Addendum to the Paper Entitled: "Structural Theorems for Topological Actions of Z2-Torion Real, Complex and Quaternionic Projective Spaces".

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In the statement of Theorem 3, there is a missing case that all connected components of $F(G, X)$ are of RP-type. The condition for case 2 should be: *$Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ and $Sq^1(\gamma)=0$*; and there should be the following third case, namely,

(c) *Case* 3, *$Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ and $Sq^1(\gamma)\neq 0$; then all connected components of $F(G, X)$ are of RP-type, i.e., $F^j\sim RP^{(k_j-1)}$ and there exist $w_j, v_1, v_2\in H^1(B_G)$ such that*

$$\gamma=v_1^2+v_1v_2+v_2^2, \qquad \alpha_j=w_j(w_j+v_1)(w_j+v_1)(w_j+v_1+v_2)$$

and

$$\iota_j^*(\zeta)=\xi_j^4+\gamma\xi_j^2+(Sq^1\gamma)\,\xi_j+\alpha_j, \qquad \iota_j^*: H_G^*(X)\to H_G^*(F^j).$$

And furthermore, the system of local weights at F^j, Ω_j, is given by

$$\Omega_j=\{(w_i+w_j), (v_1+w_i+w_j), (v_2+w_i+w_j), (v_1+v_2+w_i+w_j) \text{ multi.} k_i \ (i\neq j); \\ v_1, v_2, (v_1+v_2), 0 \text{ multi}(k_j-1)\},$$

and

$$F(x) \text{ is of} \begin{cases} RP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly independent,} \\ CP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly dependent but not all zero,} \\ QP\text{-type} & \text{if } v_1 \mid G_x = v_2 \mid G_x = 0. \end{cases}$$

Correspondingly, one should add the following proof to the proof of Theorem 3 for the above case 3:

In the case $Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ and $Sq^1(\gamma)=\delta\neq 0$, it is not difficult to show that all connected components of $F(G, X)$ are of RP-type. Let ξ_j be the generator of $H^*(F^j)$, ι_j^* be the restriction homomorphism of $H_G^*(X)$ to $H_G^*(F^j)$. Then, it follows from the following equations:

$$Sq^1(\iota_j^*\zeta)=\iota_j^*(Sq^1\zeta)=0 \quad \text{and} \quad Sq^2(\iota_j^*\zeta)=\iota_j^*(Sq^2\zeta)=\gamma\cdot(\iota_j^*\zeta)$$

that

$$\iota_j^*\zeta=\xi_j^4+\gamma\cdot\xi_j^2+\delta\xi_j+\alpha_j;$$
$$Sq^1\delta=Sq^1\alpha_j=0, \qquad Sq^2\delta=\gamma\delta, \qquad Sq^2\alpha_j=\gamma\alpha_j.$$

Let $f_j=(0,\dots,0,\xi_j^{(k_j-1)},0,\dots,0)$ be the fundamental cohomology class of F_j. Then, simple computation will show that the ideal $I_X(f_j)$ is generated by $a(f_j)=\delta^{(k_j-1)}\cdot\prod_{i\neq j}(\alpha_i+\alpha_j)$, (cf. the proof of Theorem 1). For simplicity, we may assume without loss of generality that $\alpha_1=0$.

(i) *Suppose that at least one* $k_j>1$: Then it follows from Theorem B that δ splits into product of linear factors, say $\delta=v_1\cdot v_2\cdot v_3$. Notice that

$$Sq^1\delta=(v_1+v_2+v_3)\cdot\delta=0 \quad \text{implies} \quad v_1+v_2+v_3=0,$$

i.e.,

$$\delta=v_1v_2(v_1+v_2),$$

and

$$Sq^2\delta=\gamma\cdot\delta \quad \text{implies} \quad \gamma=v_1^2+v_1v_2+v_2^2.$$

Again, it follows from Theorem B that $a(f_1)=\delta^{(k_1-1)}\cdot\prod_{j\neq 1}\alpha_j^{k_j}$ splits, and hence all α_j, $j\neq 1$, split into product of linear factors, say

$$\alpha_j=w_j\cdot(w_j+\mu_{j,1})(w_j+\mu_{j,2})\cdot(w_j+\mu_{j,3}).$$

On the other hand,

$$Sq^1\alpha_j=(\mu_{j,1}+\mu_{j,2}+\mu_{j,3})\cdot\alpha_j=0 \quad \text{implies that} \quad \mu_{j1}+\mu_{j2}+\mu_{j3}=0$$
$$Sq^2\alpha_j=(\mu_{j_1}\mu_{j_2}+\mu_{j_2}\mu_{j_3}+\mu_{j_3}\mu_{j_1})\cdot\alpha_j=\gamma\cdot\alpha_j$$

implies that

$$\{\mu_{j1},\mu_{j2},\mu_{j3}\}=\{v_1,v_2,v_1+v_2\}$$

as a set, namely

$$\alpha_j=w_j(w_j+v_1)(w_j+v_2)(w_j+v_1+v_2).$$

(ii) *Suppose all* $k_j=1$: Then $s=(n+1)>1$, $a(f_1)=\prod_{j\neq 1}\alpha_j$. Hence, α_j again splits and the above proof will show that

$$\alpha_j=w_j(w_j+\mu_{j1})(w_j+\mu_{j2})(w_j+\mu_{j3}), \qquad \mu_{j1}+\mu_{j2}+\mu_{j3}=0,$$

and

$$(\mu_{j1}\mu_{j2}+\mu_{j2}\mu_{j3}+\mu_{j3}\mu_{j1})=\gamma \quad \text{for all} \quad j>1,$$

and the assertion follows easily.

Remark. All actions of Case 3 are cohomologically modelled after the following type of linear actions on $QP^n=Sp(n+1)/Sp(1)\times Sp(n)$.

EXAMPLE 3c. Notice that the effective group of isometry on the symmetric space QP^n is $Sp(n+1)/\{\pm 1\}$ and $SO(3)=Sp(1)/\{\pm 1\}$ sits in $Sp(n+1)/\{\pm 1\}$ as diagonal unit quaternions. Let G be a $\mathbf{Z}_2$-tori of $Sp(n+1)/\{\pm 1\}$ containing a maximal $\mathbf{Z}_2$-tori of $SO(3)$. Then the restricted G-action on QP^n is of the type of Case 3.

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