Indecomposables are standard.

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Indecomposables are standard

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The study of representation-finite algebras over an algebraically closed field is more complicated in characteristic 2 because not all algebras are standard ([3], [14]). We show that this phenomenon disappears if a representation-finite algebra has a faithful indecomposable module.

It follows that the proofs of the second Brauer-Thrall conjecture via coverings work in any characteristic (for a different approach see [13]). There are two such proofs which both depend heavily on the work of Bautista, Gabriel, Roiter and Salmerón on multiplicative bases ([3]). The first proof is due to Bautista, the second to Fischbacher ([2], [10]).

Furthermore, we obtain a direct proof of an important theorem in [3], which says that a distributive algebra is representation-finite if and only if so is its standard form.

1. Statement of the results

This note is intimately related to [3]. In fact, we merely refine the results on penny-farthings given there.

For the convenience of the reader we recall briefly some of the definitions and results of [3] in a form which is sufficient for our purposes. The following notations and conventions will be kept throughout the paper.

So, let k be an algebraically closed field. We are interested in distributive categories Λ ([3], 1.3). Such a category is isomorphic to kQ/I^π, where kQ is the path category ([3], 1.1) of the Gabriel quiver $Q = Q_\Lambda$ ([3], 1.1) and I^π is an admissible ideal ([3], 1.1). It is well-known that I^π depends on the choice of some presentation π ([3], 1.1), whereas Q is given canonically. A path w of Q is stable if $w \notin I^\pi$ for all π. In that case, the depth ([3], 1.1) of the image $\bar{w}$ in Λ ([3], 1.1) is independent of π ([3], 2.1). A contour of Λ is a pair (v, w) of two stable paths in Q having the same depth and the same starting and ending points. For each distributive category Λ, there is its standard form $\Lambda^1 = kQ/I$ where I is generated by the non-stable paths and the differences $v - w$ obtained from all contours.

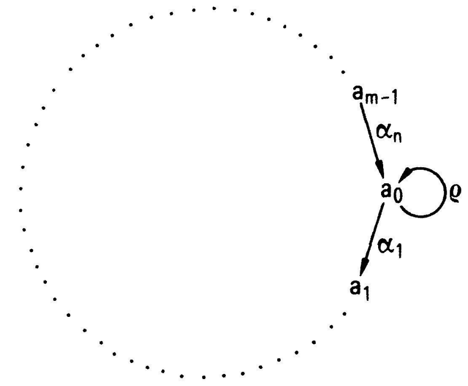


Figure 1

We are studying a special kind of contours called penny-farthings for obvious reasons. Namely, the subquiver P of Q that supports the arrows of a penny-farthing $(\alpha_n \cdots \alpha_2\alpha_1, \rho^2)$ has the shape given in Figure 1. Moreover, we ask the full subcategory of Λ living on P to be defined by P and one of the following two systems of relations

$$0 = \alpha_n \cdots \alpha_2\alpha_1 - \rho^2 = \alpha_1\alpha_n = \alpha_{f(i)} \cdots \alpha_1\rho\alpha_n \cdots \alpha_{i+1} \quad \text{or}$$

$$0 = \alpha_n \cdots \alpha_2\alpha_1 - \rho^2 = \alpha_1\alpha_n - \alpha_1\rho\alpha_n = \alpha_{f(i)} \cdots \alpha_1\rho\alpha_n \cdots \alpha_{i+1},$$

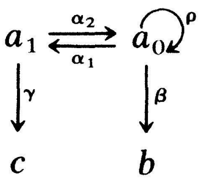
where $f:\{1, 2, \ldots, n-1\} \to \{1, 2, \ldots, n\}$ is some non-decreasing function (see [3], 2.7).

Note that we look at left modules and that we write the composition from the right to the left.

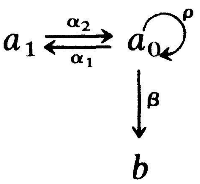
To state our main result we denote by $|x|^+$ and $|x|^-$ the number of arrows of Q starting and ending at the point x respectively.

