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Interaction of Moving Mass in Dynamic Analysis of Bridges

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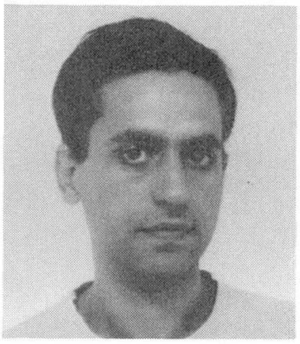

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Summary

Application of new techniques in design of bridges although reduces the structural weight of the system but it may also result in a significant reduction in stiffness of the structure. In this case the bridge would be subjected to unprecedented dynamic behavior due to movement of traffic loads. In this study a simple but accurate method for dynamic analysis of bridges has been introduced to investigate the problems associated with interaction of mass and rotary inertia of traffic loads in dynamics of flexible bridges. The effect of suspension mechanism of vehicles on deformational aspects of the bridge is also integrated into this formulation. As an example for application of this technique, the results of a brief study on the effects of speed of traffic on dynamic behavior of simply supported rail-road bridges has been reported.

1. INTRODUCTION

Using new methods in design and construction of bridges, as well as application of advanced materials in bridge systems reduces, not only the structural weight of the system but in some cases it also affects the stiffness of the structure. Flexible bridges are more likely to experience severe dynamic responses due to passage of vehicles over the system. Dynamic behavior of a bridge due to traffic loads is a major concern in defining the functionality of a system and it is also considered as an important parameter in long term performance of the structure. An accurate dynamic analysis technique for flexible bridges needs incorporation of the effects of interaction between bridge and traffic loads. In a bridge, mass and rotary inertia of the system is continually changing by movement of traffic loads on the structure. Considering the fact that, mass of the traffic load is in contact with the bridge only through the suspension mechanism of vehicles, this parameter must also be included in an accurate representation for the bridge system and in investigation of dynamic behavior of the structure.

In this work a method based on Galerkin approximation has been developed which accounts for most of the required features expected from an accurate analysis technique. The method although is quite powerful in dealing with various parameters in interaction problem, it is simple in formulation and easy in programming.

2. DYNAMICS OF A VEHICLE ON A FLEXIBLE BRIDGE

To represent the case of an ordinary vehicle, it is considered as an object supported on two axles as shown in Fig. 1. Formulation was extended for a simple case in which bridge is horizontal and vehicle moves in a uniform speed. Following relationships are equilibrium conditions of vehicle on a flexible bridge system.

$$\begin{cases}\sum F_y = 0 \\ \sum M_c = 0\end{cases} \Rightarrow \begin{cases}P + Q - mg + m\ddot{y}_c = 0 \\ P\,l_c - Q(l - l_c) - J\,\ddot{\alpha}_c = 0\end{cases} \qquad (1)$$

Where P and Q are reactions of axles of vehicle and l is the distance between two axles. Variables $\ddot{y}_c$ and $\ddot{\alpha}_c$ are vertical and rotational acceleration of center of gravity of vehicle and l_c is the distance between center of gravity and reaction P. Parameter g is the gravitational constant, m is the mass of vehicle and J is its rotary inertia. Variables $\ddot{y}_c$ and $\ddot{\alpha}_c$ can be replaced by the following expressions.

$$\ddot{y}_c = \ddot{y}_P + \frac{l_c}{l}(\ddot{y}_Q - \ddot{y}_P) \quad \text{and} \quad \ddot{\alpha}_c = tan^{-1}\frac{\ddot{y}_Q - \ddot{y}_P}{l} \qquad (2)$$

Where $\ddot{y}_P$ and $\ddot{y}_Q$ are local acceleration of bridge at the location of reactions P and Q respectively. By assuming small angle of rotation (i.e. small deformation of bridge) and in the case where center of gravity of vehicle is at the middle ($l_c = l/2$) of vehicle, solution of the equilibrium equations for P and Q results in the following expressions.

$$\begin{cases}P = \dfrac{mg}{2} + \ddot{y}_P\left[-\dfrac{J}{l^2} - \dfrac{m}{4}\right] + \ddot{y}_Q\left[\dfrac{J}{l^2} - \dfrac{m}{4}\right] \\ Q = \dfrac{mg}{2} + \ddot{y}_P\left[\dfrac{J}{l^2} - \dfrac{m}{4}\right] + \ddot{y}_Q\left[-\dfrac{J}{l^2} - \dfrac{m}{4}\right]\end{cases} \qquad (3)$$

