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On algebras of strongly unbounded representation type

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Let A be a finite dimensional associative algebra over the field k. We say that A is of *finite* (*representation*) *type* if it has only finitely many isomorphism classes of indecomposable modules. Also, A is of *bounded type* if it does not admit indecomposable modules of arbitrarily high k-dimensions. Finally, the algebra A is called of *strongly unbounded type* if there is an infinite sequence of numbers $d_1 < d_2 < \cdots < d_s < \cdots$ such that for each d_i there are infinitely many isomorphism classes of indecomposable modules with k-dimension d_i. In an old paper [J], Jans states that R. Brauer and R. M. Thrall have conjectured that algebras of bounded type are actually of finite type and that (over infinite fields) algebras of unbounded type are actually of strongly unbounded type.

The first conjecture was proved by A. V. Roiter in [R] and later on generalized for Artin rings by M. Auslander [A1]. The second conjecture was proved by Nazarova–Roiter [NR] for algebras over algebraically closed fields using matrix methods.

Here we present a new proof of the second Brauer–Thrall conjecture for algebras over algebraically closed fields of characteristic different from 2. Combining our results with the recent work of Bongartz on minimal algebras of infinite type [Bo4], a proof can be given for the general case of arbitrary characteristic.

The methods we use here are mainly geometric; they have two sources of inspiration; The work on coverings of Bongartz, Bretscher, Gabriel, Martínez and de la Peña ([BoG], [BrG], [G], [MP2]) and the Multiplicative Basis paper [BGRS].

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Throughout the whole paper k will denote an algebraically closed field. In the first section of this paper we recall basic definitions and results from [BGRS]. The proof of the main theorem is located in section 2 and rests on the more technical results of section 3.

1. Ray-categories and algebras of finite type

1.1 Let ω be a letter. The semi-group S_n with basis ω and cardinality $n+1$, $n \geqslant 1$, consists by definition of elements $1, \omega, \omega^2, \ldots, \omega^{n-1}$, 0 such that $\omega^p \cdot \omega^q = \omega^{p+q}$, $\omega^m = 0$ for $m \geqslant n$ and $\omega^0 = 1$. If S_n acts on a set M, we say that M is *cyclic* over S if $M = \{\omega^p \mu : p \in \mathbb{N}\}$ for some $\mu \in M$; such a μ is necessarily unique; if m is the smallest $p \in \mathbb{N}$ such that $\omega^p \mu = 0\mu$, M consists of the $m+1$ distinct elements $\mu, \omega\mu, \ldots, \omega^{m-1}\mu, 0\mu$.

DEFINITION. *A category P is called a* ray-category [BGRS, 1.7] *if its satisfies the following conditions* a)–f):

a) *There is a (necessarily unique) family* $({}_x0_y)x, y \in P$ *of so-called zero-morphisms* $0 = {}_x0_y : x \to y$ *such that* $\mu 0 = 0$ *and* $0\nu = 0$ *whenever the compositions make sense.*

b) *Distinct objects are not isomorphic.*

c) *For each* $x \in P$, *there are only finitely many non-zero morphisms starting or stopping at* x.

d) *For each* $x \in P$, *the semi-group of endomorphisms* $P(x, x)$ *is isomorphic to* $S_{n(x)}$ *for some* $n(x) \geqslant 1$.

e) *For all* $x, y \in P$, *the set of morphisms* $P(x, y)$ *is cyclic over* $P(x, x)$ *or over* $P(y, y)$.

f) $\mu\nu = \mu\rho\nu \neq 0$ *implies that* ρ *is an identity.*

If P is a ray-category, we denote by $k[P]$ the vector space over k which has the non-zero morphisms of P as basis-vectors. The space $k[P]$ is endowed with the following multiplication: The product of two non-zero morphisms μ and ν of P coincides with their composition if μ and ν are composable in P and have non-zero composit. The product is zero otherwise. If P is *finite*, i.e. has finitely many objects, we thus obtain a finite-dimensional associative algebra $k[P]$ with unit element $\sum_{x \in P} \mathbb{1}_x$.

