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Subgroups with projective abelianization and trivial multiplicator

MICHEAL DYER

1. Introduction

In this note we study the exact sequence

$$L \rightarrowtail G \twoheadrightarrow H$$

of groups and homomorphisms where the first homology H_1L of L is a projective H-module and the second homology H_2L is trivial. We call such subgroups *projective* subgroups. They arise as examples of projective G-crossed modules. See [R] for more details.

The motivating topological setting is as follows: let X be a connected subcomplex of a connected two-dimensional aspherical CW-complex Y and let $i: X \to Y$ be the inclusion. Let $L = \ker\{i_{\#}: \pi_1X \to \pi_1Y\}$ be the normal subgroup of $G = \pi_1X$ and $H = \operatorname{im} i_{\#}$. Then L is a projective subgroup ([D], [BD]).

Here are several examples of projective subgroups. Let X be a set and $F(X) = F$ be the free group on X. For any group H, consider $G = F * H$. Then setting elements of F equal to 1 gives an extension (which is split)

$$L \rightarrowtail G \twoheadrightarrow H$$

where L is the normal closure $\langle\langle F\rangle\rangle_G$ of F in G. It follows from the Kurosh theorem [Se, Theorem 14] that L is free on $\{hxh^{-1} \mid x \in X, h \in H\}$, that H_1L is a free H-module on X, and that $H_2L = 0$.

As a second example, consider a 1-relator presentation $\mathcal{P} = (X; r)$ of the group G. Let $F = F(X)$ and $R = \langle\langle r\rangle\rangle_F$. If the word r is *not* a proper power in F, then $H_1R \approx \mathbb{Z}G$ and $H_2R = 0$ (because R is free). This follows because the cellular 2-complex K modeled on $\mathcal{P}$ is aspherical [C].

Of course, if $L = G$ is projective, we simply mean that H_1L is free abelian and $H_2L = 0$. A projective group $L = G$ which is not a free G-crossed module would be one whose weight $>$ rank H_1L. Here the weight of G is the minimal number of normal generators of G.

Another example is $H_1L = H_2L = 0$ (L is *superperfect*). Any such L is a projective subgroup.

Projective subgroups are *hereditary* in the following sense. Suppose L is a projective subgroup of $G = K * F$. Then $\bar{L} = L \cap K$ is a projective subgroup of K. See section 3.

In this note we study the lower central series of projective subgroups. The main theorem states that if H_1L is a submodule of a free H-module and $H_2L = 0$, then each quotient L_n/L_{n+1} $(n \geq 1)$ of the lower central series $\{L_n\}$ of L is the submodule of a free H-module. The proof is an extension of the proof devised by Ralph Strebel in [S]. Applications are given to the upper central series of G.

To fix notation, we let $[a, b] = aba^{-1}b^{-1}$ and $\mathbb{Z}G$ denote the integral group ring of the group G.

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2. The lower central series of L

For any group L, define $L_1 = L$ and $L_{n+1} = [L_n, L]$ $(n \geq 1)$. This is called the *lower central series* of L. If L is a normal subgroup of G, then conjugation by elements of g ($l_n \to gl_ng^{-1}$) induces a left H ($H = G/L$)-module structure on each L_n/L_{n+1} $(n \geq 1)$. Note that $H_1L = L_1/L_2$. The graded object $\operatorname{gr} L = \{L_n/L_{n+1}\}_{n \geq 1}$ has the structure of a graded Lie-ring over $\mathbb{Z}H$ with the Lie bracket equal to [,].

If M is a left H-module, then the graded object

$$TM = \{\mathbb{Z}H, M, M \otimes M, M \otimes M \otimes M, \cdots\}$$

has the structure of a graded $\mathbb{Z}H$-algebra with multiplication given by tensoring: $m \cdot n = m \otimes n$. Here $M \otimes M$ means tensor product over $\mathbb{Z}$ with the diagonal H-action.

We now state and prove the main result of this paper.

