Remarks on the note "Generalization of a formula of C. Buchta about the convex hull of random points"

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$x^{-1} + \log x - \log(x+1) > 0$, est monotone croissante. Il faut donc, pour obtenir $\max\{p(i); p \in G_n\}$, choisir $m = n$ et $r = i/(n+i)$.

Remarque: Le théorème a un analogue sous forme continue: Si f est convolution de n densités exponentielles $g_s(x) = se^{-sx}$, la plus grande valeur de $f(x)$ est atteinte lorsque tous les facteurs ont le même paramètre $s = n/x$.

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For any convex body K in the d-dimensional Euclidean space E^d $(d \geqq 2)$ let $V_n^{(d)}(K)$ be the expected volume of the convex hull H_n of n independent random points chosen identically and uniformly from the interior of K.
For arbitrary plane convex sets, respectively three-dimensional convex bodies, Buchta [2] proves the relationships

$$V_4^{(2)}(K) = 2\, V_3^{(2)}(K) \tag{1}$$

and

$$V_5^{(3)}(K) = \tfrac{5}{2}\, V_4^{(3)}(K)\,. \tag{2}$$

In a recent note [1] we generalize Buchta's formulae (1) and (2) to

$$V_{2m}^{(2)}(K) = \sum_{k=1}^{m-1} \alpha_{2m-2k+1}\, V_{2m-2k+1}^{(2)}(K) \quad m = 2, 3, \ldots \tag{3}$$

and

$$V_{2m+1}^{(3)}(K) = \sum_{k=1}^{m-1} \beta_{2m-2k+2}\, V_{2m-2k+2}^{(3)}(K) \quad m = 2, 3, \ldots\,, \tag{4}$$

where $\alpha_{2m-2k+1}$ and $\beta_{2m-2k+2}$ are constants defined by certain recursion formulae (cf. [1], formulae (1.4′), (1.4″), (1.5′) and (1.5″)).

If we develop for instance (3) for $m = 3$ and $m = 4$ we obtain

$$V_6^{(2)}(K) = \alpha_5 V_5^{(2)}(K) + \alpha_3 V_3^{(2)}(K) \tag{5}$$

and

$$V_8^{(2)}(K) = \alpha_7 V_7^{(2)}(K) + \alpha_5 V_5^{(2)}(K) + \alpha_3 V_3^{(2)}(K). \tag{6}$$

Unfortunately, the values of α_3 and α_5 in (5) and (6) do not coincide and we have an inconsistency in our notation. This can easily be saved by defining the constants in (3) and (4) to be $\alpha_{2m,\,2m-2k+1}$ and $\beta_{2m+1,\,2m-2k+2}$, respectively.
Further, the two theorems in [1] can be stated in a simpler form, namely

Theorem. *Let K be an arbitrary d-dimensional convex body $(d=2,3)$. Then,*

$$V_{2m+d}^{(d)}(K) = \sum_{k=1}^{m} \gamma_k \binom{2m+d}{2k-1} V_{2m+d+1-2k}^{(d)}(K) \quad d = 2,3; \ m = 1,2,\dots, \tag{7}$$

where γ_k are constants defined by the recursion formula

$$\gamma_1 = \tfrac{1}{2}, \tag{7'}$$

$$\gamma_k = \tfrac{1}{2}\Big(1 - \sum_{i=1}^{k-1} \binom{2k-1}{2i-1} \gamma_i\Big) \quad \textit{for} \quad k = 2,3,\dots,m. \tag{7''}$$

Remarks

(1) This theorem is in all aspects more coherent, simpler and compacter than the two proved in [1].
(2) From (7′) and (7″) one easily verifies that the constants in (7) now only depend on k.
(3) The proof of the theorem is the same as those presented in [1].

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