Ray-categories are related to algebras of finite representation type by the following theorem [BGRS, 1.12 and 2.10]: If A is of finite type and $\operatorname{char} k \neq 2$, A is Morita equivalent to $k[P]$ for some ray-category P uniquely determined by A. By loc. cit. and [Bo4, corollary 2], the same conclusion holds in all characteristics if A satisfies the following conditions: a) A is of infinite representation-type; b) A is not strongly unbounded; c) A/I is of finite representation type for each ideal $I \neq 0$.

This reduces the proof of the second Brauer–Thrall conjecture to the case $A = k[P]$, to which we restrict in the sequel. There we say that P is a *ghost* if $k[P]$ satisfies the conditions a) and b) above. If, moreover, $k[P]$ satisfies condition c), we call it a *mild ghost.* Our problem is to show that there is no (mild) ghost.

2. Reduction of the main theorem

In the sequel, we simply refer to [BGRS] for the used terminology. Throughout we assume that the considered ray-category P admits no infinite chain [BGRS, 1.12] and that at each point of P at most 3 arrows start, at most 3 stop. These conditions are satisfied if P is a ghost (see the final remark of 2.10 and paragraph 3.7 of [BGRS]).

2.1 Given a chain $c=(\rho_1, \sigma_1, \rho_2, \ldots)$ of P [BGRS, 1.9], we call *depth* of c the sum $\mathbf{d}(c)=\mathbf{d}(\rho_1)+\mathbf{d}(\sigma_1)+\mathbf{d}(\rho_2)+\cdots$, where $\mathbf{d}(\rho_i)$ and $\mathbf{d}(\sigma_i)$ denote the depths of the morphisms ρ_i and σ_i[*BGRS*, 8.4]. The *depth* of P is the supremum $\mathbf{d}(P)=\sup_c \mathbf{d}(c)\in\mathbb{N}\cup\{\infty\}$. It satisfies $\mathbf{d}(P')\leqslant\mathbf{d}(P)$ whenever P' is a full subcategory, a residue-category or a cover of P [BGRS, 10.1]. Moreover, we have $\mathbf{d}(P)<\infty$ if P is finite.

Given $d\in\mathbb{N}$, let P range over all finite ray-categories such that $k[P]$ has finite representation type and $\mathbf{d}(P)\leqslant d$. Let M range over all indecomposable $k[P]$-modules. It follows from the work of Bongartz [B1, 2.4] that $b(d)=\sup_{M,P}\dim M<\infty$.

Now put $p(P)=\sup_{x\in P}\sup(\sum_y(|P(x,y)|-1), \sum_y(|P(y,x)|-1))$, where $|S|$ denotes the cardinality of the set S. Then we have:

LEMMA. *The quiver of a ghost P contains a non-trivial closed path of length* $\leqslant b(\mathbf{d}(P))p(P)^3$.

Proof. Assuming the lemma false, we will prove that $\dim M\leqslant b(\mathbf{d}(P))$ implies $\dim N\leqslant b(\mathbf{d}(P))$ whenever M and N are indecomposable $k[P]$-modules connected by an irreducible morphism $M\to N$ or $N\to M$. By a result of Auslander [A2, 5.8], this implies that $k[P]$ has finite representation type, a contradiction.

Assume for instance that $N\to M$ is an irreducible morphism between indecomposables such that $\dim M\leqslant b(\mathbf{d}(P))$. Our claim being trivial if M is projective, we may consider the translate $\tau M=D\operatorname{Tr}M$ which satisfied $\dim\tau M\leqslant p(P)(p(P)-1)\dim M$. The Auslander–Reiten sequence stopping at M is therefore supported by a full subcategory P' of P with at most $\dim M+\dim\tau M\leqslant(1-p(P)+p(P)^2)\dim M\leqslant p(P)^2b(\mathbf{d}(P))$ points. Since the support of N is contained in P', we obtain $\dim N\leqslant b(\mathbf{d}(P'))\leqslant b(\mathbf{d}(P))$ if we can prove that P' has finite representation type.

Assume the contrary. If the quiver of P' has a non-trivial closed path, it has one of length $\leqslant b(\mathbf{d}(P))p(P)^2$. Decomposing each arrow of this path of p' into a product of arrows of P, we obtain a non-trivial closed path of P of length $\leqslant b(\mathbf{d}(P))p(P)^2$ which contradicts the assumption that the lemma is false.