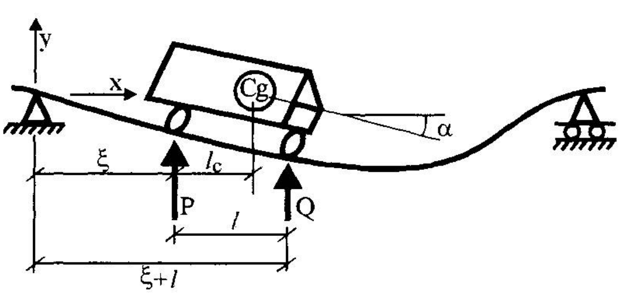

Fig. 1 - Bridge - vehicle representation

The first term in right hand side of the above equations represents the effect of static distribution of weight of vehicle on each axle. The second and third terms are representing the effects of local acceleration of bridge on reaction forces. The first term in brackets signifies the effect of mass rotary inertia while the second term (in brackets) indicates the effect of added mass of traffic load on reaction forces.

3. SUSPENSION MECHANISM

There is a similarity between suspension mechanism of vehicles and vibration isolator devices in mechanical systems. This similarity can be utilized to incorporate the effect of suspension mechanism of vehicles in dynamics of bridge by using force transmissibility factor ***TR*** (see, for example, Paz 1991). This factor in the case of bridge-vehicle system can be interpreted as the ratio between amplitude of dynamic force transmitted to the bridge with a flexible suspension mechanism and without it. Theoretically, transmissibility factor is a function of damping ratio of isolated system and frequency ratio of dynamic force, i.e.

$$TR = f(\zeta,\ \bar{\omega}/\omega) \qquad (4)$$

Where ζ is damping ratio, ω is natural frequency of system and $\bar{\omega}$ is the frequency of harmonic force. In the case of bridge, ω is natural period of suspension system and $\bar{\omega}$ could be a function of both speed of vehicle and natural frequency of bridge. An easy way to implement this simple technique is to modify rotary inertia and mass of vehicle in the second and third terms of Eq.3 by the following expressions.

$$\begin{cases}J_{TR} = TR_P\ .J \\ m_{TR} = TR_H\ .m\end{cases} \qquad (5)$$

In the above relationships parameters $\boldsymbol{TR_P}$ and $\boldsymbol{TR_H}$ are transmissibility coefficients for pitching and heaving movement of vehicle and J_{TR} and m_{TR} are modified values for rotary inertia and mass of traffic load. Transmissibility coefficients in heaving and pitching are different due to distinct natural frequencies for each of these movements. Usually this factor for pitching (TR_P) is much smaller than for heaving movement (TR_H), predominantly because of the large rotary inertia of vehicles. This is considered a simplified approach because by scaling TR_P and TR_Q differently, the coupling effect between pitching and heaving movements of vehicle would be ignored.

Considering the above modification in mass and rotary inertia, the abbreviated form of reaction forces P and Q are as follows.

$$\begin{cases} P = A_0 + A_1\ddot{y}_P + A_2\ddot{y}_Q \\ Q = A_0 + A_2\ddot{y}_P + A_1\ddot{y}_Q \end{cases} \quad \textbf{Where: } A_0 = \frac{mg}{2},\quad A_1 = \left[-\frac{J_{TR}}{l^2} - \frac{m_{TR}}{4}\right] \textbf{ and } A_2 = \left[\frac{J_{TR}}{l^2} - \frac{m_{TR}}{4}\right] \tag{6}$$

4. DYNAMICS OF BRIDGE

The simplified form of differential equation of a uniform bridge loaded with only one vehicle can be written as follows.

$$EI\frac{\partial^4 y}{\partial x^4} + \overline{m}\frac{\partial^2 y}{\partial t^2} = P\,\delta(\xi, x) + Q\,\delta(\xi + l, x) \qquad -l \le \xi \le L \tag{7}$$

In which EI is flexural stiffness, $\overline{m}$ is mass per unit length. Forcing function in this equation consists of concentrated forces P and Q, which have been applied to the system by using Dirac-delta transformation functions represented by symbol δ (see, for example, Abramowitz et al. 1974). L is the total length of bridge and ξ is the distance of reaction P from the beginning of bridge (depicted in Fig.1). Based on the speed of vehicle and the elapsed time since the front axle of vehicle (reaction Q) has entered on the bridge, parameter ξ can be evaluated.

