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# Abelian $p$ -adic Group Rings

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## Introduction

In the following we investigate the question of when two finite abelian groups  $G$  and  $H$  have isomorphic group-algebras over the  $p$ -adic integers and  $p$ -adic field.

We use the following notation:  $N$  denotes the positive integers,  $p$  and  $q$  are distinct primes in  $N$ ,  $Z_n$  is the ring of integers modulo  $n$  ( $n \in N$ ),  $P$  is the ring of  $p$ -adic integers,  $Q_p$  is the field of  $p$ -adic numbers and  $C_n$  denotes a cyclic group of order  $n$  ( $n \in N$ ).

If  $G$  is a finite group and  $R$  is a principal ideal domain,  $RG = R(G)$  will be the group algebra of  $G$  over  $R$ .

## §1

Throughout this section,  $G$  and  $H$  denote finite abelian groups of order  $q^n$ . If  $F$  is a field of characteristic  $k \neq q$ , by the theorem of Perlis-Walker [3],  $F(G) \simeq \sum_{d=1}^n (n_d/v_d) F(\zeta_d)$ , where  $\zeta_d$  is a primitive  $q^d$ th root of unity over  $F$ ,  $v_d = \text{degree}(F(\zeta_d)/F)$  and  $n_d$  is the number of elements of order  $q^d$  in  $G$ . Also,  $n_d/v_d$  is a non-negative integer.

As  $(q^d, p) = 1$ , the splitting field of the polynomial  $g(x) = x^{q^d} - 1$  over  $Q_p$ , is a totally unramified extension of  $Q_p$ . This says that  $\text{degree}(Q_p(\zeta_d)/Q_p) = \text{degree}(Z_p(\zeta_d)/Z_p)$ , where  $\zeta_d$  is a primitive  $q^d$ th root of unity over  $Z_p$ . Hence we can write

$$Q_p G \simeq \sum_{d=1}^n a_d Q_p(\zeta_d) \tag{*}$$

and

$$Z_p G \simeq \sum_{d=1}^n a_d Z_p(\zeta_d)$$

for a common collection of integers  $a_d = n_d/v_d$ ,  $d = 1, 2, \dots, n$ .

The following generalization of the Perlis-Walker result is due to Raggi Cárdenas.

**PROPOSITION 1.1.** *Let  $A$  be a local ring, with maximal ideal  $M$ , and residue field  $K$  of characteristic  $p$ . Suppose  $M$  is finite, and  $\bigcap_{n=0}^{\infty} M^n = \{0\}$ . Then  $AG \simeq \sum_{d=1}^n (n_d/v_d) A(\zeta_d)$  where  $\zeta_d$  is a primitive  $q^d$ th root of unity,  $v_d = \text{degree}(K(\zeta_d)/K)$  and  $n_d$  is the number of elements of order  $q^d$  in  $G$ .*

*Proof.* See [4].

LEMMA 1.2. *If  $a_d$ ,  $d=1, 2, \dots, n$ , are defined as in (\*), then  $PG \simeq \sum_{d=1}^n a_d P(\zeta_d)$ .*

*Proof.* If  $m \in N$ , the residue field of the local ring  $Z_{p^m}$  is  $Z_p$ . Due to Proposition 1.1 we must have

$$Z_{p^m}G \simeq \sum_{d=1}^n a_d Z_{p^m}(\zeta_d).$$

By taking injective limits, (the injective limit of  $Z_{p^m}$  is  $P$ ), the above isomorphism leads to an isomorphism

$$PG \simeq \sum_{d=1}^n a_d P(\zeta_d).$$

COROLLARY 1.3. *If  $a_d$ ,  $d=1, 2, \dots, n$ , are defined as in (\*), and  $R$  is either  $Z_p$ ,  $P$  or  $Q_p$  then*

$$RG \simeq \sum_{d=1}^n a_d R(\zeta_d)$$

where  $\zeta_d$  is a primitive  $q^d$ th root of unity.