So P' has no non-trivial closed path. Being finite, P' is intervall-finite, and so is its universal cover $\tilde{P}'$. By the criterion of Bongartz [BGRS, 10.3], $\tilde{P}'$ contains a critical convex subcategory K. For arbitrarily large d, $k[K]$ admits an infinite family (M_α) of non-isomorphic indecomposable modules of dimension d. Extending the M_a to $\tilde{P}'$ by zero and pushing these extensions down to P', we obtain an infinite family of indecomposable $k[P']$-modules of dimension d (compare with [G, 3.3]). The Kan-extensions of these $k[P']$-modules to $k[P]$ are indecomposable and non-isomorphic, and their dimensions lie between d and $dp(P)$. This contradicts the assumption that P is a ghost.

2.2 Assume that the quiver of P is connected. Denote by Π the fundamental group of P at some point p [BGRS, 10.1], by Π' the subgroup of Π generated by the commutators. We call the quotient $P^{ab} = \tilde{P}/\Pi'$ the *universal abelian cover* of P and denote by $\pi^{ab}: P^{ab} \to P$ the covering induced by the universal covering $\pi: \tilde{P} \to P$. The action of Π on $\tilde{P}$ induces an action of the free abelian group $\Pi^{ab} = \Pi/\Pi'$ [BGRS, 10.4] on P^{ab}, the quotient P^{ab}/Π^{ab} being identified with P.

LEMMA. *If P is a ghost with connected quiver, P^{ab} admits a residue-category of the form $P_1 \coprod \Delta$ where P_1 is a mild ghost and Δ a ray-category without arrows.*

Proof. We first claim that P' contains a finite full subcategory P' of infinite representation-type. Otherwise, each indecomposable (finite-dimensional) representation M of the linearization $k(P')$ [BGRS, 1.8] satisfies $\dim M \leqslant b(\mathbf{d}(P^{ab})) = b(\mathbf{d}(P))$. So, given a point $x \in P^{ab}$, all indecomposables M with $M(x) \neq 0$ are supported by a finite full subcategory of P^{ab}. It follows that P^{ab} is locally representation-finite in contradiction to [G, 3.6] or to [MP1].

Now, if P' is as above, denote by I the ideal [BGRS, 1.13] of P^{ab} formed by the non-identical morphisms $\mu: x \to y$ such that $P(t, x)\mu P(y, z) = 0$ for all $t, z \in P'$. Then P^{ab}/I has the form $P'' \coprod \Delta$, where Δ has no arrow and P'' is a finite ray-category containing p' as a full subcategory. As P' is of infinite representation type, so is P''. As P is not strongly unbounded P'' is not (see the argument at the end of 2.1). The wanted category P_1 is a suitable quotient of P''.

2.3 Suppose that the quiver Q of P is connected. A *well-knotted* path of P is a non-identical closed path of Q whose liftings to Q^{ab} (= quiver of P^{ab}) are closed. In other words, the well-knotted paths of P are the images of the non-identical closed paths of Q^{ab}.

LEMMA. *A well-knotted path of a mild ghost cannot be simple* [BGRS, 2.6].

The proof of this lemma is given in section 3 below.

2.4 THEOREM. *Finite-dimensional algebras of infinite representation-type over algebraically closed fields are strongly unbounded.*

Proof. It suffices to prove that there is no mild ghost: Assume that P is one. Starting with $P_0 = P$ we can construct an infinite sequence $P_0, P_1, \ldots$ of mild ghosts such that $P_{n+1} \amalg \Delta_n$ is a residue-category of P_n^{ab} for some ray-category Δ_n without arrows (2.2). All these categories P_n obviously satisfy $p(P_n) \leq p(P)$ and $\mathbf{d}(P_n) \leq \mathbf{d}(P)$.