Forces P and Q are both functions of local acceleration of bridge y_p and y_Q as it is shown in Eq. 6. Interaction between load and deformational aspects of structure differentiate this problem from the ordinary problems in classical dynamic analysis. Therefore the available techniques in dynamic analysis of structures (modal analysis, for example) are not applicable to this particular case because of the interaction problem. In this study an algorithm based on Galerkin approximation (see, for example, Mikhlin 1964) has been chosen to address the problem. According to this approach deformation of the structure is approximated by a set of shape functions as follows.

$$y(x,t) = \sum_{i=1}^{n} \phi_i(x).q_i(t) \tag{8}$$

In this equation $y(x,t)$ is vertical displacement of bridge while $\phi_i(x)$ are predefined displacement shape functions. Parameters $q_i(t)$ are shape function coefficients to be calculated at time t and n is the number of these shape functions. Replacing this approximation for deformation of the bridge into differential equation of system results in the following expression.

$$\begin{aligned} EI\sum_{i=1}^{n}\phi_i^{IV}(x)q_i(t) + \overline{m}\sum_{i=1}^{n}\phi_i(x)\ddot{q}_i(t) = &\\ &\left[A_0 + A_1\sum_{i=1}^{n}\phi_i(\xi)\ddot{q}_i(t) + A_2\sum_{i=1}^{n}\phi_i(\xi+l)\ddot{q}_i(t)\right]\delta(\xi, x) \\ +&\left[A_0 + A_2\sum_{i=1}^{n}\phi_i(\xi)\ddot{q}_i(t) + A_1\sum_{i=1}^{n}\phi_i(\xi+l)\ddot{q}_i(t)\right]\delta(\xi+l, x) \end{aligned} \tag{9}$$

If shape functions ϕ_i are chosen to be a set of orthogonal functions, it can be shown that Dirac-delta equations are expandable based on the following relationships.

$$\delta(\xi,x)=\sum_{i=1}^{n}\phi_i(\xi)\phi_i(x) \qquad \text{and} \qquad \delta(\xi+l,x)=\sum_{i=1}^{n}\phi_i(\xi+l)\phi_i(x) \tag{10}$$

In a simplified approach, natural mode shapes of system can be used as shape functions for Galerkin approximation. By using the relational properties associated with application of a set of normalized modal shapes (see, for example, Paz.1991) and by pursuing Galerkin procedure, the final result will be the following set of equations.

$$\ddot{q}_m(t)+\omega_m^2.q_m(t)=$$
$$\frac{1}{\overline{m}}\left\{\begin{aligned}&\left[A_0+A_1\sum_{i=1}^{n}\phi_i(\xi).\ddot{q}_i(t)+A_2\sum_{i=1}^{n}\phi_i(\xi+l).\ddot{q}_i(t)\right].\phi_m(\xi)+\\&\left[A_0+A_2\sum_{i=1}^{n}\phi_i(\xi).\ddot{q}_i(t)+A_1\sum_{i=1}^{n}\phi_i(\xi+l).\ddot{q}_i(t)\right].\phi_m(\xi+l)\end{aligned}\right\} \quad m=1,2,..n \tag{11}$$

Where ω_m's are natural periods of the bridge. The above relationship is a set of n simultaneous coupled differential equations in time domain (ξ can be evaluated based on time).

The above relationships represents a case where only one vehicle with two axles is passing over the bridge. If the number of vehicles are more than one and system is subjected to damping forces, it can be easily shown that the system of equations will be changed to the following form.

$$\ddot{q}_m(t)+2\omega_m\zeta_m\dot{q}_m(t)+\omega^2{}_m q_m(t)=$$
$$\sum_{j=1}^{k}\alpha_m(\xi_j)+\sum_{i=1}^{n}\ddot{q}_i(t)\sum_{j=1}^{k}\phi_i(\xi_j)\,\beta_m(\xi_j)+\sum_{i=1}^{n}\ddot{q}_i(t)\sum_{j=1}^{k}\phi_i(\xi_j+l)\lambda_m(\xi_j) \tag{12}$$
$$m=1,2,\dots n$$

