$$V = W + P \tag{3}$$

and the horizontal component of the tension is

$$H = (P + W/2)/\tan\alpha \tag{4}$$

For a given γ_c, the actual configuration depends on A_c and ℓ_0, which determine the value of W. It can be found by using the iteration procedure shown in the next section.

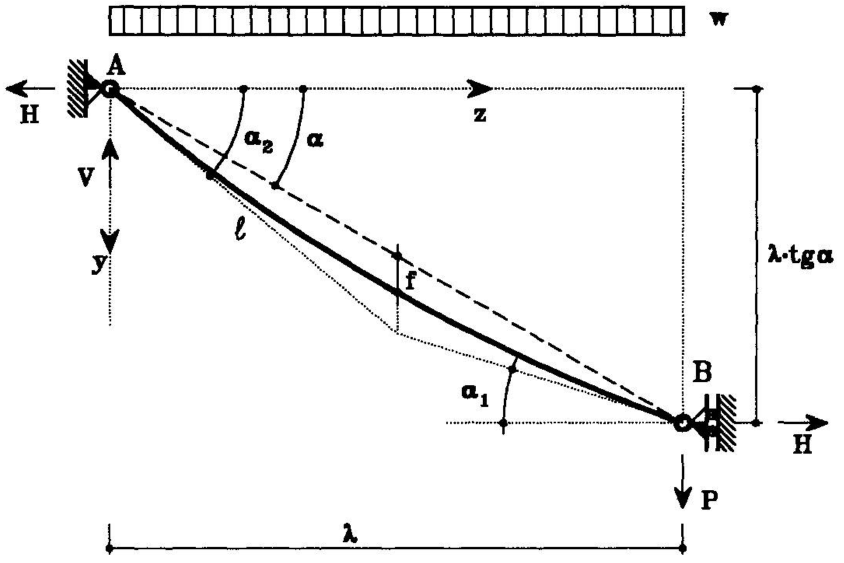

Fig. 2 Cable in the actual configuration

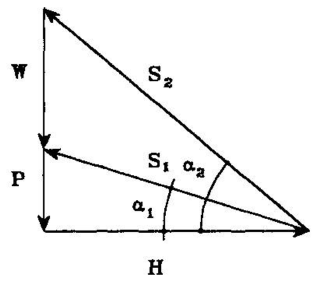

Fig.3 Equilibrium at A and B

3. The Limit Span

While the direct solution of the equilibrium of the cable is almost hard, stresses in the cable can be found, for a fixed configuration, in a simplest way. Consider the cable of Fig. 2, subject to its self-weight and to load P at its lower end B. Suppose the geometrical configuration of the cable to be fixed. In the hypothesis of parabolic shape, it is defined by the sag f at the mid-span:

$$y = -\left(4f/\lambda^2\right)\cdot z^2 + \left[\tan\alpha + \left(4f/\lambda\right)\right]\cdot z \tag{5}$$

and the angles at A and B are defined, respectively, by the relations:

$$\tan\alpha_1 = \tan\alpha - 4f/\lambda \qquad \tan\alpha_2 = \tan\alpha + 4f/\lambda \tag{6}$$

The actual length of the cable, in this configuration, can be approximately estimated with the relation-ship

$$\ell \approx \lambda\cdot\left[1 + \frac{8}{3}\left(\frac{f}{\lambda}\right)^2 + \frac{\tan^2\alpha}{2} - \frac{32}{5}\left(\frac{f}{\lambda}\right)^4 + \frac{\tan^4\alpha}{8}\right] \tag{7}$$

The horizontal component of the tension

$$H = P/\tan\alpha_1 \tag{8}$$

is independent of the span λ. It depends only on the geometrical shape of the cable. The resultant of the self-weight W is applied at a horizontal distance from A equal to

$$z_W = \frac{\int_0^\lambda z\cdot\left[1+y'^2\right]^{1/2} dz}{\int_0^\lambda \left[1+y'^2\right]^{1/2} dz} \tag{9}$$

which, in the assumption of uniformly distributed self-weight, can be supposed to be equal to $\lambda/2$. From the rotational equilibrium equation around A, the total weight of the cable can be deduced