Let Q_n be the quiver of P_n and C_n the finite set of all non-trivial closed paths of Q_n of length $\leq b(\mathbf{d}(P)p(P)^3$. The sets C_n are not empty by 2.1, and the natural maps $Q_{n+1} \to Q_n$ give rise to an inverse system $\cdots C_{n+1} \to C_n \to \cdots \to C_0$. By König's graph theorem, $\varprojlim_n C_n$ is not empty. This means that there is an infinite sequence of paths $p_n \in C_n$ such that p_{n+1} is mapped onto p_n. In particular, each p_n is well knotted and therefore non simple (2.3). But the number $i(p)$ of "self-intersections" of a path path p of length l satisfies $i(p) \leq \binom{l+1}{2} - 1$. On the other hand, since a simple path is not well knotted (2.3), we have $i(p_{n+1}) < i(p_n)$, hence $i(p_n) < 0$ for large values of n in contradiction to the definition of $i(p)$.

3. Abelian coverings and proof of Lemma 2.3

Throughout section 3, the ray-category P is supposed to have a connected quiver Q and to satisfy the conditions stated at the beginning of section 2, with Z we denote an abelian group.

3.1 Denote by $\dot{C}_1(P)$ the free abelian group generated by the arrows of the Q. Each path $P = \alpha_1 \cdots \alpha_s$ of Q (with composing arrows α_i) then yields an element $\sigma(p) = \alpha_1 + \cdots + \alpha_s \in \dot{C}_1(P)$. We denote by $\dot{B}_1(P)$ the subgroup of $\dot{C}_1(P)$ generated by the differences $\sigma(p) - \sigma(q)$, where (p, q) ranges over the essential contours of P [BGRS, 2.7]. The group $\dot{B}_1(P)$ is contained in the subgroup $\dot{Z}_1(P)$ formed by the $\mathbb{Z}$-linear combinations $\sum_\alpha c_\alpha \alpha$ such that $\sum_{\beta \in x^-} c_\beta = \sum_{\gamma \in x^+} c_\gamma$ for all points x (by x^- and x^+ we denote the sets of arrows stopping and starting at x respectively).

A Z-valued *cocycle* of P is a group homomorphism $f: \dot{C}_1(P) \to Z$ which vanishes on $\dot{B}_1(P)$. The last condition permits to define f on all non-zero morphisms μ of P by setting $f(\mu) = f(\sigma(p))$ if p is a path of Q with composition $\vec{p} = \mu$[BGRS, 1.14]. We can even extend f to the walks of P [BGRS, 10.1], i.e. to the formal compositions $\lambda\, |\mu^*|\, \nu\, |\pi| \cdots$ of non-zero morphisms ξ and their "formal inverses" ξ^*: Just set $f(\lambda\, |\mu^*|\, |\pi| \cdots) = f(\lambda) - f(\mu) + f(\nu) + f(\pi) \cdots$. Two homotopic walks [BGRS, 10.1] then have the same image in Z. As a consequence, f induces a group homomorphism $f^*: \pi \to Z$ (2.2).

Let $K^f = \operatorname{Ker} f^*$. We denote by P^f the quotient $\tilde{P}/K^f$, by $\pi f: P^f \to P$ the covering of P induced by the universal covering $\pi: \tilde{P} \to P$ (2.2). According to the construction of P given in [BGRS, 10.1], the points of P are identified with the f-equivalence classes of walks of p with fixed terminus p; here we say that two walks w and v are *f-equivalent* if they have the same origin, the same terminus and $f(v)=f(w)$. As a consequence, two paths p' and q' of Q^f(= the quiver of P^f) which have the same origin also have the same terminus iff their projections p and q on Q satisfy $f(p)=f(q)$, or equivalently iff $\sigma(p)-\sigma(q)\in \operatorname{Ker} f \subset \dot{C}_1(P)$.

Let $f, g:\dot{C}_1(P)\to Z$ now be two cocycles. Our description of the points of P^f shows that $P^f=p^g$ whenever $f\mid \dot{Z}_1(P)=g\mid \dot{Z}_1(P)$, i.e. whenever f and g induce the same cohomology class $\bar{f}=\bar{g}\in \operatorname{Hom}_Z(\dot{Z}_1(P)/\dot{B}_1(P), Z)$. In particular, let $\varphi:\dot{C}_1(P)\leftarrow \dot{Z}_1(P)/\dot{B}_1(P)$ be an extension of the canonical projection $\dot{Z}_1(P)\to \dot{Z}_1(P)/\dot{B}_1(P)$ (such a φ exists because $\dot{Z}_1(P)$ is defined as the kernel of a homomorphism of $\dot{C}_1(P)$ into a free abelian group). Then P^ϕ does not depend on the choice of the extension ϕ. In fact:

LEMMA. *P^ϕ equals the universal abelian cover P^{ab}* (3.2). *Accordingly, a closed path $p=\alpha_1\cdots\alpha_s$ of Q is well-knotted* (3.3) *iff $\sigma(p)=\alpha_1+\cdots+\alpha_s\in \dot{B}_1(P)$.*

Proof. ϕ induces a homomorphism $\phi^*:\Pi\to \dot{Z}_1(P)/\dot{B}_1(P)$, hence a homomorphism $\bar{\phi}^*:\Pi/\Pi'\to \dot{Z}_1(P)/\dot{B}_1(P)$. It suffices to prove that $\bar{\phi}^*$ is bijective (notice that $\sigma(p)\in \dot{Z}_1(P)$ and that $\dot{Z}_1(P)\cap \operatorname{Ker}\phi=\dot{B}_1(P)$). We just produce an inverse map (compare with lemma 10.1 of [BGRS]): For each $x\in P$, choose a walk w_x from p to x. For each arrow $x\xrightarrow{\alpha} y$ of Q, denote by $f(\alpha)\in \Pi/\Pi'$ the class mod Π' of the homotopy class of the closed walk $w_x\,|\alpha|\,w_y^*$. The map f extends to a homomorphism $\dot{C}_1(P)\to \Pi/\Pi'$, which induces the wanted inverse $\dot{Z}_1(P)/\dot{B}_1(P)\to \Pi/\Pi'$.

3.2 A cocycle $f:\dot{C}_1(P)\to \mathbb{Z}$ with integral values is called *positive* if $f(\alpha)>0$ for each arrow α. If f is positive and p' is a non-trivial path of P^f with origin a' and projection $p=\alpha_1\cdots\alpha_s$ on P, we have $f(p)=f(\alpha_1)+\cdots+f(\alpha_s)>0$. If follows that p' and $\Pi_{a'}$ have different termini (3.1), hence that Q^f admits no non-trivial closed path. Moreover, since P^f is a quotient of P^{ab}, it follows that Q^{ab} has no non-trivial closed path.

If $k(P)$ is locally representation-finite, we know from [BrG, 1.5] that there exists a positive cocycle, namely the *grade-function gr* which maps each arrow of P onto its grade. This proves the first part of the following lemma.

LEMMA. *Suppose that $k(P)$ is locally representation-finite. Then Q^{ab} has no non-trivial closed path. If p and q are parallel paths* [*BGRS*, 1.2] *of Q^{ab} with compositions $\vec{p}$ and $\vec{q}$ in P^{ab}, the condition $\vec{p}\neq 0$ implies $\vec{q}\neq 0$.*

Proof. Since the universal covering $\pi:\tilde{P}\to P$ induces a covering $P^{ab}\to P^{gr}$, it suffices to prove the corresponding statement for P^{gr}. By [BrLRi, 5.1] or [P], it suffices to prove that the Auslander–Reiten quiver Γ^{gr} of $P^{gr}=\tilde{P}/K^{gr}$ has no non-trivial closed path.

Let Γ and $\tilde{\Gamma}$ be the Auslander–Reiten quivers of $k(P)$ and $k(\tilde{P})$. By [H, 3.5], we have $\tilde{\Gamma}/K^{gr} \xrightarrow{\sim} \Gamma^{gr}$. By [BrG, 3.1], $\tilde{\Gamma}$ is identified with the universal cover of Γ [BoG, 1.3] and Π with the fundamental group of Γ. Using this identification, we can describe $gr^*: \Pi \to \mathbb{Z}$ (3.1) as follows: If $\bar{w} \in \Pi$ is the homotopy class of a closed walk of Γ, say $w = \alpha\,|\beta^*|\,\gamma\,|\delta|\,\varepsilon^*$, we have $gr^*(\bar{w}) = +1 -1 +1 -1$. Since the points of $\tilde{\Gamma}$ are defined as homotopy classes of walks of Γ, it follows that two paths of $\Gamma^{gr} = \tilde{\Gamma}/K^{gr}$ with common origin and parallel projections on Γ have the same termini iff they have the same length. So Γ^{gr} has no non-trivial closed path.