MAIN THEOREM 2.1. *Suppose $L \rightarrowtail G \twoheadrightarrow H$ is an exact sequence of groups with H_1L a submodule of a free H-module and H_2L a torsion group. Then each successive quotient L_n/L_{n+1} of the lower central series L_n $(1 \leq n < \omega)$ is a submodule of a free H-module.*

Proof. We follow the proof of Theorem 1 of [S], p. 149, and check that at each stage the maps defined are isomorphisms of H-modules. This yields a graded Lie

$\mathbb{Z}H$-algebra isomorphism $\alpha : \operatorname{gr} L \to TH_1L$ from the graded Lie $\mathbb{Z}H$-algebra $\operatorname{gr} L$ associated with L *onto* the Lie $\mathbb{Z}H$-subalgebra of TH_1L generated by $H_1L = T^1H_1L$. It is clear that if H_1L is a submodule of a free H-module F, then TH_1L is a subalgebra of TF, with each $T^iF = F \otimes \cdots \otimes F$ (i times) a free H-module. Hence, each $\alpha(L_n/L_{n+1})$ is a submodule of T^nH_1L, which in turn is a submodule of T^nF.

Let $I = IL$ be the augmentation ideal of $\mathbb{Z}L$ and define a descending chain of normal subgroups of L by setting

$$D^i(L) = \{l \in L \mid l - 1 \in I^i\}$$

This series is a central series, and we can form the associated graded Lie $\mathbb{Z}H$-algebra $\operatorname{gr}\{D(L)\}$, because each $D^i(L)$ is *normal in G*.

In order to see that H acts on D^i/D^{i+1} via conjugation by elements of G, it is enough to show that if $l \in D^i(L)$, then, for any $\omega \in L$, $\omega l \omega^{-1} \equiv l \bmod D^{i+1}(L)$; i.e., $\omega l \omega^{-1} l^{-1}$ is a member of $D^{i+1}(L)$. But an easy calculation shows that $\omega l \omega^{-1} l^{-1} - 1 = (\omega - 1)(l-1)l^{-1} - \omega(l-1)(\omega^{-1} - 1)l^{-1}$. So if $l - 1 \in I^i$, then $\omega l \omega^{-1} l^{-1} - 1 \in I^{i+1}$ and we have verified H acts on $\{D^i/D^{i+1}\}$ via conjugation.

The inclusion $L_j \subseteq D^j(L)$ allows one to define a Lie $\mathbb{Z}H$-algebra homomorphism $i : \operatorname{gr} L \to \operatorname{gr}(D(L))$.

Let $\operatorname{gr} \mathbb{Z}L$ denote the graded $\mathbb{Z}H$-algebra associated to the filtration $\{I^j\}_{0 \le j < \omega}$ of $\mathbb{Z}L$. Here H acts on I^j/I^{j+1} via conjugation by elements of G. This is well-defined because conjugation by elements of L is trivial $\bmod I^{j+1}$. The function $g \mapsto g - 1$ then defines an *injective* Lie $\mathbb{Z}H$-algebra homomorphism $\beta : \operatorname{gr}(D(L)) \to \operatorname{gr} \mathbb{Z}L$.

Finally, we use the isomorphism $\mu : H_1L \approx I/I^2$ $(l \cdot L' \mapsto (l-1) + I^2)$ to extend to a homomorphism $\mu : TH_1L \to \operatorname{gr} \mathbb{Z}L$ of graded associative $\mathbb{Z}H$-algebras, given in degree j by (H acts diagonally on TH_1L)

$$l_1L_2 \otimes l_2L_2 \otimes \cdots \otimes l_jL_2 \mapsto (l_1 - 1)(l_2 - 1) \cdots (l_j - 1) + I^{j+1}.$$

Clearly, μ is always surjective; it is also injective if H_1L is torsion free and H_2L is a torsion group (see [S], p. 150). The Lie $\mathbb{Z}H$-algebra homomorphism $\alpha : \operatorname{gr} L \to TH_1L$ is defined by $\alpha = \mu^{-1}\beta i$ in the following diagram:

$$\operatorname{gr} L \xrightarrow{i} \operatorname{gr}(D(L)) \xrightarrow{\beta} \operatorname{gr} \mathbb{Z}L \xleftarrow{\mu} TH_1L.$$

