Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 82 (1999)

Artikel: Cable finite element of high accuracy

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DOI: https://doi.org/10.5169/seals-62129

Cable Finite Element of High Accuracy

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Abstract

A long taut cable is a structure having transversally a low stiffness: applying a perpendicular load to the cable induces important transverse displacements, even if the load amount is low versus the cable tension. It is for that reason that a long taut cable with a large slope of the chord versus the vertical axis has a non-negligible sag having influence on the extension stiffness of the cable (i.e. the stiffness versus an extension of the chord joining the two supports). Moreover, this stiffness depends on the cable tension and the loads applied along the cable and following a non-linear rule.

In the simplest cases where the loads are uniformly distributed along the cable, excellent formula can be found giving the extension stiffness of the cable. In the same way, regarding the static problems encountered in the analysis of large structures, it is convenient to replace a complete cable by an equivalent truss element whose stiffness is tuned at each time step. This technique avoids to subdivide the cable in smaller elements and offers the advantage to not introduce nodes with a low stiffness inducing a slow, even a tricky, convergence in the iterative methods. Regarding dynamic analysis, the previous model in which each cable is replaced by a unique truss element with an appropriate stiffness, is not always acceptable because, for some frequencies, the kinetic energy of a specific cable can be largely underestimated. It is always possible to simulate a given cable by a number of truss elements (the simplest finite element available in all the finite element codes for structural analysis). But, in this case, to obtain a good accuracy on the frequency, a large number of truss elements is required. For instance, for a given accuracy of 0.3 percent on the frequency of the first in-plane or out-of-plane mode, a cable requires twelve truss element of equal length when using classical mass matrices.

Cable-supported structures become more and more large and flexible. The convenient assumption of a linear behaviour, as well regarding static or dynamic analysis, becomes less and less acceptable. Linear elasticity assumption remains applicable to the constitutive materials of the structures, but the displacements have to be taken into account in the equations of equilibrium, including geometric non-linearity. Computations are then carried out using iterative methods which, although they present no theoretical difficulties, are however computation time consuming because the structure is assembled at each iteration (or at each time step in dynamic analysis). It is therefore very worthwhile to have the use of finite elements of high accuracy allowing computations with the minimum number of nodes compatible with the requested accuracy.

This work outlined the equations of equilibrium of a very long cable subjected to a load varying linearly along the chord (i.e. the line joining the supports) and its computed tangent stiffness matrix. For all static structural analyses, it is convenient to use only one single innovative cable finite element, whatever its length is.

A very specific and unusual method has been developed in order to obtain the mass matrix of this finite element. The accuracy is so good that in order to obtain the first k^{th} in-plane or out-of-plane natural frequencies with an accuracy better than 0.3 percent, only $2k$ innovative finite elements are required to model the cable. This gives a saving higher than six times that obtained via modelling using classical truss elements. Regarding a step by step integration dynamic analysis of a cable subjected to support excitation, for instance a parametric excitation, the fact to divide by six the number of finite elements leads to divide the computation time by a high factor.

This innovative technique proposed for computing mass matrices has also been applied to the very simple finite elements that are truss element having a constant cross-section or straight beams having constant mechanical properties. Regarding truss elements, the saving in number of elements to have to use for a given accuracy is again 6, and regarding beams, this value is between 3 and 4. For instance, the quite accurate value of the first bending eigenvalue of a cantilever beam (or of a simply supported beam) can be obtained with only one single two nodes innovative element.

The drastic reduction of the number of degrees of freedom resulting from our study open the way of step-by-step dynamic analyses of large structure without using truncated modal decomposition.