We define the function  $\gamma: N \rightarrow N$  by  $\gamma_q(m)$  is the least positive integer such that  $p^{\gamma_q(m)} \equiv 1 \pmod{q^m}$ . Then  $\gamma_q(1) \leq \gamma_q(2) \leq \dots$  and  $\gamma_q(d) \rightarrow \infty$ . If  $d \in N$ , and  $\zeta_d$  is a primitive  $q^d$ th root of unity over  $Z_p$ ,  $1 = \zeta^{q^d} = \zeta^{p^{v_d} a^{-1}}$ , so that  $\gamma_q(d) = v_d$ , and  $\gamma_q(d) \mid \gamma_q(d+1)$ ,  $d=1, 2, \dots$

Define the sequence  $\{\alpha_m\}$ ,  $m=1, 2, \dots$ , by  $\alpha_1=1$ , and if  $\alpha_m$  is defined,  $\alpha_{m+1}$  is the smallest positive integer such that  $\gamma_q(\alpha_{m+1}) > \gamma_q(\alpha_m)$ . Thus  $\gamma_q(\alpha_1) < \gamma_q(\alpha_2) < \dots$  and

$$\{\gamma_q(\alpha_i) \mid i \in N\} = \{\gamma_q(i) \mid i \in N\}.$$

If  $R$  is  $Z_p$ ,  $P$  or  $Q_p$ , then  $R(\zeta_d) = R(\zeta_{d+1}) \Leftrightarrow \gamma_q(d) = \gamma_q(d+1)$  so that we define the sequence  $b_1, b_2, \dots, b_n$  by

$$b_i = \sum_{j=\alpha_i}^{\alpha_{i+1}-1} a_j$$

where  $a_r = 0$  if  $r > n$  and  $a_1, a_2, \dots, a_n$  are as in (\*).

In terms of this notation, we can restate Corollary 1.3 as

COROLLARY 1.3'. *If  $R = Z_p$ ,  $P$  or  $Q_p$ , then there exist non-negative integers  $b_1, b_2, \dots, b_n$  such that*

$$RG \simeq \sum_{i=1}^n b_i R(\zeta_{\alpha_i})$$

where  $\zeta_{\alpha_i}$  is a primitive  $q^{\alpha_i}$ th root of unity.

We now show that the sequence  $b_1, b_2, \dots, b_n$  are invariants of  $RG$ .

If  $m \in \mathbb{N}$ , and  $r$  is a non-negative integer, by the symbol  $rC_m$  we mean the direct sum of  $r$  copies of  $C_m$ . Let  $s_1, s_2, \dots, s_n$  be a sequences of non-negative integers such that  $s_i \mid s_{i+1}$ ,  $i=1, \dots, n-1$ . Suppose that  $A$  and  $B$  are finite abelian groups of the the same order and  $A \simeq \sum_{i=1}^n r_i C_{p^{s_i-1}}$   $B \simeq \sum_{i=1}^n \bar{r}_i C_{p^{s_i-1}}$  where  $r_i, \bar{r}_i$  are non-negative integers for  $i=1, \dots, n$ . Since  $(p^{s_i}-1) \mid (p^{s_{i+1}}-1)$ , the fundamental theorem of abelian groups tells us that  $A \simeq B$  if and only if  $r_i = \bar{r}_i$ ,  $i=1, \dots, n$ .

If  $S$  is a commutative ring with identity, let  $S^* = \{\delta \in S \mid \delta \text{ is of finite multiplicative order}\}$ .

**THEOREM 1.4.** *Let  $R$  be  $Z_p$ ,  $P$ , or  $Q_p$ . If  $G$  and  $H$  are finite abelian groups of order  $q^n$ , then*

$$RG \simeq RH \Leftrightarrow (RG)^* \simeq (RH)^*.$$

*Proof.* Since adjoining  $\zeta_{\alpha_i}$  to  $Q_p$  gives a totally unramified extension of  $Q_p$ ,  $(Q_p(\zeta_{\alpha_i}))^* \simeq (Z_p(\zeta_{\alpha_i}))^* \simeq C_{p^{\nu_q(\alpha_i)}-1}$ . But any element of finite order in  $Q_p(\zeta_{\alpha_i})$  is in fact in  $P(\zeta_{\alpha_i})$  so that  $(P(\zeta_{\alpha_i}))^* \simeq C_{p^{\nu_q(\alpha_i)}-1}$ .