THEOREM. *Let $\Lambda = \Lambda^1$ be a mild* ([3], 1.4) *category so that the associated ray-category $\vec{\Lambda}$* ([3], 1.7) *contains no infinite chain* ([3], 1.12). *Suppose that a penny-farthing as in Figure* 1 *occurs as a contour of Λ.*

a) *If there exists an arrow $\beta : a_0 \to b$ not contained in P, then $n = 2$ and we are in one of the following two situations*:

i) 

is a subquiver of Q. We have $|a_0|^+ + |a_0|^- = 5$, $|a_1|^+ + |a_1|^- = 3$, $|b|^- = |c|^- = 1$ and $0 = \beta\alpha_2 = \gamma\alpha_1 = \beta\rho = \delta\beta = \delta\gamma$ for all arrows δ.

ii) 

is a subquiver and we have $|a_0|^+ + |a_0|^- = 5$, $|a_1|^+ + |a_1|^- = 2$, $|b|^- = 1$ and $0 = \beta\rho = \delta\beta$ for all δ.

b) *If* $|a_0|^+ + |a_0|^- = 4$ *and* $\Lambda(a_0, y) \neq 0$ *for some* $y \notin P$, *then* $n = 2$ *and the following holds:*

$$\begin{array}{ccc} a_1 & \underset{\alpha_1}{\overset{\alpha_2}{\rightleftarrows}} & a_0 \circlearrowleft^{\rho} \\ \downarrow{\scriptstyle\gamma} & & \\ c & & \end{array}$$

is a subquiver of Q, $|a_1|^+ + |a_1|^- = 3$, $|c|^- = 1$ *and* $0 = \gamma\alpha_1\rho = \delta\gamma$ *for all* δ.

The proof of the theorem is given in Section 2. It is a lengthy exercise in Galois coverings ([11]). The so called BHV-bazar (see [12], [6] and [3], 10.8) is used to detect some representation-infinite full subcategories of the universal cover of Λ. Of course one can also use the technique of cleaving diagrams ([3], 3). Indeed, to condense the argument we use lemma 7.7 of [3] which is obtained by this technique. However, in a first approach the criterion of [6] and the lists of [5] convinced me of the truth of the theorem.

To end this introduction, we mention some more or less immediate but interesting consequences which are proved in the last paragraph.

COROLLARY 1. *A distributive category is locally representation-finite iff so is its standard form. In that case the Auslander-Reiten quivers coincide.*

COROLLARY 2. *A mild representation-infinite category contains no penny-farthing provided its ray-category has no infinite chain.*

Therefore, such a category is standard by [3], 9.6.

COROLLARY 3. *A representation-finite algebra of finite dimension that has a faithful indecomposable is standard.*

This shows that the determination of the almost split sequences of representation-finite algebras is reduced to the standard case, hence by Galois-coverings to simply connected faithful algebras. Namely, let X be an indecomposable of maximal dimension occurring in the almost split sequence $0 \to M \to N \to L \to 0$. Then the annihilator of X annihilates M, N and L.

2. The proof of the theorem

Throughout this section, Λ satisfies the requirements of the theorem stated before a).

2.1. We remind the reader how to construct the universal cover $\tilde{\Lambda}$ of Λ ([7], 3.3).

First one takes the universal cover $\tilde{Q}_\Lambda$ of Q_Λ with respect to some base point x_0 (we can assume that Λ is connected!). Given an essential contour (v, w) ([3], 2.7) of Λ, one chooses a walk u from x_0 to the start of v. The Gabriel quiver $Q_{\tilde{\Lambda}}$ of $\tilde{\Lambda}$ is the quotient of $\tilde{Q}_\Lambda$ by the normal subgroup of the fundamental group of Q_Λ generated by all $u^{-1}v^{-1}wu$ gotten in the above manner. Finally, the ideal of $kQ_{\tilde{\Lambda}}$ defining $\tilde{\Lambda}$ is generated by all lifted non-stable paths and differences $v-w$ obtained from contours. In the terminology of [3], 10.1–2, $\tilde{\Lambda}$ is the linearization $k(\tilde{\vec{\Lambda}})$ of the universal cover of the ray-category $\vec{\Lambda}$.

The point is that $\tilde{\Lambda}$ is Schurian, directed and interval-finite and it satisfies $H_1\tilde{\Lambda} = 0$ (see [7] for the definitions). This is shown in [7] if Λ is locally representation-finite. In the other case we look at $\bar{\Lambda} = \Lambda/I$ where I is the ideal generated by $\bar{\rho}^3$. Since Λ and $\bar{\Lambda}$ have the same essential contours we get $Q_{\tilde{\bar{\Lambda}}} = Q_{\tilde{\Lambda}}$, whence $\tilde{\Lambda}$ is directed and interval-finite. It is Schurian because Λ is distributive, and $H_1\tilde{\Lambda} = 0$ follows from [3], 10.1.