On page 151 of [S], Strebel shows that α is a monomorphism. ■

The following example shows that even if H_1L is a free H-module and

$L \rightarrowtail G \twoheadrightarrow H$ is *split*, the quotients L_n/L_{n+1} are not necessarily *projective*. Let $H=\mathbb{Z}_2=\{e, h\}$ and $G=\mathbb{Z} * H$, where $\mathbb{Z}$ generated by x. Then $L=\langle\langle \mathbb{Z}\rangle\rangle_G$ is the free group of rank 2 with basis x and $y=hxh^{-1}$. We order $x<y$, as weight one basic commutators. The only basic commutator of weight 2 is $c_1=[y, x]$. The action of H on $L_2/L_3 \cong \mathbb{Z}$, generated by $\bar{c}_1$, is non-trivial, because $h[y, x]h^{-1}=[hyh^{-1}, hxh^{-1}]=[x, y]=[y, x]^{-1}$. Hence L_2/L_3 is not projective, but is still a submodule of $\mathbb{Z}H$, as guaranteed by Theorem 2.1.

It is intriguing to ask just when the L_n/L_{n+1} might themselves be projective. The next theorem gives a partial result in this direction.

A group H is said to be *ordered* if there is a linear ordering $\leq$ on H such that if $a \leq b$ in H, then $ah \leq bh$ and $ha \leq hb$ for all h in H. Note that $1 \leq a$ in H iff $a^{-1} \leq 1$. For example, any torsion free nilpotent group is ordered [P, p. 581].

THEOREM 2.2. *Suppose $L \rightarrowtail G \twoheadrightarrow H$ is a split exact sequence of groups, with $H_2L=0$, H_1L a free H-module, and H an ordered group. Then* gr L *is a free graded H-module.*

First, we prove the following.

LEMMA 2.3. *Suppose $F=F(X)$ is a free group with basis X and H is any ordered group. Form the group $G=F * H$ and the split exact sequence $L \overset{i}{\rightarrowtail} G \overset{\varphi}{\twoheadrightarrow} H$ where φ is obtained by setting elements of F equal to 1 and L is the normal closure $\langle\langle F\rangle\rangle_G$ of F in G. Then each free abelian group L_n/L_{n+1} $(n \geq 1)$ is a free H-module.*

Proof. If X is a basis for F, then $\bar{X}=\{hxh^{-1} \mid x \in X, h \in H\}$ is a basis for the free group $L=\langle\langle F\rangle\rangle_G$. We order X arbitrarily and $\bar{X}$ lexicographically according to the pairing (x, h). We consider the basic commutators in $\bar{X}$ (see [H], p. 166). Each basic commutator c_k of weight k is of the form (uniquely, as L is free)

$$c_k=[c_i, c_j]$$

where $wt(c_i)+wt(c_j)=k$, c_i, c_j are basic commutators and $c_i>c_j$. If $c_i=[c_r, c_s]$, then $c_j \geq c_s$. We *order* the basic commutators of weight k *lexicographically* by using the pairing (c_i, c_j).

We now will prove inductively the following: if c_l, c_m are basic commutators of weight k and $h \in H$, then (1) hc_lh^{-1} is a basic commutator of weight k and (2) $c_l>c_m$ implies that $hc_lh^{-1}>hc_mh^{-1}$, using the lexicographic ordering given above. It is clearly true for $k=1$, using the ordering on $\bar{X}$ and that H is an ordered group. If $c_l=[c_i, c_j]$ with $c_i>c_j$, then $hc_lh^{-1}=[hc_ih^{-1}, hc_jh^{-1}]$, with $hc_ih^{-1}>hc_jh^{-1}$. Also if $c_i=[c_r, c_s]$ and $c_j \geq c_s$, then $hc_jh^{-1} \geq hc_sh^{-1}$, by induction. Finally, we must show that if $c_l > c_m$, then $hc_lh^{-1} > hc_mh^{-1}$. Let $c_m = [c_a, c_b]$ with $c_a > c_b$. If $c_i > c_a$, then $hc_ih^{-1} > hc_ah^{-1}$, while if $c_i = c_a$ and $c_j > c_b$, then $hc_jh^{-1} > hc_bh^{-1}$ is true by induction. Thus (1) and (2) are true for all basic commutators.

