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Ultimate Strength and Collapse Mechanism of Composite Frames under Seismic Loading

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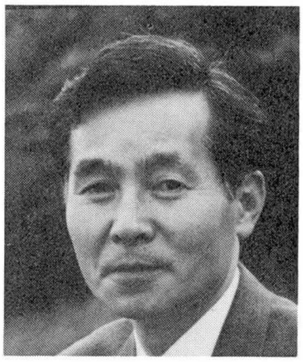

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Summary

In seismic design of composite building frames in Japan, the weak beam strong column moment frame system is usually adopted. In the case of composite low rise building frames, the plastic hinges formed at beams and columns show not only the rotational deformation but also the axial elongation. Considering these axial and rotational deformations, the real collapse mechanism is somewhat different from the expected one. In this paper, the ultimate strength and the collapse mechanism of the composite frames under the vertical and horizontal loads are discussed. Using geometric compatibility condition, a recurrence formula is derived between the rotation angles of the columns and beams, and a procedure for obtaining the ultimate strength and the collapse mechanism is shown. The analytical results show that plastic hinges should be formed not only at the beam ends and column bases but also at almost all the lower ends of the columns.

1. Introduction

In seismic design of composite moment frames, the weak beam strong column system is usually adopted. In this case, plastic hinges are supposed to form at all beam ends and column bases. A plastic hinge formed in the composite members shows not only the rotational deformation but also the axial deformation. The effect of this axial deformation at the plastic hinges on the collapse mechanism is important, but only a few researches have been made[1,2,4]. In this paper, the ultimate strength and the collapse mechanism of the composite frames are discussed, considering both the rotational and axial deformations at the plastic hinges. A recurrence formula is obtained between the rotation angles of the members. Using the formula, a procedure to obtain the ultimate strength and the collapse mechanism is shown, and some numerical analyses are carried out.

2. Ultimate Strength and Plastic Deformation of Composite Members

Fig.1 shows a composite member having a rectangular concrete cross section with an ideal I-section steel. For simplicity, the cross section is idealized into three point model such as shown in the figure. Considering rigid perfect plastic stress-strain relationship for both concrete and steel, such as shown in Fig.2, the yield condition of the idealized model cross section can be obtained as a hexagonal shape under the axial force (N) and bending moment (M), such as shown in Fig.3(a)[3,4]. The associated flow rule shows that the ratio between axial deformation e and rotation ω is constant when the generalized stress (N,M) moves on each line of the yield locus, such as shown in Fig.3(b).

$$e = -(D/3)\omega \qquad \text{for} \quad -{}_sN_o \le N \le {}_cN_o/3$$
$$e = 0 \qquad \text{for} \quad {}_cN_o \le N \le 2{}_cN_o/3$$
$$e = (D/3)\omega \qquad \text{for} \quad 2{}_cN_o \le N \le {}_cN_o + {}_sN_o$$

where ${}_sN_o = 2{}_s\sigma_oA_f$ and ${}_cN_o = {}_c\sigma_oBD$. If the axial load level is low $(-{}_sN_o \le N \le {}_cN_o/3)$, axial elongation occurs when the plastic hinge rotates, such as shown in Fig.3(c).
The yield condition of this range is

$$f = N - ({}_sN_o/{}_sM_o)M + {}_sN_o = 0$$

Rate of energy dissipation D can be obtained as a simple form as follows:

$$D = N \cdot e + M \cdot \omega = {}_sM_o \cdot \omega$$

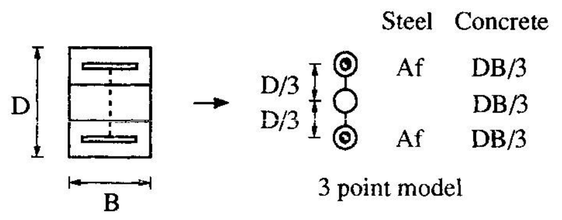


Fig.1 Model of composite cross section

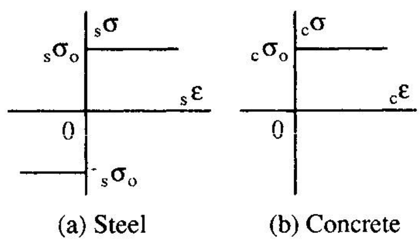
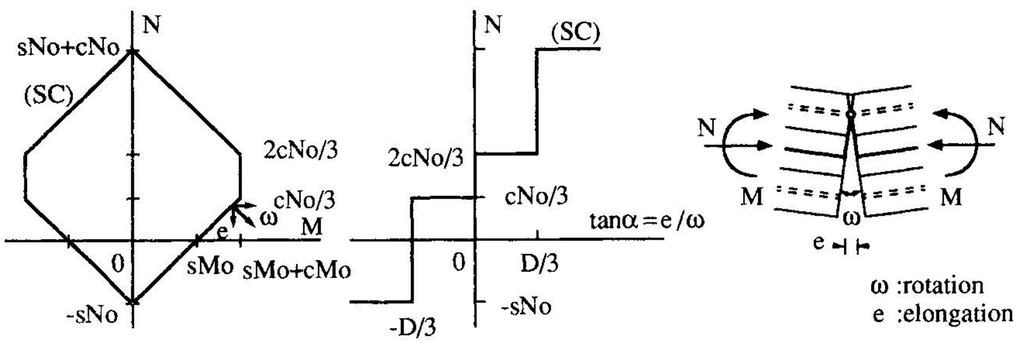