Clearly, we may assume that $\tilde{\Lambda}$ and any full subcategory is defined by commutativity and zero-relations. To prove the theorem we have to show that certain cases cannot occur, e.g. $n \geq 3$ in our penny-farthing. To this aim we pick always an appropriate representation-infinite full subcategory M of $\tilde{\Lambda}$ which does not contain ρ^3. This condition implies that M fully embeds into $\tilde{\bar{\Lambda}}$ (use for instance [3], 7.7). Usually, we describe M by its Gabriel quiver and we indicate the zero-relations by - - - -.

The next useful lemma is well-known for representation-finite algebras whose Auslander–Reiten quiver has no oriented cycle ([8], 5.1).

LEMMA. *Under the above hypotheses any path from x to y in $Q_{\tilde{\Lambda}}$ is stable provided $\tilde{\Lambda}(x, y) \neq 0$.*

Proof. Suppose not. We choose $x, y \in \tilde{\Lambda}$ not satisfying the statement so that the convex hull M of x and y is as small as possible. Since $\tilde{\Lambda}$ has no closed chain, $H_1M = 0$ by [6], 2.2. Thus all points of M are separating ([7], [6]). In particular, the radical R of the indecomposable projective corresponding to x has to be indecomposable. So we can suppose that there is no arrow $x \to y$.

By the minimality of M, the arrows starting in x fall into two non-empty disjoint subsets S_1 and S_2, where $\alpha \in S_1$ (resp. $\alpha \in S_2$) if $\overline{v\alpha} \neq 0$ (resp. $\overline{v\alpha} = 0$) for all paths leading from the end of α to y. Let R_i be the submodule of R generated by all $\bar{\alpha}$, $\alpha \in S_i$. We claim $R = R_1 \oplus R_2$ which is the wanted contradiction.

Of course, $R = R_1 + R_2$. If the sum is not direct, there are two arrows $\alpha_i : x \to z_i$ in S_i and two paths $v_i : z_i \to t$ so that $\overline{v_i\alpha_i} \neq 0$ for $i = 1, 2$. Choosing a path $w : t \to y$ we get $0 \neq \overline{wv_1\alpha_1} = wv_2\alpha_2$, which is impossible.

2.2. In this paragraph we reduce to the case $n=2$.

First we claim that $\tilde{\Lambda}$ contains the full subcategory M given in Figure 2.2.1, where $\pi(a_i^j)=a_i$ for all i and j.

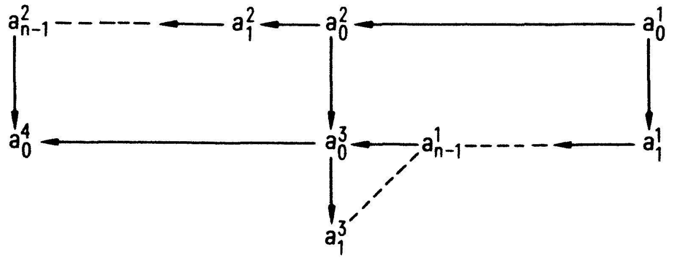


Figure 2.2.1

Indeed, $Q_{\tilde{\Lambda}}$ has the above quiver as a subquiver and it only remains to show that Q_M has no additional arrows.

By Lemma 2.1 and the zero-relation $a_{n-1}^1 \to a_0^3 \to a_1^3$ we have $M(a_0^1, a_1^3)=0$. Any non-zero morphism from a_0^1 to any other point factors through $a_0^1 \to a_0^2$ or through $a_0^1 \to a_1^1$, and any arrow $x \to a_0^1$ gives rise to an oriented cycle. From $a_0^2 \to a_0^3$ and Lemma 2.1 we infer $M(a_k^2, a_j^1)=0$ for all $0 \le k \le n-1$, $1 \le j \le n-1$. Similarly, $M(a_j^1, a_0^2) \ne 0$ would contradict $a_0^1 \to a_0^2$. Since the left rectangle in Figure 2.2.1 is conjugate to the right one there are also no maps between a_0^3 and the a_j^2's, $1 \le j \le n-1$.

Now $M(a_j^1, a_k^2) \ne 0$ for some $1 \le j, k \le n-1$ gives rise to a closed chain $a_0^2 \to a_k^2 \leftarrow a_j^1 \to a_0^3 \leftarrow a_0^2 \to a_k^2$ in $\vec{M}$. In the same vein, $M(a_j^2, a_1^3) \ne 0$ induces $a_0^3 \to a_0^4 \leftarrow a_j^2 \to a_1^3 \leftarrow a_0^3 \to a_0^4$.

