Kleine Mitteilungen

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vectors defining any level curve can be constructed in the Euclidean sense. In practice, a combination of both geometric and arithmetic methods is most convenient to sketch any level curve quickly yet accurately.

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An identity involving Ramanujan's sum

Let f be an arithmetical function and let $f'=\mu * f$ denote the Dirichlet convolution of f and the Möbius function μ:

$$f'(n)=\sum_{d\mid n}\mu(d)f\left(\frac{n}{d}\right), \qquad n\geqslant 1\,.$$

A Ramanujan series is a series of the form

$$\sum_{q=1}^{\infty} a_q c_q(n),$$

where $c_q(n)$ is Ramanujan's sum,

$$c_q(n)=\sum_{\substack{h=1\\(h,q)=1}}^{q}\exp\left(2\pi i\frac{hn}{q}\right),$$

and where

$$a_q=\sum_{m=1}^{\infty}\frac{f'(mq)}{mq}\,.$$

H. Delange proved [1] the following.

Theorem. *If* $\sum_{n=1}^{\infty} 2^{\omega(n)} |f'(n)|/n < \infty$, *where* $\omega(n)$ *is the number of distinct prime divisors of n, then* $\sum_{q=1}^{\infty} |a_q c_q(n)| < \infty$ *for every n, and* $\sum_{q=1}^{\infty} a_q c_q(n) = f(n)$.

In his proof, Delange used the inequality

$$\sum_{d|k} |c_d(n)| \leq 2^{\omega(k)} n$$

(see [1], p. 263).

In this note we establish the following identity:

For positive integers k and n,

$$\sum_{d|k} |c_d(n)| = 2^{\omega\left(\frac{k}{(k,n)}\right)} (k,n). \qquad (*)$$

Proof: For any fixed positive integer n, the arithmetical function $g(k)=(k,n)$ is multiplicative:

$$(kl,n)=(k,n)(l,n) \quad \text{if} \quad (k,l)=1.$$

Hence if F is any multiplicative arithmetical function, then (for any fixed $n \geqslant 1$), $F(k/(k,n))$ is multiplicative in k:

$$F\left(\frac{kl}{(kl,n)}\right)=F\left(\frac{k}{(k,n)}\right)F\left(\frac{l}{(l,n)}\right) \quad \text{if} \quad (k,l)=1.$$

It follows that the right hand side of $(*)$ is multiplicative in k, for any fixed integer $n \geqslant 1$.

So is the left hand side, since $c_k(n)$, and therefore also $|c_k(n)|$ has this property ([2], theorem 67).

Thus in order to prove $(*)$, it suffices to verify it when $k=p^a$, a prime power. And indeed in this case, each side is equal to 2 if $p \nmid n$; if $p^\beta \| n$ for some $\beta \geqslant 1$, then each side is equal to $2p^\beta$ if $1 \leqslant \beta < a$, and to p^a if $\beta \geqslant a$.

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