In which k is the number of vehicles on the bridge and parameters α , β and λ are defined as:

$$\begin{cases}\alpha_m(\xi)=A_0\left[\phi_m(\xi)+\phi_m(\xi+l)\right]/\overline{m}\\ \beta_m(\xi)=\left[A_1\phi_m(\xi)+A_2\phi_m(\xi+l)\right]/\overline{m}\\ \lambda_m(\xi)=\left[A_2\phi_m(\xi)+A_1\phi_m(\xi+l)\right]/\overline{m}\end{cases} \tag{13}$$

The following is the matrix form of the equation set No. 12.

$$[A_\xi]\{\ddot{q}_t\}+[2\omega\zeta]\{\dot{q}_t\}+[\omega^2]\{q_t\}=\{B_\xi\} \tag{14}$$

Where $[2\omega\ \zeta]$ is a diagonal matrix representing damping contribution in the system. Matrix $[A_\xi]$ and vector $\{B_\xi\}$are coefficient matrix and load vector, respectively. These two terms must be evaluated at time t based on location parameter ξ.

In comparison with classic dynamic analysis, in the above equation matrix $[A_\xi]$ is not a diagonal matrix. This clearly indicates that the equation set is in a coupled system. Among numerous time integration method applicable to this problem, a method based on Runge-Kutta formulas of order five and six (see Hull et al. 1976) has been adopted as the solution algorithm in this study. To be able to use Runge-Kutta formulas, differential equations must be transformed to a set of first order differential equations. It can be shown that, such transformation is possible by only using a simple change in the variables (see Hull et al. 1976).

8. CASE STUDY FOR HIGH-SPEED TRAINS

The capability of this technique is shown in an example in railway bridges. A train system as shown in Fig.2 is considered as traffic load on the bridge. Dynamic analysis was carried out for a bridge with 50 meters in length (shown in the same figure). The length of bridge is chosen less than the train, to investigate the case in which the whole bridge is loaded steadily with traffic. In this case, bridge is considered as simply supported single span structure.