By Lemma 2.1 and the zero-relation $a_{n-1}^1 \to a_0^3 \to a_1^3$, we get $M(a_j^1, a_1^3)=0=M(a_1^3, a_j^2)$ for all $0 \le j \le n-1$. The remaining possibilities are excluded by obvious reasons.

Next, we treat case a) of the theorem, i.e. there is an arrow $\beta: a_0 \to b$. We show that the full subcategory N of $\tilde{\Lambda}$ supported by the subquiver of $Q_{\tilde{\Lambda}}$ given in Figure 2.2.2 is representation-infinite provided $n \ge 3$.

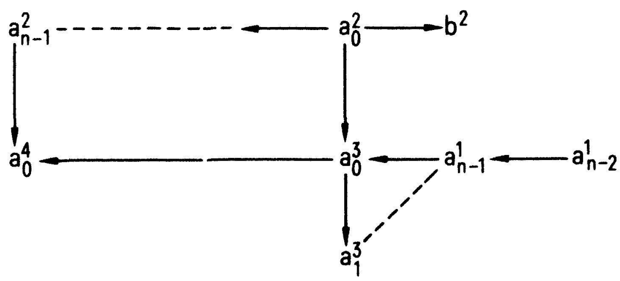


Figure 2.2.2

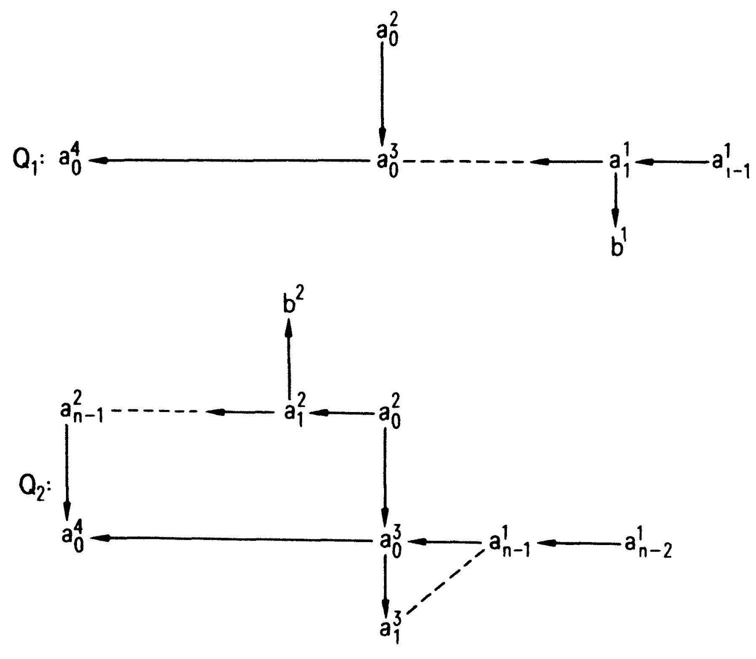


Figure 2.2.3

To abbreviate the proof that this figure gives also the quiver of N, we use the fact that $\Lambda(b, a_i) = 0$. This is contained in the dual of Lemma 7.7 in [3]. In the sequel we refer to it as Lemma 7.7.

The arrow $a_0^2 \to b^2$ implies $N(a_i^j, b^2) = 0$ for all $j \geq 2$, and the closed chain $a_j^1 \to b^2 \leftarrow a_0^2 \to a_0^3 \leftarrow a_j^1 \to b^2$ excludes the case $a_j^1 \to b^2$. For $n \geq 4$, N contains a representation-infinite category of type 2 in the list of [3], 10.8. If n equals 3, N belongs to the family 40.

In the case b) of the theorem we choose a stable path $a_0 \to a_1 \to \cdots \to a_i \to c$ with $c \notin P$ so that i is as small as possible. For $j = 1, 2$, let N_j be the full subcategory of $\tilde{\Lambda}$ supported by Q_j (see Figure 2.2.3). If $i \geq 2$, N_1 is given by Q_1 and no relation. In the other case N_2 has quiver Q_2 and it is defined by the indicated relation (see Figure 2.2.3). Therefore, it contains an algebra of the family 4 or it belongs to the family 43. The verification of the details is left to the reader. One needs the same type of arguments as in case a).