3.3. *Proof of Lemma* 2.3. Suppose that $c = \phi_1 \cdots \phi_s$ is a simple well-knotted path of a mild ghost P. Up to duality, we can suppose that P admits an efficient tackle ending with a *hook* $x \xrightarrow{\delta} y$ [BGRS, 8.3 and 8.4]. Denote by p a path of Q of maximal length which passes through δ and has a non-zero composition $\vec{p}$. Set $P = P/\vec{p}$ and consider the following cases:

a) $p = \delta$: In this case, the quiver of $\bar{P}$ is $Q\backslash\{\delta\}$. Since $k(\bar{P})$ has finite representation-type, $\bar{P}$ admits a positive ($\mathbb{Z}$-valued) cocycle $\bar{f}$ (3.2). We extend $\bar{f}$ to a homomorphism $f: \dot{C}_1(P) \to \mathbb{Z}$ such that $f(\delta) > 0$. Since all essential contours of P [BGRS, 2.7] are contained in $\bar{P}$, f is a positive cocycle of P. Jence Q^{ab} has no non-trivial closed path in contradiction to 2.1 and 2.2.

b) *p has length $\geqslant 2$ and δ belongs to no essential contour of P*: Then $\bar{P}$ has the same quiver as P and $\dot{C}_1(\bar{P})$ equals $\dot{C}_1(P)$. A positive cocycle for $\bar{P}$ is also a positive cocycle for P. As in case a), we obtain a contradiction.

c) *There are essential contours (v, w) of P such that $\vec{v} = \vec{w} = \vec{p}$, and they all contain δ*: Let $(\sigma q, r)$ be such a contour (see [BGRS, 8.4]. Then $\dot{B}_1(P) = \dot{B}_1(\bar{P}) \oplus \mathbb{Z}(\delta + \sigma(q) - \sigma(r))$ and $\dot{C}_1(P) = \dot{C}_1(\bar{P})$. Let $\bar{f}$ be a positive cocycle of $\bar{P}$, and define a cocycle $f: \dot{C}_1(P) \to \mathbb{Z}$ as follows: $f(\delta) = \bar{f}(\alpha(r)) - \bar{f}(\sigma(q))$ and $f(\alpha) = \bar{f}(\alpha) > 0$ if the arrow α is not δ. Since c is well-knotted, its liftings to P^{ab} and P^f are closed. So we have $f(c) = f(\phi_1) + \cdots + f(\phi_s) = 0$ (3.1), hence $f(\phi_i) \leqslant 0$ and $\phi_i = \delta$ for some i, say for $i = 1$.

Since $\sigma(c) \in \dot{B}_1(P)$ and $\phi_j \neq \delta$ for $j \neq 1$, we have $\sigma(c) = \delta + \phi_2 + \cdots + \phi_s = (\delta + \sigma(q) - \sigma(r)) + \rho$ for some $\rho \in \dot{B}_1(\bar{P})$. It follows that $(\phi_2 + \cdots + \phi_s + \sigma(r)) - \sigma(q) \in B_1(\bar{P})$. This means by 3.1 that the paths $\phi_2 \cdots \phi_s r$ and q can be lifted to parallel paths of $\bar{P}^{ab}$. Since we have $\vec{\phi}_2 \cdots \vec{\phi}_s \vec{r} = \vec{\phi}_2 \cdots \vec{\phi}_s \vec{p} = 0$ and $\vec{q} \neq 0$ in $\bar{P}$, we obtained a contradiction to 3.2.

(d) *There is an essential contour (v, w) of P which satisfies $\vec{v} = \vec{w} = \vec{p}$ and does not contain δ*: By [BGRS, 8.5], this means that 3 arrows β, γ, δ start at x, and that we have 3 paths βu, γvt, δwt with common composition $\vec{p}$ which give rise to two essential contours $(\beta u, \gamma vt)$ and $(\gamma v, \delta w)$. Then we have $\dot{B}_1(P) = \dot{B}_1(\bar{P}) \oplus \mathbb{Z}(\delta + \sigma(w) - \sigma(\beta u)) \oplus \mathbb{Z}(\gamma + \sigma(v) - \sigma(\beta u))$ if t is trivial, and $\dot{B}_1(P) = \dot{B}_1(\bar{P}) \oplus \mathbb{Z}(\delta + \sigma(w) - \sigma(\beta u)$ otherwise.

