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## Kleine Mitteilungen

### On a Problem of Rotkiewicz on Pseudoprime Numbers

A composite number  $k$  is called pseudoprime with respect to  $a > 1$  if  $k \mid a^k - a$ . A pseudoprime number with respect to 2 is called simply a pseudoprime number. Let  $P_n$  be the  $n$ -th pseudoprime number. K. Szymiczek [3] proved that the series  $\sum 1/P_n$  is convergent. A. Rotkiewicz in his survey [2] asked (problem 47) whether the series  $\sum 1/\log P_n$  is convergent. Below we prove that the series  $\sum 1/\log P_n(a)$ , where  $P_n(a)$  is the  $n$ -th pseudoprime number with respect to  $a$ , is divergent.

M. Cipolla [1] proved that if  $p$  is an odd prime number not dividing  $a^2 - 1$  (e.g.  $p > a^2$ ), then  $(a^{2p} - 1)/(a^2 - 1)$  is a pseudoprime with respect to  $a$ . Therefore

$$\sum \frac{1}{\log P_n(a)} \geq \sum_{p > a^2} \frac{1}{\log (a^{2p} - 1)/(a^2 - 1)} \geq \sum_{p > a^2} \frac{1}{\log a^{2p}} = \frac{1}{\log a^2} \sum_{p > a^2} \frac{1}{p},$$

but the last series is divergent (see, e.g. E. Trost [4], theorem 23, p. 49), which completes the proof.

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- [3] K. SZYMICZEK, *On Pseudoprimes which are Products of Distinct Primes*, Amer. Math. Monthly 74, 35–37 (1967).
- [4] E. TROST, *Primzahlen*, Birkhäuser, Basel 1953.

### On the Equation $\varphi(n + k) = 2\varphi(n)$

W. Sierpiński [1] proved that the equation  $\varphi(n + k) = a\varphi(n)$  for  $a = 1$  and for every positive integer  $k$  has at least one solution ( $\varphi$  denotes the Euler totient function). Here we prove the analogous result for  $a = 2$ .

Indeed, if  $(k, 6) = 1$  we can take  $n = 2k$ ; if  $k$  is even:  $k = 2^l u$  ( $l \geq 1$ ,  $u$  odd), we can take  $n = 2^l u$ ; if  $k$  is odd and divisible by 3:  $k = 6l + 3$ , we can take  $n = 2l + 1$ .

The proof follows from the equalities

$$\varphi(2k + k) = \varphi(3k) = 2\varphi(k) = 2\varphi(2k) \text{ for } (k, 6) = 1,$$

$$\varphi(2^l u + 2^l u) = \varphi(2^{l+1} u) = 2^l \varphi(u) = 2\varphi(2^l u)$$

$$\varphi(2l + 1 + 6l + 3) = \varphi(4(2l + 1)) = 2\varphi(2l + 1).$$

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- [1] W. SIERPIŃSKI, *Sur une propriété de la fonction  $\varphi(n)$* . Publ. Math. Debrecen 4, 184–185 (1956).