Now it is easy to find basic commutators of wt k which form an H-basis for L_k/L_{k+1}. These will consist of basic commutators of wt k whose first occurrence of an element of $\bar{X}$ is actually an element of X. For example, basic commutators of weight 2 look like $[hxh^{-1}, gyg^{-1}]$ where $x, y \in X$, $g, h \in H$ and $(x, h) > (y, g)$. Then the set $\{[x, hyh^{-1}] \mid x, y \in X,\ h \in H \text{ and } (x, 1) > (y, h)\}$ is a $\mathbb{Z}H$-basis because

$$\{h_1[x, hyh^{-1}]h_1^{-1} \mid h_1, h \in H, x, y \in X, (x, 1) > (y, h)\}$$

yields all weight 2 basic commutators with no repetitions. ∎

Proof of Theorem 2.2. Let $s: H \to G$ denote a splitting of the sequence $L \rightarrowtail G \twoheadrightarrow H$. Because H_1L is a free H-module and $H_2L = 0$, we may choose a subset X of L so that the image of X in H_1L is a $\mathbb{Z}H$-basis for H_1L and so that the corresponding $\bar{X} = \{(sh)x(sh^{-1}) \mid L \in H, x \in X\}$ (see [HS], p. 204) is the basis for a free subgroup $\bar{L} < L$. Let $F = F(X)$ be the free subgroup of L generated by X. Let $\bar{G} = F * H$ and consider the split exact sequence and commutative diagram:

$$\begin{tikzcd} \bar{L} = \langle\langle F \rangle\rangle_{\bar{G}} \arrow[r, tail] \arrow[d, tail, "i"] & \bar{G} \arrow[r, two heads, shift left] \arrow[d, "\omega"] & H \arrow[l, shift left] \arrow[d, equal] \\ L \arrow[r, tail] & G \arrow[r, shift right] & H \arrow[l, shift right, "s"'] \end{tikzcd}$$

The map ω is defined by the inclusion $i: F \to L$ and the splitting s. Because $H_1(i)$ is an isomorphism and $H_2(i)$ is zero we have that the map

$$\bar{L}_n/\bar{L}_{n+1} \to L_n/L_{n+1}$$

is an isomorphism of $\mathbb{Z}H$-modules, which are free by the lemma. ∎

3. Applications to groups

In this section we apply the main theorem about the structure of gr L to show that often the center $\mathscr{Z}G$ of G must be "buried" inside L; i.e., $\mathscr{Z}G \subset \bigcap_{n<\omega} L_n = L_\omega$.

First we state a simple lemma about certain elements in group rings which are not zero divisors.

LEMMA 3.1. *Let H be a group and h be an element in H. Then $(h-1) \in \mathbb{Z}H$ is a zero divisor iff the order of h is finite: $(h+1)$ is a zero divisor in $\mathbb{Z}H$ iff the order of h is even. If $|n| \neq 1$, then $(h-n)$ is never a zero divisor in $\mathbb{Z}H$.* ∎

THEOREM 3.2. *Let* $\rightarrowtail G \overset{\varphi}{\twoheadrightarrow} H$ *be an exact sequence of groups with* H *torsion free,* H_1L *isomorphic to a submodule of a free* H*-module and* $H_2L=0$. *Let* $g \in G-L$. *Then any* $l \in L$ *which commutes with* g *must live in* $L_\omega = \bigcap L_n$.

