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Correction to 'The Boolean algebra of spectra'

Commentarii Mathematici Helvetici 54/3 (1979) pp. 368–377

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J. M. Boardman and others have pointed out an error in my proof of Proposition 1.5 [1]. Namely, the presentation of the CW-spectrum B_λ/A as the homotopy cofibre of $1-g$ on p. 371 is incorrect for a general limit ordinal λ. The error arose when I wrongly simplified an earlier proof, and the proposition remains valid. As suggested by Boardman, the proof can be repaired by using the equivalence of B_λ/A with the homotopy colimit C of the transfinite sequence $\{B_s/A\}_{s<\lambda}$. The required theory of homotopy colimits can be found in [3], [4], [5]. In more detail, C can be obtained by imposing appropriate face identifications on the wedge of the $B_{s_0}/A\wedge(\Delta^n\cup *)$ running over all $(n+1)$-tuples of ordinals $s_0<s_1<\cdots<s_n<\lambda$ for all $n\geq 0$. Thus, C is a CW-spectrum with an increasing filtration by closed subspectra $\{F_nC\}$ such that $F_nC/F_{n-1}C$ is the wedge of the $B_{s_0}/A\wedge S^n$ running over all $(n+1)$-tuples of ordinals $s_0<s_1<\cdots<s_n<\lambda$. The associated spectral sequence for π_*C has $E^2_{n,t}\approx\operatorname{colim}^n\{\pi_tB_s/A\}$, and this derived colimit vanishes for $n>0$ because it is indexed by a directed set. Thus there is an edge isomorphism $\operatorname{colim}_{s<\lambda}\pi_*B_s/A\approx\pi_*C$ and the canonical map $C\to B_\lambda/A$ is a weak equivalence of CW-spectra. Consequently $C\simeq B_\lambda/A$. This equivalence can also be shown by using the isomorphisms $\operatorname{colim}_T\pi_*C_T\approx\pi_*C$ and $\pi_*C_T\approx\pi_*B_{m(T)}/A$ where T runs over all finite nonempty sets of ordinals less than λ, where $C_T\subset C$ is the homotopy colimit of the finite sequence $\{B_t/A\}_{t\in T}$, and where $m(T)$ is the largest ordinal in T. Having shown $C\simeq B_\lambda/A$ one uses the Milnor cofibering $\bigvee_{n\geq 0}F_nC\to\bigvee_{n\geq 0}F_nC\to C$ together with the above wedge decomposition of $F_nC/F_{n-1}C$ to deduce that B_λ/A is $[E,\ ]_*$-colocal and belongs to Class-E as required for the proof of Proposition 1.5 [1] and for subsequent applications. A similar error appeared in the proof of Lemma 1.13 [2] and can be repaired similarly.

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