$$W = \left(\lambda/z_W\right)\cdot\left(H\cdot\tan\alpha - P\right) \tag{10}$$

and, from this, the stay cross-sectional area

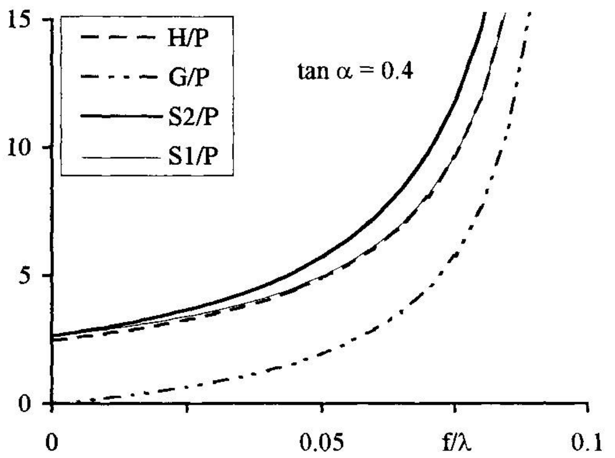

Fig. 4 H/P, G/P, S_1/P, and S_2/P versus f/λ

$$A_c = W/\gamma_c \ell \tag{11}$$

It is worth to note that, the ratio λ/z_W depending on f/λ only, W is independent of λ and so is A_c.
The force in the cable varies from the minimum value at the bottom (Fig. 3)

$$S_1 = H/cos\alpha_1 \tag{12}$$

to the maximum value at the top

$$S_2 = H/cos\alpha_2 \tag{13}$$

Also S_1 and S_2, are independent of λ. They are correlated to the horizontal component of the tension H and to the shape of the cable.
In Fig. 4 the diagrams of non-dimensional parameters H/P, W/P, S_2/P and S_1/P versus f/λ are plotted for the case $tan\alpha$=0.4. They all are independent of λ and γ_c. As one can see all the parameters increase very much for $f/\lambda>0.05$. For $f/\lambda=0$, it is $S_1=S_2=H/\cos\alpha=P/\sin\alpha$. This relations, which are usually used for the preliminary design, are approximately valid only for very low values of f/λ, i.e. when W is negligible. When f/λ increases, W becomes comparable to P and the stresses in the cable get higher. The difference between S_1 and S_2 increases and $S_1 \to H$. When $f/\lambda \to 0.1$ all the parameters tend to infinite. In fact if $f/\lambda=0.1$ then $\alpha_1=0$, and the equilibrium at B is impossible. The minimum and maximum stresses are respectively:

$$\sigma_1 = S_1/A_c \qquad \sigma_2 = S_2/A_c \tag{14}$$

The stresses can be evaluated in all the sections. So the variation $\Delta\ell$ of the cable length and finally the natural length of the cable can be calculated.
The described procedure is very suitable to find out the limit span λ_{lim} of a cable-stay, this being the span for which the whole capacity of the cable is required to carry its own self-weight. If λ is fixed, for each value of f/λ the corresponding maximum stress $\sigma=\sigma_2$ can be calculated and so the apparent tangent modulus E^*:

$$E^* = E\cdot\left[1+E/12\sigma\cdot(\gamma\lambda/\sigma_m)^2\right]^{-1/2} \tag{15}$$

where $\sigma_m = (\sigma_1 + \sigma_2)/2$.
The following assumption were made: γ_c=0.078 MN/m^3 and E=200000 MPa and P=1 MN. In Fig. 5 the curves of σ versus f/λ are plotted, for different values of λ. First of all it is to note that all the curves stop at $f/\lambda=0.1$, that is the limit value for the assumed value of α. If $f/\lambda \to 0.1$, $\alpha_1 \to 0$ and $H \to \infty$. If $f/\lambda>0.1$, then α_1 becomes negative. As a result H is negative too, and a negative value of W would be needed for the equilibrium. With regard to this limit case, the limit value of the span λ can be defined. In fact, only one curve intersects the straight line $f/\lambda=0.1$ in correspondence of the fixed maximum value of σ. The value of λ, which characterises the individualised curve is the limit value of λ, for the given σ.
Fig. 6 shows the diagrams of E^*/E versus f/λ. The deterioration of the apparent modulus is more evident for high values of f/λ and low values of λ. This paradox can be explain with the following consideration: for high values of λ, W being independent of λ, A_c must be lower and therefore σ_m is higher. As a result the stay behaves harder.
In Fig. 5 the curves relative to given values of E^*/E are also plotted. From a technical point of view it is important to fix a minimum value of E^*/E. So the technical limit value λ of can be defined as follows. Suppose that a value of σ has been fixed and a value of E^*/E has been chosen. These two values define a point in the diagram. The curve $\sigma(f/\lambda)$ passing through this point is the curve relative to the maximum value of λ.
The assumed hypothesis about the cable shape were tested with reference to the limit case $f/\lambda=0.1$. The difference between the co-ordinate y at mid-span between the assumed parabolic shape and the catenary is 1.15%. The error in evaluating z_W is about 1.7%. Obviously the errors are lower when $f/\lambda<0.1$.