Proof. Let the abelianization $L \to H_1L$ be denoted by $l \mapsto \bar{l}$ and $\varphi(g)=\hat{g}$ $(l \in L, g \in G)$. Then $glg^{-1}l^{-1}=1 \Rightarrow \hat{g}\bar{l}-\bar{l}=0$ in $H_1L \Rightarrow (\hat{g}-1)\bar{l}=0$ in $H_1L \subset \bigoplus_{i\in I} \mathbb{Z}H_i$. Write $\bar{l}=(\bar{l}_i)_{i\in I}$, where each $\bar{l}_i \in \mathbb{Z}H$. Thus $((\hat{g}-1)\bar{l}_i)=0$ and it follows from the lemma that each $\bar{l}_i=0$; hence $\bar{l}=0$. So $l \in L_2$. We use Theorem 2.1 and a similar argument to show that $l \in L_n$ for all $n \geq 2$. ■

Note 3.3. (a) A similar argument will show that if $g \in G-L$ and $l \in L$ satisfies $glg^{-1}=l^n$ $(n \in \mathbb{Z})$, then $l \in L_\omega$.

(b) Other identities may be used. For example, if $g_1, g_2 \in G-L$ and $l \in L$ with $[[g_1, l], g_2]=1$, then $l \in L_\omega$. This follows because $\overline{[[g_1, l], g_2]}=(1-\hat{g}_2)\overline{[g_1, l]}=(1-\hat{g}_2)(\hat{g}_1-1)\bar{l}=0$. Then apply the argument twice.

Recall that, if G is a group, then the nth order center of G, $\mathfrak{X}^nG$ $(n \geq 1)$, is inductively defined as $\mathfrak{X}^1G=$ center of G, $\mathfrak{X}^{n+1}G=\{g \in G \mid \varphi_n(g) \in \mathfrak{X}^1(G/\mathfrak{X}^nG)$, where $\varphi: G \to G/\mathfrak{X}^nG$ is the natural map$\}$.

COROLLARY 3.4. *Suppose that the exact sequence of groups is as in Theorem 3.2 with* $G-L \neq \varnothing$ *and* $H_1L \neq 0$. *Then all the centers* $\mathfrak{X}^nG$ $(n \geq 1)$ *are contained in* L_ω.

Proof. (a) We show that $\mathfrak{X}^1G \subset L_\omega$. Suppose there is an element $g \in (G-L) \cap \mathfrak{X}^1G$. Choose any $l \in L$ and observe that $[g, l]=1$. By Theorem 3.2, we see that $l \in L_\omega$. Hence $L=L_\omega$, which contradicts the hypothesis that $H_1L \neq 0$. Hence $\mathfrak{X}G \subset L$.

Now choose any $g \in G-L$ and $l \in \mathfrak{X}G \subset L$. Again, the proposition shows that $l \in L_\omega$. Hence $\mathfrak{X}^1G \subset L_\omega$.

(b) We observe that $g \in \mathfrak{X}^2G$ iff for all $g_1, g_2 \in G$ the commutator $[[g_1, g], g_2]=1$. Now suppose $g \in G-L \cap \mathfrak{X}^2G$. Choose any $l \in L$ and observe that $[[g, l], g_2]=1$ for all $g_2 \in G$. Choosing $g_2 \in G-L$, we see that $l \in L_\omega$ for all $l \in L$ by note (b). The rest of the argument is similar to (a).

(c) One can either prove $\mathfrak{X}^nG \subset L$ for $n \geq 2$ by induction or by studying the higher commutators $[\ldots, [[g_1, l], g_2] \ldots, g_n]$. ■

Note 3.5. Corollary 3.4 also follows without using 3.2 because L/L_ω is a non-abelian free group (under the assumption imposed above) and in that case $\mathfrak{X}^nG \subseteq \mathfrak{X}^nL \subseteq L_\omega$.

Note 3.6. The above corollary is false if $H_1L=0$ or if $G=L$. Let L be a superperfect group, let $G=\mathbb{Z}\times L$ and $G\to\mathbb{Z}$ be the projection with kernel L. Then $\mathbb{Z}\subset\mathscr{Z}G$ is not contained in L. Also, if $G=L=\mathbb{Z}$, then $\mathscr{Z}G=\mathbb{Z}$ is not contained in $L_\omega=0$.