2.3. To make the computations easier, we introduce the category $\bar{\Lambda} = \Lambda/I$ where I is the ideal defined by $I(x, y) = 0$ if (x, y) belongs to $X := \{(a_0, a_0), (a_0, a_1), (a_1, a_0)\}$ and $I(x, y) = (\mathrm{Rad}^2 \Lambda)\ (x, y)$ otherwise.

To see that I is an ideal, we only have to show $\Lambda(v, y) I(u, v)\ \Lambda(x, u) = 0$ for $(x, y) \in X$ and u, v arbitrary. First take $x = a_0$. If $\{u, v\} \not\subseteq \{a_0, a_1\}$ we are done by Lemma 7.7. In the other case we have $I(u, v) = 0$ or $\mathrm{Rad}^2 \Lambda(a_1, a_1) \Lambda(a_0, a_1) = 0$. So, take $x = a_1$ and $y = a_0$. For $v = a_0$ we obtain $I(u, v) = 0$ or $I(u, a_0) \Lambda(a_1, u) \subseteq \Lambda(a_1, a_0) \Lambda(u, a_1) \Lambda(a_1, u) = 0$, where $I(u, a_0) \subseteq \Lambda(a_1, a_0)\ \Lambda(u, a_1)$ because α_2 and

ρ are the only arrows ending at a_0 (look at the separated quiver in case a)). If $v \neq a_0$ we get similarly $\Lambda(v, a_0)I(u, v)\Lambda(a_1, u) \subseteq \Lambda(a_1, a_0)(\mathrm{Rad}^2 \Lambda)$ (x, y) which is zero.

The point is that the universal cover $\tilde{\bar{\Lambda}}$ of $\bar{\Lambda}$ is easy to handle because $\bar{\Lambda}$ is defined by zero relations and by $\rho^2 = \alpha_2\alpha_1$. Of course, $\bar{\Lambda}$ has the same Gabriel quiver as Λ.

2.4. We are ready to prove part a) of the theorem. Looking at the separated quiver, we infer $|a_0|^+ + |a_0|^- = 5$ and that there exists no arrow $a_1 \to b$. Assume that there is an arrow $d \to b$, $d \to a_1$ or an additional arrow $a_1 \to d$. Then Figure 2.4 shows representation-infinite full convex subcategories of $\tilde{\bar{\Lambda}}$ which do not contain ρ^3. They belong to the families 12, 24 and $\tilde{D}_{6,1,0}$ of the list in [3], 10.8.

Now we restrict to case i), that means there exists an arrow $a_1 \to c$. If there would be an arrow $d \to c$ in Q_Λ, $\tilde{\bar{\Lambda}}$ would contain a member of family 20. Therefore, the quiver of the full subcategory M of $\bar{\Lambda}$ with support $\{a_0, a_1, b, c\}$ is given by the figure in part a) i) of the theorem. We infer $0 = \beta\rho = \beta\alpha_2 = \gamma\alpha_1$ because otherwise $\tilde{M}$ contains a category of type $\tilde{D}_{5,1,0}$, 23 or 11.

If $\overline{\delta\gamma} \neq 0$ for some $\delta : c \to d$ we consider the full subcategory N of Λ with support $\{a_0, a_1, b, c, d\}$ and we divide it by the ideal generated by all $\overline{\varepsilon\beta}$. Then $\overline{\delta\gamma}$ is not annihilated in the residue-category $\bar{N}$ and $\tilde{\bar{N}}$ contains an algebra of the family 20. An analogus argument using a member of family 12 shows that $\overline{\delta\beta} = 0$ for all arrows δ. The proof of part i) is complete. In fact, we have also shown part ii) as one verifies easily.