We may suppose that γ is the hook of an essential contour: Indeed, otherwise,

β is so by [BGRS, 8.5]. So we may exchange β and γ if t is trivial. If t is not trivial, we may replace δ by β and are reduced to case c).

Now let $\bar{f}$ be a positive cocycle of $\bar{P}$. Define a cocycle $f: \dot{C}_1(P) \to \mathbb{Z}$ as follows: $f(\gamma) = \bar{f}(\sigma(\beta u)) - \bar{f}(\sigma(vt))$, $f(\delta) = \bar{f}(\sigma(\beta u)) - \bar{f}(\sigma(wt))$ and $f(\alpha) = \bar{f}(\alpha) > 0$ if the arrow α is not γ or δ. As in c), we have $f(c) = f(\phi_1) + \cdots + f(\phi_s) = 0$, hence $f(\phi_i) \leq 0$ and $\phi_i = \gamma$ or δ for some i, say $\phi_1 = \delta$.

Setting $r = \beta u$ and $q = wt$, we conclude as in c) above.

BIBLIOGRAPHY

[A1] AUSLANDER, M., *Representation theory of Artin algebras II*, Comm. Algebra *2* (1974), 269–310.

[A2] AUSLANDER, M., *Applications of morphisms determined by objects*. Proc. Conf. On Representation Theory. Philadelphia (1976) Marcel Dekker (1978), 245–327.

[BGRS] BAUTISTA, R., GABRIEL, P., ROITER, A. V. and SALMERÓN, L., *Representation-finite algebras and multiplicative bases*. To appear in Inv. Math. 1985.

[Bo1] BONGARTZ, K., *Treue einfach zusammenhängende Algebren I.*, Comment. Math. Helv. *57* (1982), 282–330.

[Bo2] BONGARTZ, K., *A criterion for finite representation type*, Math. Ann. *269* (1984), 1–12

[Bo3] BONGARTZ, K. *Critical simply connected algebras*, Manus. Math., *46* (1984), 117–136.

[Bo4] BONGARTZ, K. *Indecomposable modules are standard*, Comment. Math. Helv., this volume.

[BoG] BONGARTZ, K. and GABRIEL, P., *Covering spaces in representation theory*, Inv. Math. *65* (1982), 331–378.

[BrG] BRETSCHER, O. and GABRIEL, P., *The standard form of a representation-finite algebra*, Bull. Soc. Math. France, *111* (1983), 21–40.

[BrLRi] BRETSCHER, O., LÄSER, C. and RIEDTMANN CHR. Manus. Math. 36 (1981), 253–301.

[G] GABRIEL, P. *The Universal Cover of a Representation-Finite Algebra. Representations of Algebras*. Proceedings, Puebla, México 1980 Lecture Notes in Mathematics 903 Springer Verlag pp. 68–105.

[J] JANS, J., *On the indecomposable representations of algebras*, Ann. of Math. *66* (1957), 418–429.

[MP1] MARTÍNEZ-VILLA, R. and DE LA PEÑA, J. A. *Automorphisms of representation-finite algebras*, Invent. Math. 72 (1983), 359–362.

[MP2] MARTINEZ-VILLA, R. and DE LA PEÑA, J. A., *Multiplicative basis for algebras whose universal cover has no oriented cycles*. J. Alg. *87* (1984), 389–395.

[NR] NAZAROVA, L. A. and ROITER A. V., *Categorical matrix problems and the Brauer-Thrall conjecture*. Preprint Kiev (1973). German version in Mitt. Math. Sem. Giessen *115* (1975), 1–153.

[P] DE LA PEÑA, J. A., *Representation-finite algebras whose Auslander-Reiten quiver is planar*. Preprint Publicaciones Preliminares Inst. Matemáticas UNAM 65 (1983).

[R] ROITER, A. V., *The unboundedness of the dimension of the indecomposable representations of algebras that have a finite number of indecomposable representations*. Izv. Acad. Nauk SSR, *32* (1968), 1275–1282.

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