Fig.2 Stress strain relations



(a) Yield Condition (b) Flow Rule (c) Plastic Hinge

Fig.3 Yield condition and associated flow rule

3. Ultimate Strength and Collapse Mechanism of Composite Frames

3.1 Design of Composite Frames

The composite frames subjected to vertical and horizontal forces are designed to collapse by forming plastic hinges at all beam ends and column bases, using simple plastic analysis. To realize the weak beam and strong column frames, the total amount of plastic moment of the columns is 1.2 times as large as that of beams gathering at each beam to column joint. The vertical and horizontal forces are applied at each beam to column joint such as shown in Fig.4. The horizontal force is specified in conformance with the building standard law of Japan. The base shear coefficient is assumed to 0.3 for the ultimate strength stage, in this case. For simplicity, all the column concrete cross sections are the same (D_cxD_c) and also all the beam cross sections are the same $(D_b/2xD_b)$ such as shown in Fig.5. The required plastic moment capacity of each member ${}_sM_o$ is adjusted by the cross sectional area of the steel I section. The plastic moment of the outer column is assumed to be 0.6 times to that of the inner column.

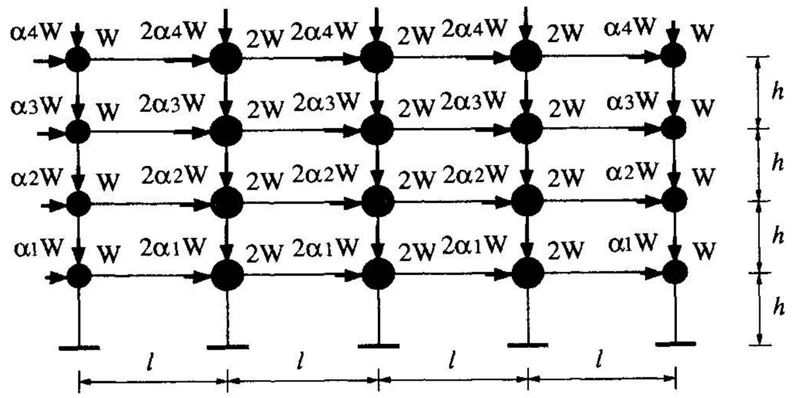


Fig.4 Composite frames subjected to vertical and horizontal loads

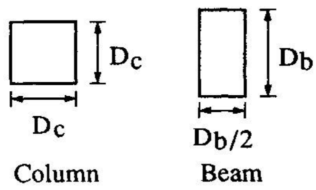


Fig.5 Cross section

3.2 Rotation Angle of the Members

The plastic hinge shows not only rotation but also elongation when the axial load revel is low. According to the axial deformation at the plastic hinges, elongation of the beam or the columns occurs. Fig.6(a) shows a possible collapse mechanism of a 2-span 3-storied frame. The plastic hinges are supposed to be formed at all beam ends and column bases, according to the simple plastic analysis. But this collapse mode (collapse mechanism A) shows the additional plastic hinges formed at some lower ends of the columns, due to the axial elongation of the beams. And elongation of the column plastic hinges brings the rotational deformation to the beams. On the basis of geometric compatibility condition, the rotation angle of the columns $\omega_c(i, j)$ can be expressed as follows :

$$\omega_c(2,3) = \omega_c(3,3) = \omega_c(1,3)$$

$$\omega_c(1,j) = [1-(2D_b/3hK_r)]\omega_c(1,j+1)$$

$$\omega_c(2,j) = [1-(2D_b/3hK_l)]\omega_c(2,j+1) + [\omega_c(1,j+1) - \omega_c(1,j)]/K_l$$

$$\omega_c(3,j) = [1-(2D_b/3hK_l)]\omega_c(3,j+1) + \sum_{i-1}^{2}[\omega_c(i,j+1) - \omega_c(i,j)]/K_l$$

where $K_l = 1+(D_b/3h)[1-(2D_c/3l)]$ and for j=1,2.

And the rotation angle of the beams $\omega_b(i, j)$ can be expressed as follows:

$$\omega_b(i,j) = (D_c/l)[\omega_c(i,j) - \omega_c(i,j-1)]$$

for i=1,2,3 and j=1,2.