By Corollary 1.3', there exist non-negative integers  $b_1, b_2, \dots, b_n$  such that  $RG \simeq \sum_{i=1}^n b_i R(\zeta_{\alpha_i})$ . Thus  $(RG)^* \simeq \sum_{i=1}^n b_i C_{p^{\nu_q(\alpha_i)}-1}$ . If  $\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n$  is another sequence of non-negative integers such that  $RG \simeq \sum_{i=1}^n \bar{b}_i R(\zeta_{\alpha_i})$ ,  $(RG)^* \simeq \sum_{i=1}^n \bar{b}_i C_{p^{\nu_q(\alpha_i)}-1}$ . By the fundamental theorem of abelian groups, we must have  $\bar{b}_i = b_i$ ,  $i=1, \dots, n$ . Thus the sequence  $b_1, b_2, \dots, b_n$  is determined by  $RG$  and in turn determines, via a one-one correspondence, the group  $(RG)^*$ . Therefore  $RG \simeq RH \Leftrightarrow (RG)^* \simeq (RH)^*$ .

**COROLLARY 1.5.** *Let  $R$  be  $Z_p$ ,  $P$  or  $Q_p$ . If  $G$  is a finite abelian group of order  $q^n$ , and  $RG \simeq \sum_{i=1}^n b_i R(\zeta_{\alpha_i})$ ,  $RG \simeq \sum_{i=1}^n \bar{b}_i R(\zeta_{\alpha_i})$ , where  $b_i, \bar{b}_i$ ,  $i=1, \dots, n$ , are non-negative integers then  $b_i = \bar{b}_i$ ,  $i=1, 2, \dots, n$ .*

**COROLLARY 1.6.** *Suppose  $G$  and  $H$  are finite abelian groups of order  $q^n$ . The following are equivalent.*

- (i)  $Z_p G \simeq Z_p H$ , (ii)  $P G \simeq P H$ , (iii)  $Q_p G \simeq Q_p H$ .

*Proof.* Note that  $(Z_p G)^* \simeq (P G)^* \simeq (Q_p G)^*$  and use Theorem 1.4

## §2

If  $A$  is a finite abelian group of order  $n = p^e q_1^{e_1} q_2^{e_2} \dots q_r^{e_r}$ , we let  $A_p$  be the  $p$ -Sylow subgroup of  $A$  and  $A_{q_i}$  the  $q_i$ -Sylow subgroup of  $A$ . Again we start from a result of Perlis-Walker.

**PROPOSITION 2.1.** *If  $A$  and  $B$  are finite abelian groups of order  $n = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$  where  $p_i, i = 1, \dots, r$ , are distinct primes, and  $F$  is a field of characteristic  $k$ , where  $k = 0$  or  $(k, n) = 1$ , then  $FA \simeq FB \Leftrightarrow FA_{p_i} \simeq FB_{p_i}, i = 1, \dots, r$ .*

*Proof.* See [3] and [1].

**PROPOSITION 2.2.** *Let  $A$  and  $B$  be finite abelian groups of order  $p^n$ . Then  $Q_p A \simeq Q_p B \Leftrightarrow A \simeq B$ .*

*Proof.* By the result of Perlis-Walker (of the last section),  $Q_p A \simeq \sum_{d=1}^n a_d Q_p(\zeta_d)$  where  $\zeta_d$  is a primitive  $p^d$ th root of unity,  $a_d = n_d/v_d$ ,  $n_d$  is the number of elements in  $A$  of order  $p^d$ , and  $v_d = \text{degree}(Q_p(\zeta_d)/Q_p)$ . Similarly,  $Q_p B \simeq \sum_{d=1}^n \bar{a}_d Q_p(\zeta_d)$ , where  $\bar{a}_d = \bar{n}_d/v_d$ , and  $\bar{n}_d$  is the number of elements in  $B$  of order  $p^d$ .  $Q_p A \simeq Q_p B$  implies

$$(Q_p A)^* \simeq \sum_{d=1}^n a_d (Q_p(\zeta_d))^* \simeq \sum_{d=1}^n \bar{a}_d (Q_p(\zeta_d))^* \simeq (Q_p B)^*.$$