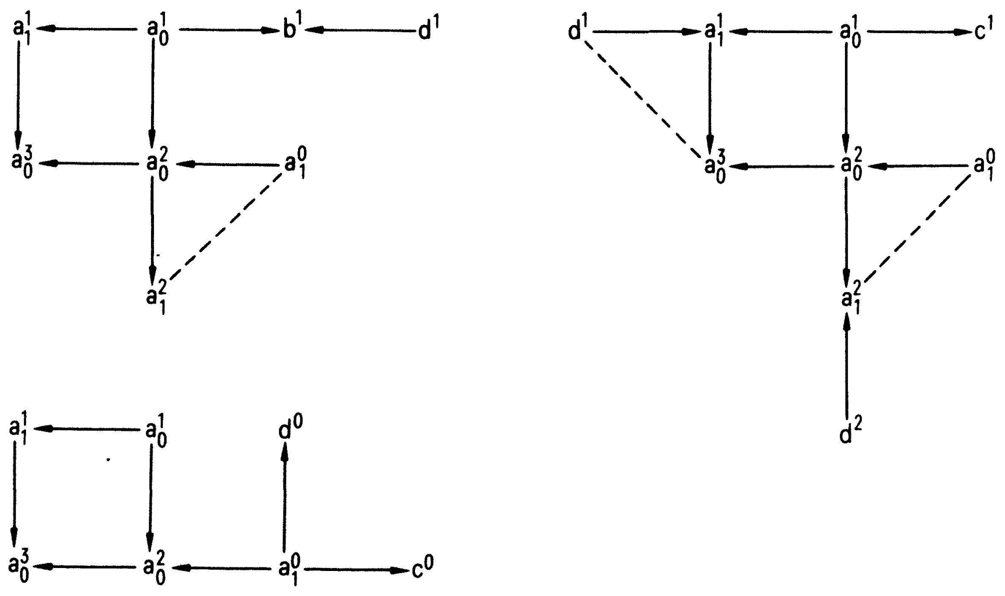


Figure 2.4

2.5. In the proof of part b), we look first at the same residue-category as in 2.3. We infer $|a_1|^+=2$ in the same way as in the beginning of 2.4.

Next, we claim that $\dim \Lambda(c, c)=1$. Otherwise, by Lemma 7.7 the quiver Q_M of the full subcategory M supported by $\{a_0, a_1, c\}$ is obtained by adding a loop δ in c to the subquiver of Q given in part b). The residue-category defined by $\bar{\delta}^2=\overline{\delta\gamma}=0$ has in its universal cover an algebra of type 21.

We define $I(x, y)=0$ for $(x, y)\in Y=\{(a_0, a_0), (a_0, a_1), (a_0, c), (a_1, a_0)\}$ and $I(x, y)=(\mathrm{Rad}^2 \Lambda)\ (x, y)$ otherwise. Similarly to 2.3 one verifies that I is an ideal. To see $\Lambda(v, c)I(u, v)\Lambda(a_0, u)=0$ one has to use $|a_1|^+=2$ and $\dim \Lambda(c, c)=1$.

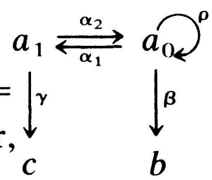
Considering the universal cover of $\bar{\Lambda}=\Lambda/I$ we infer that $|a_1|^-=|c|^-=1$, because $\tilde{\bar{\Lambda}}$ cannot contain a member of the family 21. For the same reason we have $\overline{\gamma\alpha_1\rho}=0$.

To obtain $\overline{\delta_\gamma}=0$ for any arrow $\delta: c\to d$ one looks at the full subcategory with support $\{a_0, a_1, c, d\}$. Its universal cover contains an algebra of type 21 again, if $\overline{\delta\gamma}\neq 0$.

The proof of the theorem is complete.

3. Consequences of the theorem

3.1. Let Λ be the category with Gabriel quiver defined by the relations $0=\alpha_2\alpha_1-\rho^2=\beta\alpha_2=\gamma\alpha_1=\beta\rho=\rho^4=\alpha_1(1-\rho)\alpha_2$. Then its standard form Λ^1 has the same Gabriel quiver, but it is defined by $0=\alpha_2\alpha_1-\rho^2=\beta\alpha_2=\gamma\alpha_1=\beta\rho=\alpha_1\alpha_2$.



As usual we write $P(x)$ or $I(x)$ for the projective or injective indecomposable corresponding to a point x.

We claim that Figure 3.1 gives for Λ and Λ^1 that part of the Auslander–Reiten quiver which contains all indecomposables not annihilated by ρ^3, namely $P(a_0)$ and $I(a_0)$. Here------›indicates the translation, X denotes $I(a_0)/\mathrm{Soc}\, I(a_0)$, Y is the obvious indecomposable and φ, ψ are the obvious irreducible maps.