Fig.6(b) shows another possible collapse mechanism (collapse mechanism B) of the same frame.

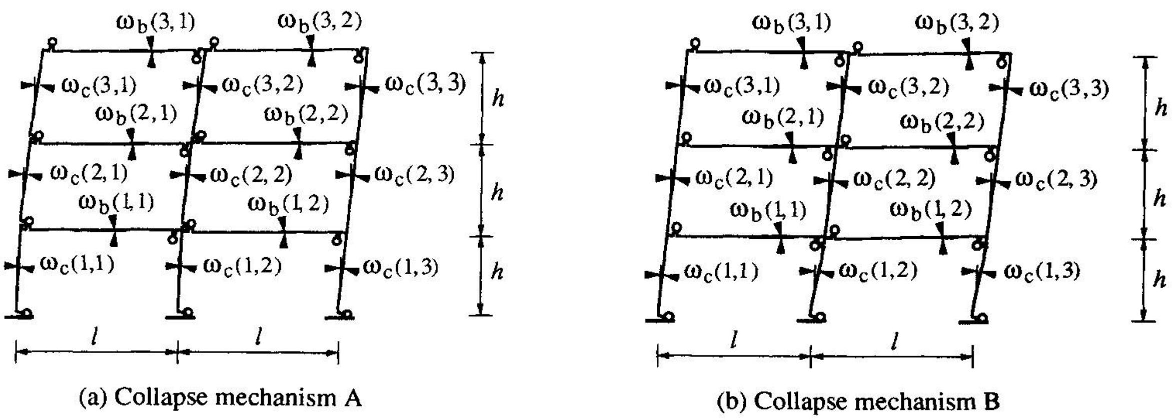


Fig.6 Collapse mechanism of 2-span 3-storied composite frame

The left hand side column forms a plastic hinge only at the column base. In this case, additional plastic hinges are formed at the upper end of another first-story columns. Using geometric compatibility condition, the rotation angle of the columns and beams can be expressed by the following equations:

$$\omega_c(1,2) = \omega_c(1,3) = [1 + (2D_b / 3hK_r)]\omega_c(1,1)$$
$$\omega_c(2,j) = \omega_c(1,1)$$
$$\omega_c(3,j) = \omega_c(1,1)$$
$$\omega_b(i,j) = 4D_cD_b /(9hlK_r)$$

where $K_r = 1 - (4D_cD_b / 9hl)$ and for i=1,2,3 and j=1,2.

Using the rotation angle of the column having a plastic hinge only at the column base, $\omega_c(1,1)$ in this case and $\omega_c(1,3)$ in the former case, the rotation angles of another columns and beams can be expressed by the recurrence formula shown above. The recurrence formula can be obtained for any composite frames designed according to the conditions shown in section 3.1.

3.3 Ultimate Load and Collapse Mechanism

The ultimate load and the collapse mechanism of the frames (from 2-span 2-storied to 6-span 6-storied frames) will be discussed in this section. Assuming the position of a column with a plastic hinge only at the column base, rotation angles of another columns and beams can be expressed, using the recurrence formula. The ultimate load factor λ associated with an assumed collapse mechanism can be obtained using the principle of virtual work as follows:

$$\lambda \sum_{j=1}^{m+1} \sum_{i=1}^{n} \alpha(i,j)W(i,j)u(i,j) - \sum_{j=1}^{m+1} \sum_{i=1}^{n} W(i,j)v(i,j)$$
$$= \sum_{j=1}^{m+1} \sum_{i=1}^{n} \{\xi(i,j,l)_c\,\omega_o(i,j,l) + \xi(i,j,u)_c\,\omega_o(i,j,u)\}_{sc} M_o(i,j)$$
$$+ \sum_{j=1}^{m} \sum_{i=1}^{n} \{\xi(i,j,l)_b\,\omega_o(i,j,l) + \xi(i,j,r)_b\,\omega_o(i,j,r)\}_{sb} M_o(i,j)$$