$Q_p$  contains only the  $(p-1)$ st roots of unity, and  $Q_p(\zeta_d)$  is a totally ramified extension of  $Q_p$ , so that  $(Q_p(\zeta_d))^* \simeq C_{p^d(p-1)}$ . By the fundamental theorem of abelian groups we have  $a_d = \bar{a}_d, d = 1, \dots, n$ , and so  $n_d = \bar{n}_d, d = 1, \dots, n$ . Thus  $A \simeq B$ .

**COROLLARY 2.3.** *If  $A$  and  $B$  are finite abelian groups of order  $n$ , and  $Q$  is the field of rational numbers, then  $QA \simeq QB \Leftrightarrow A \simeq B$ .*

*Proof.* By Proposition 2.1, it is sufficient to suppose that  $n = p^m$ . If  $QA \simeq QB$ , then  $QA \otimes Q_p \simeq QB \otimes Q_p$ ; i.e.,  $Q_p A \simeq Q_p B$ . But by Proposition 2.2,  $A \simeq B$ .

**COROLLARY 2.4.** *If  $A$  and  $B$  are finite abelian groups of order  $p^n$ , then  $PA \simeq PB \Leftrightarrow A \simeq B$ .*

*Proof.*  $PA \simeq PB$  implies  $PA \otimes Q_p \simeq PB \otimes Q_p$ . By Proposition 2.2  $A \simeq B$ .

Note that if  $A$  and  $B$  are finite abelian groups of order  $p^n$ , then  $Z_p A \simeq Z_p B \Leftrightarrow A \simeq B$ . See [2], e.g.

**THEOREM 2.5.** *Suppose  $R$  is  $Z_p, P$  or  $Q_p$ . If  $G$  and  $H$  are finite abelian groups of order  $n = p^e q_1^{e_1} \cdots q_r^{e_r}$  where  $p, q_1, \dots, q_r$  are distinct primes, then*

$$RG \simeq RH \Leftrightarrow RG_p \simeq RH_p \quad \text{and} \quad RG_{q_i} \simeq RH_{q_i} \quad i = 1, \dots, r.$$

*Proof.* If  $R = Q_p$ , the result follows immediately by Proposition 2.1.

Suppose  $R = Z_p$ , and  $Z_p G \simeq Z_p H$ . By the results in May [2],  $G_p \simeq H_p$ . Suppose  $|G_p| = p^{c_1}$ ,  $|H_p| = p^{c_2}$ , and  $c = \max(c_1, c_2)$ . Let  $(Z_p G)^{p^c} = \{\delta^{p^c} \mid \delta \in Z_p G\}$ .  $(Z_p G)^{p^c} \simeq (Z_p H)^{p^c}$ . As  $Z_p^p \simeq Z_p$ , we have  $(Z_p G)^{p^c} \simeq Z_p(G/G_p) \simeq (Z_p H)^{p^c} \simeq Z_p(H/H_p)$ . By Proposition 2.1,  $Z_p(G/G_p) \simeq Z_p(H/H_p) \Rightarrow Z_p(G_{q_i}) \simeq Z_p(H_{q_i}), i = 1, \dots, r$ . Thus  $Z_p(G) \simeq Z_p(H) \Rightarrow Z_p G_p \simeq Z_p H_p$  and  $Z_p(G_{q_i}) \simeq Z_p(H_{q_i}), i = 1, \dots, r$ . The opposite implication follows from the tensor product.