Indeed, $P(b)$ is simple projective so that the almost split sequence starting at $P(b)$ is the drawn one. Thus X is the translate of $P(a_0)$ and the almost split

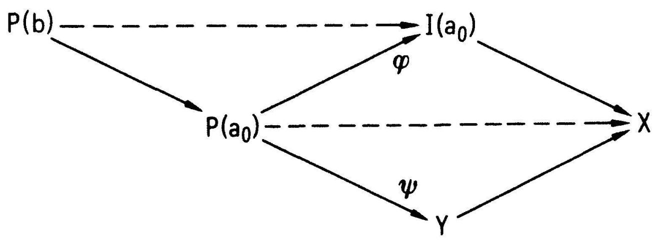
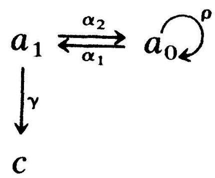


Figure 3.1

sequence starting at $P(a_0)$ is also correct. If an indecomposable U is not killed by ρ^3, it admits a map $\chi: P(a_0) \to U$ with $0 \neq \chi(\rho^3) \in U(a_0)$. By the definition of almost split sequences we get $\chi = \varphi'\varphi + \psi'\psi$ for some maps $\varphi': I(a_0) \to U$ and $\psi': Y \to U$ provided $U \to P(a_0)$. From $\psi(\rho^3) = 0$ we conclude that $\varphi'\varphi(\rho^3) = \chi(\rho^3) \neq 0$, hence that $U \xrightarrow{\sim} I(a_0)$.

Since $\Lambda/\langle\bar{\rho}^3\rangle \xrightarrow{\sim} \Lambda^1/\langle\bar{\rho}^3\rangle$ it follows that Λ and Λ^1 are both representation-finite and have the same Auslander-Reiten quivers. The same reasoning applies for the two algebras arising in case ii) of part a).



3.2. Similarly, look at the quiver and define Λ^1 by $0 = \rho^2 - \alpha_2\alpha_1 = \gamma\alpha_1\rho = \alpha_1\alpha_2$ and Λ by $0 = \rho^2 - \alpha_2\alpha_1 = \gamma\alpha_1\rho = \rho^4 = \alpha_1\alpha_2 - \alpha_1\rho\alpha_2$.

This time, it is easy to describe the part of the Auslander–Reiten quivers containing all modules not killed by $\alpha_1\rho\alpha_2$, namely $P(a_1)$ and $I(a_1)$. The result is given in Figure 3.2 and the proof proceeds as before. From $\Lambda/\langle\overline{\alpha_1\rho\alpha_2}\rangle \xrightarrow{\sim} \Lambda^1/\langle\overline{\alpha_1\rho\alpha_2}\rangle$ we infer again that the Auslander Reiten quivers coincide.

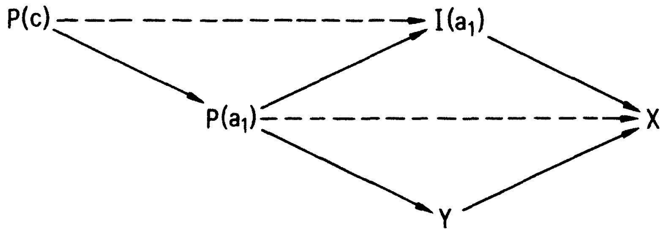


Figure 3.2

3.3. In this section we prove Corollary 1.

Arguing as in the proof of [3], 9.7 we can assume that Λ is finite. Thus, let Λ or Λ^1 be representation-finite. Then $\vec{\Lambda} \xrightarrow{\sim} \overrightarrow{\Lambda^1}$ has no infinite chain. Moreover, proceeding by induction on dim Λ and using the correpondence between ideals of Λ and Λ^1 (see [3]), we conclude that Λ and Λ^1 are mild.

If Λ^1 contains no penny-farthing we have $\Lambda \xrightarrow{\sim} \Lambda^1$ by [3], 9.5. So, let P be a penny-farthing of Λ^1. We use the notation of the introduction.

Now, $\Lambda(a_0, y) \neq 0$ for some $y \notin P$ iff $\Lambda^1(a_0, y) \neq 0$ for some $y \notin P$ (use the construction of Λ^1 given in [3], 1.7 and 1.11). In that case, part a) or part b) of our theorem applies to Λ^1. Thus we can split off the points b and c into a "receiver" and an "emitter" ([4]). The resulting categories Λ' and $\Lambda^{1\prime}$ are decomposed into direct sums $\Lambda' = \Lambda_1 \amalg \Lambda_2$ and $\Lambda^{1\prime} = \Lambda_1^1 \amalg \Lambda_2^1$. Here we are in the situation of 3.1, 3.2 or induction applies. Our statement about Λ and Λ^1 follows from the close relationship between mod Λ and mod Λ' respectively mod Λ^1 and mod $\Lambda^{1\prime}$ (see [4]). It is even so that Λ and Λ^1 have the same number of isomorphism classes of indecomposables.