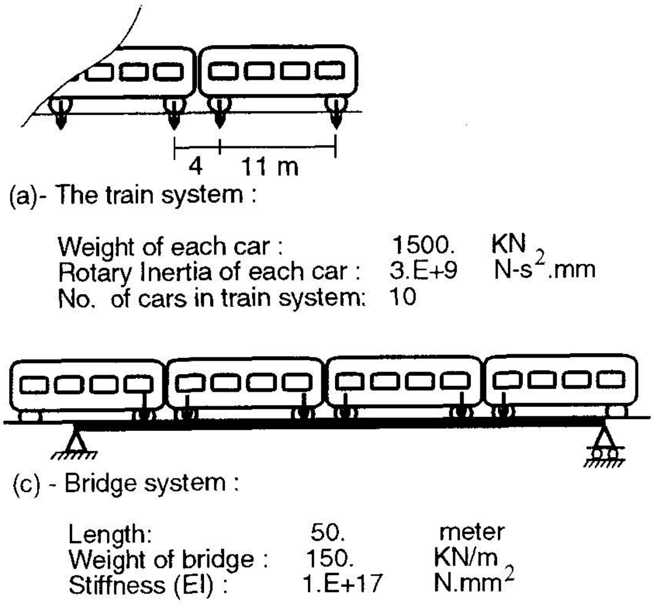

Fig. 2- The bridge - traffic model

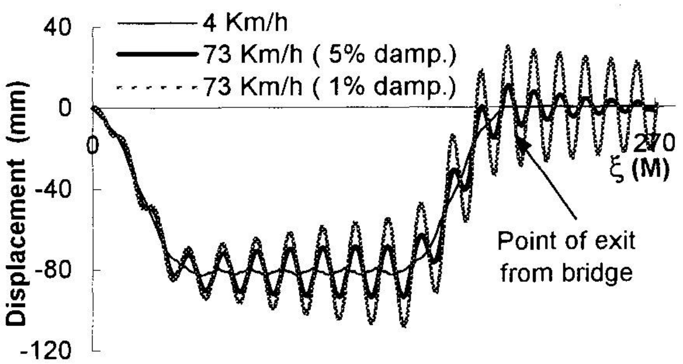

Fig. 3 - Displacement at mid-span during resonance

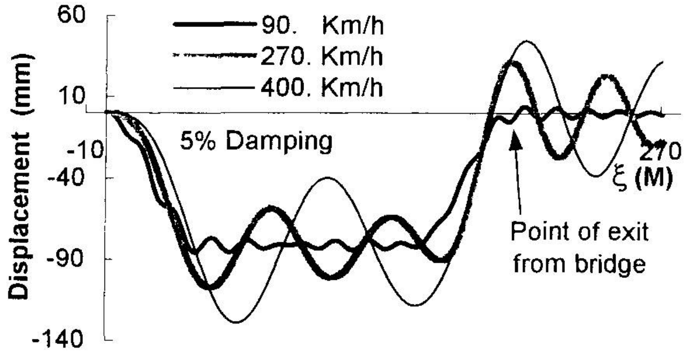

Fig. 4 - Displacement at mid-span at high speed

It is assumed that all the natural modes have a damping ratio of 5% (ζ_m=0.05). The parameters TR_P and TR_H have been chosen intuitively to represent a system with characteristics of soft suspension mechanism (0.1 and 0.9 respectively). Figure 3 illustrates mid-span displacement of bridge when resonance occurs at speed of 73 Km/h. In this figure, horizontal axes represents the distance of front axle of the first car from the left support of the bridge (ξ+l). This figure is similar to classical *Influence Line* in bridge engineering. Figures with this type of horizontal axis hereinafter are referred to as ***Dynamic Influence Line***.

A case of low speed train (4Km/h) is also included in the same figure to simulate a behavior similar to static analysis of the bridge. As it is shown in the figure, a lower damping ratio causes larger amplitude of vibration in the structure. In the case of resonance the amplitude of vibration grows steadily with continuation of vibration process, thus its maximum magnitude depends on the number of cars in the train system.

By further increase on the speed of train there will be a substantial increase on maximum displacement of bridge as it is shown in Fig. 4.

The result of analysis for shear force at a point close to support (at a distance of 1% of span from the left support) is illustrated in Fig. 5. The importance of traffic load

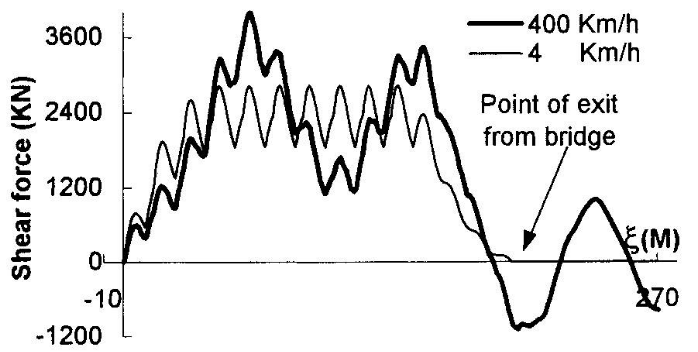

Fig. 5 - Shear force at a point close to support

representation as point loads in this formulation is illustrated in the same figure. According to this figure, even in the case of static loading (speed of 4Km/h) there is a periodic variation on the level of shear force in structure which aggravates by increase on the speed of traffic. If, for example, a detail information on stress cycle history of bridge for fatigue design is required, such accuracy in analysis is quite important.

9. CONCLUSION

To study the effects of reduction in stiffness of bridges on functionality and long term performance of these systems, a technique has been proposed for accurate dynamic analysis of these structures. This method is capable of representing the effects of movement of vehicles on bridge with a reasonable accuracy. The method is based on Galerkin approximation and it offers a phenomenal simplicity in formulation and also high efficiency in the computational efforts. However, since this method relies only on the general structural parameters of system (such as mass and flexural stiffness), it does not have the required generality to be applied to the detail analysis of bridges. In other words, this method can only provide a general view on dynamic behavior of those bridges with complicated structural system. In such cases the results of dynamic analysis by this approach can be used to extrapolate the response of the structure obtained from a detailed static analysis (by, for example, finite element method). It is believed that, this technique can be extended to a more general formulation to serve wider range of applications.

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REFERENCES

Abramowitz, M. and Stegun, I.A. (1974). *Handbook of Mathematical Functions.* National Bureau of Standards, Dover Publications Inc., New York.

Hull, T.E., Enright, W.H. and Jackson, K.R. (1976). *User's guide for DVERK - A subroutine for solving non-stiff ODEs.* Department of Computer Science, Technical Report 100, University of Toronto.

Mikhlin, S.C. (1964). *Variational Methods in Mathematical Physics.* Macmillan.

Paz, M. (1991), *Structural Dynamics Theory and Computation*, Van Nostrand Reinhold.