If  $R=P$ , and  $PG \simeq PH$ , we let  $J=(p)$  be the ideal of  $P$  generated by  $p$ . The epimorphism  $P \rightarrow P/J \simeq Z_p$ , induces an isomorphism  $Z_p G \simeq Z_p H$ . By the previous case  $Z_p G \simeq Z_p H \Rightarrow G_p \simeq H_p$  and  $Z_p G_{q_i} \simeq Z_p H_{q_i}$ ,  $i=1, \dots, r$ . But by Corollary 1.6  $Z_p G_{q_i} \simeq Z_p H_{q_i} \Leftrightarrow PG_{q_i} \simeq PH_{q_i}$ , so that  $PG \simeq PH \Rightarrow PG_p = PH_p$  and  $PG_{q_i} \simeq PH_{q_i}$ ,  $i=1, \dots, r$ . The opposite implication follows from the tensor product.

**COROLLARY 2.6.** *Let  $G$  and  $H$  be finite abelian groups of order  $n$ . The following are equivalent.*

- (i)  $Z_p G \simeq Z_p H$ , (ii)  $PG \simeq PH$ , (iii)  $Q_p G \simeq Q_p H$ .

### §3

In this section we investigate for which integers  $m \in N$ , is it true that two abelian groups  $G$  and  $H$  of order  $m$  have isomorphic  $p$ -adic group rings if and only if  $G$  and  $H$  are isomorphic. To study this question, it is sufficient, by Theorem 2.5, to again suppose that  $G$  and  $H$  are abelian groups of order  $q^n$ . We will use the notation of Section 1.

**DEFINITION.** We let  $\mathcal{J}(q) = r$ , if there is an  $r \in N$  such that  $\alpha_r = r$  and  $\alpha_{r+1} \neq (r+1)$ . Otherwise, we define  $\mathcal{J}(q) = \infty$ . We will call  $\mathcal{J}(q)$ , the index of  $q$ , (relative to  $p$ ).

**THEOREM 3.1.** *Let  $R$  be  $Z_p, P$  or  $Q_p$ . Let  $n \in N$ .*

- (i) *If  $n < 2\mathcal{J}(q)$ , and  $G$  and  $H$  are abelian groups of order  $q^n$ , then*

$$RG \simeq RH \Leftrightarrow G \simeq H.$$

- (ii) *If  $n \geq 2\mathcal{J}(q)$ , there exist non-isomorphic abelian groups  $G$  and  $H$ , of order  $q^n$  such that  $RG \simeq RH$ .*

*Proof.* (i) Suppose  $G \simeq C_{q^{y_1}} \times C_{q^{y_2}} \times \dots \times C_{q^{y_r}}$  and  $H \simeq C_{q^{z_1}} \times C_{q^{z_2}} \times \dots \times C_{q^{z_r}}$  where  $y_1 \geq y_2 \geq \dots \geq y_r \geq 0$ ,  $z_1 \geq z_2 \geq \dots \geq z_r \geq 0$  and  $y_1 + y_2 + \dots + y_r = z_1 + z_2 + \dots + z_r = n$ . By Corollary 1.3', there exist non-negative integers  $b_1, b_2, \dots, b_n$  and  $\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n$  such that  $RG \simeq \sum_{i=1}^n b_i R(\zeta_{q^i})$  and  $RH \simeq \sum_{i=1}^n \bar{b}_i R(\zeta_{q^i})$ . By Corollary 1.5,  $RG \simeq RH \Leftrightarrow b_i = \bar{b}_i$ ,  $i=1, \dots, n$ . We suppose  $RG \simeq RH$ .  $b_i = n_i / \gamma_q(i)$  for  $i=1, \dots, \mathcal{J}(q)-1$ , where  $n_i$  denotes the number of elements of  $G$  of order  $q^i$ . Similarly,  $\bar{b}_i = m_i / \gamma_q(i)$ , where  $m_i$  denotes the number of elements of  $H$  of order  $q^i$ . Thus if  $y_1 < \mathcal{J}(q)$  or  $z_1 < \mathcal{J}(q)$  we must have  $n_i = m_i$ ,  $i=1, \dots, n$ , and  $G \simeq H$ .