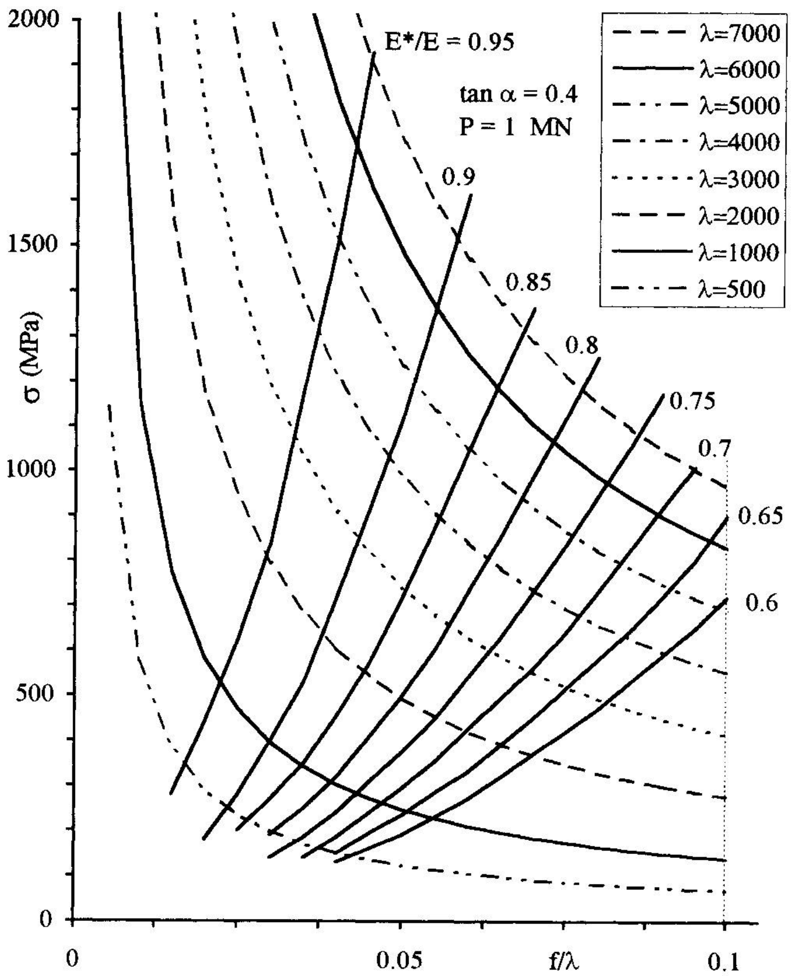

Fig. 5 σ versus f/λ

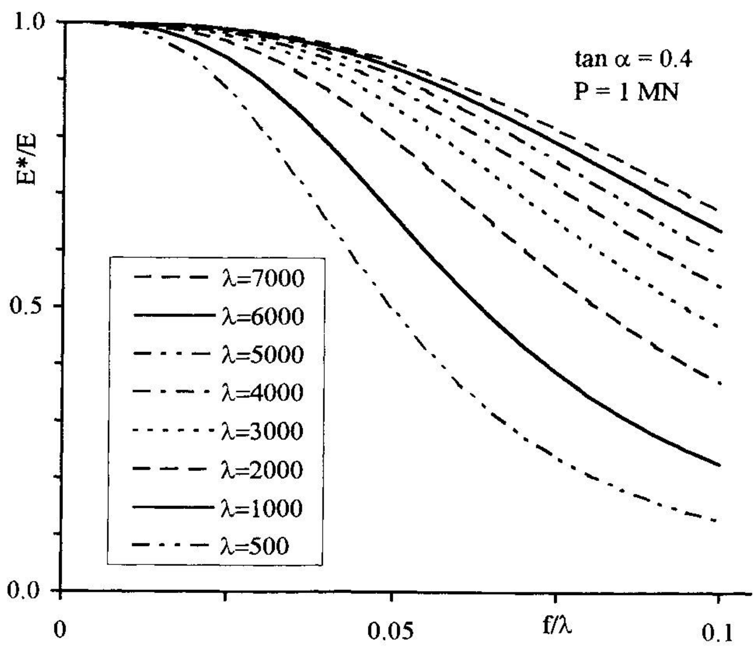

Fig. 6 E^*/E versus f/λ

The limit value of the span λ depends mainly on the stress σ. The value of σ, which can be assumed for the preliminary design, is related to the ratio p/w between live and dead loads. If f_s is the limit stress of the cable, the allowable stress is between $f_s/3$ and $f_s/2$. This study being relative to the analysis of very long-span cable-stays, low values of p/w were considered. Therefore values of σ very close to the allowable ones were assumed.