In order to give the next application, we need the notion of a C-subgroup. We say that a subgroup $N<G$ is a *C-subgroup* if there is a G-projective resolution $P_* \twoheadrightarrow \mathbb{Z}$ of the trivial module $\mathbb{Z}$ for which the homomorphism $N\otimes\partial_3:\mathbb{Z}\otimes_N P_3\to \mathbb{Z}\otimes_N P_2$ is trivial. See [BDS] for properties and applications of this concept.

The next result shows that, in some sense, H is close to being a two-dimensional group, the "closeness" being measured by H_2L.

THEOREM 3.7. *Let $L \rightarrowtail G \twoheadrightarrow H$ be an exact sequence of groups with H_1L a projective H-module, $H_2L=0$ and L a C-subgroup. Then the cohomological dimension of H is ≤ 2.*

Proof. Let $P_* \to\mathbb{Z}$ be the resolution assured by L being a C-subgroup of G. By tensoring P_* with $\mathbb{Z}\otimes_L -$, using that H_1L is projective as an H-module and that L is a C-subgroup, we obtain an exact sequence of H-modules

$$H_2L \rightarrowtail \mathbb{Z}\otimes_L P_2\oplus H_1L\to\mathbb{Z}\otimes_L P_1\to\mathbb{Z}\otimes_L P_0\to\mathbb{Z}$$

with the inner three terms projective. The hypothesis that $H_2L=0$ does the rest. ■

The next result yields new information about the aspherical question of J. H. C. Whitehead ([BD] and [BDS]): Are subcomplexes of aspherical 2-complexes aspherical?

PROPOSITION 3.8. *Suppose $G=K*N$ is a free product of groups K and N and that L is a projective subgroup of G. Then $\bar{L}=K\cap L$ is a projective subgroup of K.*

Proof. We observe that $\bar{L}$ is a normal subgroup of K. It follows from the Kurosh structure theorem about subgroups of a free product [Se, Theorem 14] that $\bar{L}$ is a free summand of $L=\bar{L}*M$. It is easy to check that, if $\bar{H}=K/\bar{L}$ ($\bar{H}<G/L$), then H_1L is isomorphic to $H_1\bar{L}\oplus H_1M$ as $\bar{H}$-modules. Because H_1L is a projective G/L-module, it is a projective $\bar{H}$-module. We have $H_2\bar{L}=0$ because $H_2L\cong H_2\bar{L}\oplus H_2M=0$. Thus $\bar{L}$ is projective. ■

A $[G,2]$-complex X is a connected, 2-dimensional CW-complex with fundamental group isomorphic to G and a single zero cell. If X is a $[G,2]$-complex which is a subcomplex of an *aspherical* $[K,2]$-complex Y, we let $\bar{X}=X\cup Y^1$ be the union of X together with the 1-skeleton of Y and $i: X \hookrightarrow Y$, and $i_1: \bar{X} \hookrightarrow Y$ denote the inclusion maps. Furthermore, let $L = \ker \pi_1(i_1)$ and $\bar{L} = \ker \pi_1(i)$. Then the fundamental group $\pi_1(\bar{X}) \cong G * F$, where F is a free group whose rank corresponds to the number of 1-cells of Y outside of X. It is well known (see [BD]) that $L \rightarrowtail G * F \twoheadrightarrow \operatorname{im} \pi_1(i_1) = H$ is a free $G * F$-crossed module. Thus $H_1 L$ is a free H-module and $H_2 L = 0$. Now $\bar{L} = \ker \pi_1(i) = L \cap G$ is a projective $\bar{H} = G/\bar{L}$-module by the previous proposition (see [D]). Furthermore (and this seems to be new), the sequence

$$\hat{L} = F \cap L \rightarrowtail F \twoheadrightarrow F/F \cap L = \hat{H} < H$$

is also a projective subgroup of the free group F, with $\hat{H}$ a 2-dimensional group.

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