We will assume now  $y_1 \geq \mathcal{J}(q)$  and  $z_1 \geq \mathcal{J}(q)$ .

Suppose  $y_1 > z_1 \geq \mathcal{J}(q)$ . Let  $\bar{G} \simeq C_{q^{y_2}} \times C_{q^{y_3}} \times \dots \times C_{q^{y_r}}$  and  $\bar{H} \simeq C_{q^{z_2}} \times C_{q^{z_3}} \times \dots \times C_{q^{z_r}}$ . Then  $G \simeq C_{q^{y_1}} \times \bar{G}$ ,  $H \simeq C_{q^{z_1}} \times \bar{H}$ ,  $|\bar{H}| > |\bar{G}|$ ,  $|\bar{G}| \leq q^{n - \mathcal{J}(q) - 1} \leq q^{\mathcal{J}(q) - 2}$  and

$$|\bar{H}| \leq q^{n-\mathcal{J}(q)} \leq q^{\mathcal{J}(q)-1}$$

$$b_{\mathcal{J}(q)-1} = \frac{n_{\mathcal{J}(q)-1}}{\gamma_{q(\mathcal{J}(q)-1)}} = \bar{b}_{\mathcal{J}(q)-1} = \frac{m_{\mathcal{J}(q)-1}}{\gamma_{(q^{\mathcal{J}(q)-1})}}$$

Clearly,  $n_{\mathcal{J}(q)-1} = \phi(q^{\mathcal{J}(q)-1})|\bar{G}|$  ( $\phi$  = Euler “phi” functions) while  $m_{\mathcal{J}(q)-1} \geq \phi(q^{\mathcal{J}(q)-1})|\bar{H}| > \phi(q^{\mathcal{J}(q)-1})|\bar{G}| = n_{\mathcal{J}(q)-1}$ . This contradiction shows  $y_1 = z_1$ .

Finally, we assume  $y_i = z_i, i = 1, \dots, t, t \geq 1$ , and  $z_{t+1} < y_{t+1} < \mathcal{J}(q)$ . Let

$$G' = C_{q^{y_1}} \times C_{q^{y_2}} \times \dots \times C_{q^{y_r}}$$

$$G'' = C_{q^{y_{t+1}}} \times \dots \times C_{q^{y_r}}$$

$$H'' = C_{q^{z_{y+1}}} \times \dots \times C_{q^{z_r}}$$

then  $G \simeq G' \times G'', H \simeq G' \times H''$  and  $|G''| = |H''| < q^{\mathcal{J}(q)}$ .

Let  $\mu = y_{t+1}$ . Then

$$b_\mu = \frac{n_\mu}{\gamma_{q(\mu)}} = \frac{m_\mu}{\gamma_{q(\mu)}} = \bar{b}_\mu.$$

If  $\lambda$  is the number of elements of  $G'$  of order  $q^\mu$ , then  $m_\mu = \lambda|H''|$ . But  $n_\mu \geq \lambda|G''| + |G'|(\phi(q^\mu)) > \lambda|G''| = m_\mu$ . This is the desired contradiction, and we conclude  $y_i = z_i, i = 1, \dots, r$ , and  $GW = H$ .

(ii) If  $n = 2\mathcal{J}(q)$ , let

$$G \simeq C_{q^{\mathcal{J}(q)}} \times C_{q^{\mathcal{J}(q)}}$$

and

$$H \simeq C_{q^{\mathcal{J}(q)+1}} \times C_{q^{\mathcal{J}(q)-1}}.$$

A straightforward verification shows that  $b_i = \bar{b}_i, i = 1, \dots, \mathcal{J}(q)-1$  and  $b_i = \bar{b}_i = 0, i > \mathcal{J}(q)$ . Since  $|G| = |H|$ , we must have  $b_{\mathcal{J}(q)} = \bar{b}_{\mathcal{J}(q)}$  and  $RG \simeq RH$ .

If  $n > 2\mathcal{J}(q)$ , we merely tack on a sufficient number of copies of  $C_q$  to each of the above examples.

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