In Fig. 7 the diagrams of the deflection f/λ versus λ are plotted, for different values of σ. It is apparent that f/λ varies almost linearly with λ and significant reduction of it can be obtained by increasing σ. The sag ratio f/λ is correlated to the deformability of the cable and so is the ratio E^*/E, which is also plotted in Fig. 7 versus λ, for usual values of σ. As obvious E^*/E decreases when λ gets higher. It decreases more rapidly for lower values of the stress σ.

In Fig. 8 the diagram of the cable cross-section area A_c versus λ is plotted for different values of σ. All the curves show a slight increment of A_c with λ, that becomes very rapid for high values of λ. The value of λ, for which the slope in the diagram changes, increases with λ. It is evident that to obtain a reduction of A_c, σ must be increased. For a given value of A_c the curves show the same tangent, which happens to be independent of λ. Therefore fixing a value of $\Delta A_c/\Delta\lambda$ is equivalent to fix a value of A_c.

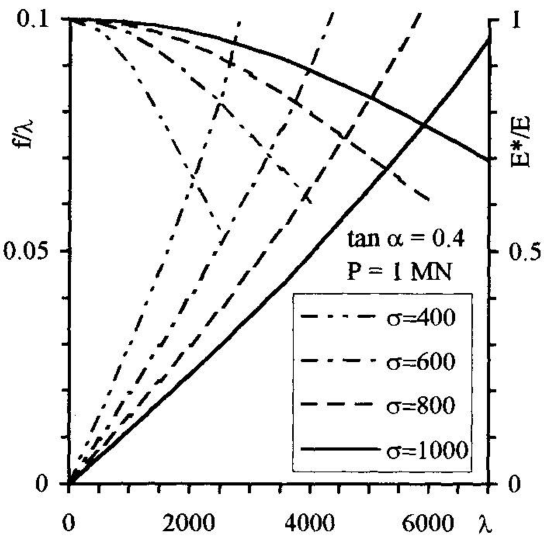

Fig. 7 f/λ and E^*/E versus λ

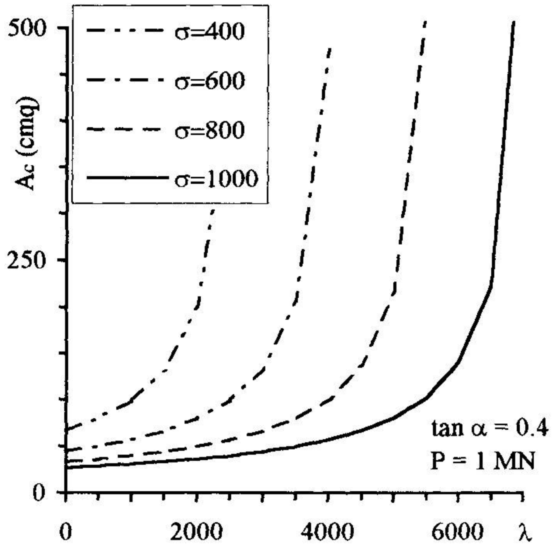

Fig. 8 A_c versus λ

4. Conclusions

In the design of a cable-stay its own self-weight is usually ignored and the actual configuration is confused with the straight line connecting A and B. As a result the force in the cable is supposed to be constant and equal to $P/\sin\alpha$. This assumption is acceptable only if $f/\lambda \approx 0$ and, therefore, if W is very low. This condition is satisfied only for short spans.
In the case of very long-spans the cable weight becomes very high and the sag ratio f/λ is not negligible. The force in the cable is everywhere higher than $P/\sin\alpha$. Both the differences $S_1-P/\sin\alpha$ and S_2-S_1 increase with f/λ. As a result the self-weight of the stay cannot be ignored in the cable design. It is worth to point out that stress in the cable being higher than $P/(A_c \sin\alpha)$ the apparent modulus is higher than one could expect.
The numerical results shown in this paper are relative to steel cables, but they can be easily generalised to other materials. In particular, from the values of σ in Fig. 5, those of the ratio σ/γ_c can be deduced, γ_c being equal to 0.078 MN/m^3.
Materials, characterised by low values of σ/γ_c, have the potential to cover longer distances in the next future. Carbon fiber composite cables seem to be very good because of their high strength ($\approx$2000 *MPa*) and their very low unit weight (0.015 MN/m^3), but may have a lower Young's modulus. The aerodynamic behaviour of light stays is also to be investigated. New high performance materials are available also for the beam and pylon, which translate to reduced weight and thus loading but they are too expensive at the present time.

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