By duality, we are reduced to the case $\Lambda(a_0, y) = \Lambda(y, a_0) = \Lambda^1(a_0, y) = \Lambda^1(y, a_0) = 0$ for all $y \notin P$. But then the $P(a_0)$'s are injective in mod Λ and mod Λ^1. By [9], page 404, the $P(a_0)$'s are the only indecomposables not annihilated by $\bar{\rho}^3$. Because of $(\Lambda/\langle\bar{\gamma}^3\rangle)^1 \xrightarrow{\sim} \Lambda^1/\langle\bar{\rho}^3\rangle$ induction applies.

The statement about the Auslander–Reiten quivers is well-known. But with a little more effort, the above proof can be refined to show this directly.

3.4. In the proof of Corollary 2 we can assume that Λ is standard, because Λ is mild iff Λ^1 is so (see [3], 9.7). Moreover, there is no projective-injective by [9]. Thus, if Λ contains a penny-farthing, we can suppose that up to duality we have a decomposition $\Lambda' = \Lambda_1 \coprod \Lambda_2$ as in 3.3 above, where Λ_2 is a proper quotient of Λ and Λ_1 a quotient of the categories considered in 3.1 and 3.2. But these are representation-finite by [6] and so is Λ'. We conclude that Λ is representation-finite by [4] in contradiction to our hypothesis.

3.5. Finally, let U be a faithful indecomposable of a representation-finite non-standard algebra Λ. Then Λ contains a Riedtmann-contour ([3], 9.2) by [3], 9.5. Up to duality, we are dealing with a quotient of one of the categories in 3.1 or 3.2. But then U is annihilated by $\bar{\rho}^3$ or by $\overline{\alpha_1\rho\alpha_2}$.

Note. In the meantime, Fischbacher has shown that the universal cover of a ray-category having no infinite chain is interval-finite. Combining this with [3], [6] and this paper one gets a numerical version of the Second Brauer–Thrall conjecture. Moreover, one can skip parts of Section 2.1 of this note. Also, as was pointed out by the referee, one can give an elementary proof of the main theorem of this article which uses only some Galois-coverings of representation-finite full subcategories of quotients of Λ.

REFERENCES

[1] Auslander, M. and Reiten, I., *Representation theory of artin algebras IV*, Communications in algebra *5* (5) (1977), 443–518.

[2] Bautista, R., *On algebras of strongly unbounded representation type*, Comment. Math. Helv., this volume.

[3] Bautista, R., Gabriel, P., Roiter, A. V. and Salmerón, L., *Representation-finite algebras and multiplicative bases*, preprint (1984), 1–109.

[4] Bongartz, K. and Riedtmann, Ch., *Algèbres stablement héréditaires*, C.R. Acad. Sci. Paris Sér. A–B, *288* (1979), 703–706.

[5] Bongartz, K., *Treue einfach zusammenhängende Algebren I*, Comment. Math. Helv. *57* (1982), 282–330.

[6] Bongartz, K., *A criterion for finite representation type*, Math. Ann. *269* (1984), 1–12.

[7] BRETSCHER, O. and GABRIEL, P., *The standard form of a representation-finite algebra*, Bull. Soc. Math. France, *111* (1983), 21–40.
[8] BRETSCHER, O., LÄSER, Ch. and RIEDTMANN, Ch., *Selfinjective and simply connected algebras*, Manuscripta Math. *36* (1981), 153–307.
[9] CURTIS, C. W. and REINER, I., *Representation theory of finite groups and associative algebras*, Interscience publishers (1962).
[10] FISCHBACHER, U., *A new proof of a theorem of Nazarova and Roiter*, C.R. Acad. Sc. Paris, t 300, Série I, n° *9*, 1985, 259–262.
[11] GABRIEL, P., *The universal cover of a representation finite algebra*, Proc. Puebla 1980, Springer Lect. Notes 903, 68–105.
[12] HAPPEL, D. and VOSSIECK, D., *Minimal algebras of infinite representation type with preprojective component*, Manuscripta Math. *42* (1984), 221–243.
[13] NAZAROVA, L. A. and ROITER, A. V., *Categorical matrix problems and the Brauer-Thrall conjecture*, preprint Kiev (1972), German version in Mitt. Math. Sem. Giessen *115* (1975), 1–153.
[14] RIEDTMANN, Ch., *Selfinjective representation-finite algebras of class D_n*, Compositio Mathematica *49* (1983), 231–282.

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