where α(i,j) shows the ratio of the lateral load to the vertical load W(i,j) and u(i,j) and v(i,j) are horizontal and vertical displacements at the (i,j)th joint. ${}_c\omega_o(i,j,l)$ and ${}_c\omega_o(i,j,u)$ show the rotation of the plastic hinges at the lower end (l) and the upper end (u) of the (i,j)th column, and ${}_b\omega_o(i,j,l)$ and ${}_b\omega_o(i,j,r)$ the rotation of the plastic hinges at the left side (l) and the right side (r) of the (i,j)th beam. ξ=1 is for active plastic hinge and ξ=0 without plastic hinge. Changing the position of the column with plastic hinge only at its base in turn from left to right, the corresponding ultimate load factor can be obtained. According to the upper bound theorem of plasticity, the minimum load factor obtained is the real collapse load factor and the associated mechanism is the real collapse mechanism. Table 1 shows the collapse load factors calculated. λ=1 shows the load factor obtained using simple plastic analysis. As the number of stories increases, the collapse load factor decreases, and as the number of spans increases, the collapse load factor increases in a gentle manner, because the effect of the axial force of the beams on the bending capacity is small. Table 2 shows the position of the column forming plastic hinge only at the column base. The position of the column is usually at the right part of the frame, and as the number of stories increases and as the number of spans decreases, the position moves to the right end. So almost all the additional plastic hinges are formed at the lower end of the columns. Fig.7 shows an example of the collapse mechanism. The column having a plastic hinge only at the column base is the 2nd column from the right (j=6). The bar graph shows the ratio of the rotation angle of the plastic hinges formed. In the upper part of the frame, rotation of plastic hinges at the lower end of the columns is very small. In the lower part of the frame, rotation of plastic hinges at the beam ends or the column bases decreases as the number of spans (j) decreases but rotation increases at the

Table 1 Collapse load factor λ

number of stories \ number of spans	2	3	4	5	6
2	1.123	1.126	1.131	1.136	1.141
3	1.082	1.084	1.086	1.089	1.090
4	1.065	1.067	1.069	1.071	1.072
5	1.053	1.055	1.056	1.058	1.059
6	1.047	1.046	1.047	1.049	1.050

Table 2 Position of the column forming plastic hinge only at the column base (j)

number of stories \ number of spans	2	3	4	5	6
2	j=2	3	4	5	5
3	3	4	5	5	6
4	3	4	5	5	6
5	3	4	5	6	6
6	3	4	5	6	6

lower end of the columns as j decreases. Fig.7 shows not only the result of the 6-span 6-storied frame but also the information of another frames. For example, the collapse mechanism of the 4-span 4-storied frame can be shown by the dotted region in Fig.7. On the basis of the results shown in Table 1, the collapse mechanism and the rotation angles of the plastic hinges of the composite frames ranging from 2-span 2-storied to 6-span 6-storied can be obtained.

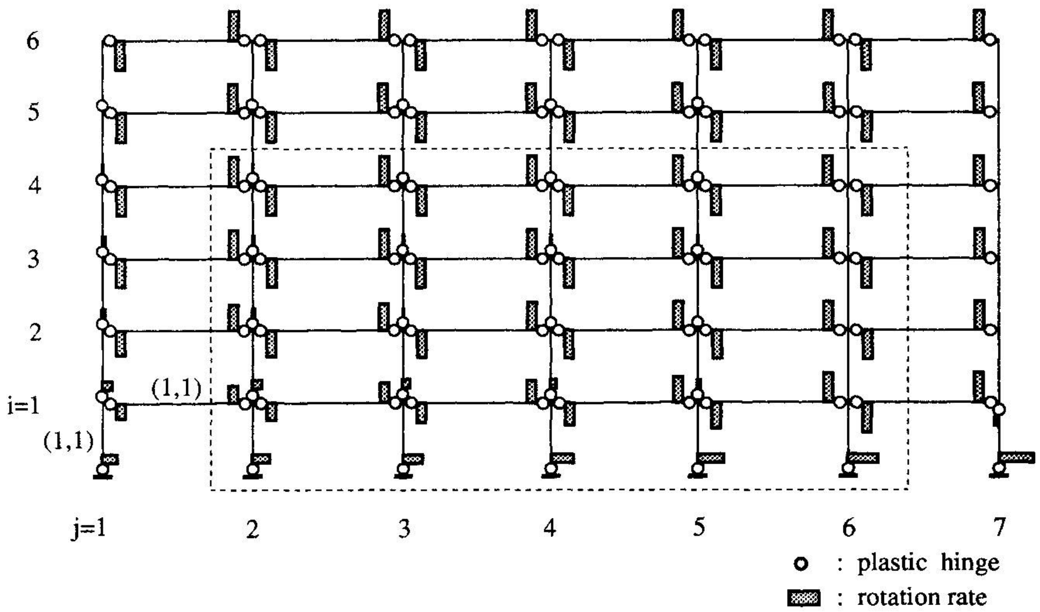


Fig.7 Collapse mechanism and rotation angle of plastic hinges

4. Conclusions

This paper presented a method for obtaining the ultimate strength and the collapse mechanism of the composite frames subjected to seismic force, considering the yield condition of the composite members and the associated flow rule. A recurrence formula is derived between the rotation angles of the plastic hinges formed at the beams and columns. Using the formula, the collapse mechanism of the composite frames composed of strong columns and weak beams is clarified, in the case of the low rise buildings. The real collapse mechanism shows that the plastic hinges are formed not only at the beam ends and column bases but also at almost all the lower end of the columns. The distribution of rotation at the plastic hinges changing from hinge to hinge, is also clarified.

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