

Zeitschrift: Elemente der Mathematik
Herausgeber: Schweizerische Mathematische Gesellschaft
Band: 43 (1988)
Heft: 5

Artikel: Remarks on the note "Generalization of a formula of C. Buchta about the convex hull of random points"
Autor: Affentranger, Fernando
DOI: <https://doi.org/10.5169/seals-40812>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 23.12.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

$x^{-1} + \log x - \log(x+1) > 0$, est monotone croissante. Il faut donc, pour obtenir $\max \{p(i); p \in G_n\}$, choisir $m = n$ et $r = i/(n+i)$.

Remarque: Le théorème a un analogue sous forme continue: Si f est convolution de n densités exponentielles $g_s(x) = se^{-sx}$, la plus grande valeur de $f(x)$ est atteinte lorsque tous les facteurs ont le même paramètre $s = n/x$.

H. Carnal, Institut für math. Statistik der Universität Bern

REFERENCES

- 1 Rätz J. et Russell D.: An extremal problem related to probability, Aeq. Math. 34, 316–324 (1987).

© 1988 Birkhäuser Verlag, Basel

0013-6018/88/050149-03\$1.50 + 0.20/0

Remarks on the note “Generalization of a formula of C. Buchta about the convex hull of random points”

For any convex body K in the d -dimensional Euclidean space E^d ($d \geq 2$) let $V_n^{(d)}(K)$ be the expected volume of the convex hull H_n of n independent random points chosen identically and uniformly from the interior of K .

For arbitrary plane convex sets, respectively three-dimensional convex bodies, Buchta [2] proves the relationships

$$V_4^{(2)}(K) = 2 V_3^{(2)}(K) \quad (1)$$

and

$$V_5^{(3)}(K) = \frac{5}{2} V_4^{(3)}(K). \quad (2)$$

In a recent note [1] we generalize Buchta's formulae (1) and (2) to

$$V_{2m}^{(2)}(K) = \sum_{k=1}^{m-1} \alpha_{2m-2k+1} V_{2m-2k+1}^{(2)}(K) \quad m = 2, 3, \dots \quad (3)$$

and

$$V_{2m+1}^{(3)}(K) = \sum_{k=1}^{m-1} \beta_{2m-2k+2} V_{2m-2k+2}^{(3)}(K) \quad m = 2, 3, \dots, \quad (4)$$

where $\alpha_{2m-2k+1}$ and $\beta_{2m-2k+2}$ are constants defined by certain recursion formulae (cf. [1], formulae (1.4'), (1.4''), (1.5') and (1.5'')).

If we develop for instance (3) for $m = 3$ and $m = 4$ we obtain

$$V_6^{(2)}(K) = \alpha_5 V_5^{(2)}(K) + \alpha_3 V_3^{(2)}(K) \quad (5)$$

and

$$V_8^{(2)}(K) = \alpha_7 V_7^{(2)}(K) + \alpha_5 V_5^{(2)}(K) + \alpha_3 V_3^{(2)}(K). \quad (6)$$

Unfortunately, the values of α_3 and α_5 in (5) and (6) do not coincide and we have an inconsistency in our notation. This can easily be saved by defining the constants in (3) and (4) to be $\alpha_{2m, 2m-2k+1}$ and $\beta_{2m+1, 2m-2k+2}$, respectively.

Further, the two theorems in [1] can be stated in a simpler form, namely

Theorem. *Let K be an arbitrary d -dimensional convex body ($d=2, 3$). Then,*

$$V_{2m+d}^{(d)}(K) = \sum_{k=1}^m \gamma_k \binom{2m+d}{2k-1} V_{2m+d+1-2k}^{(d)}(K) \quad d=2, 3; \quad m=1, 2, \dots, \quad (7)$$

where γ_k are constants defined by the recursion formula

$$\gamma_1 = \frac{1}{2}, \quad (7')$$

$$\gamma_k = \frac{1}{2} \left(1 - \sum_{i=1}^{k-1} \binom{2k-1}{2i-1} \gamma_i \right) \quad \text{for } k=2, 3, \dots, m. \quad (7'')$$

Remarks

- (1) This theorem is in all aspects more coherent, simpler and compacter than the two proved in [1].
- (2) From (7') and (7'') one easily verifies that the constants in (7) now only depend on k .
- (3) The proof of the theorem is the same as those presented in [1].

Fernando Affentranger, Department of Mathematics, University of Buenos Aires

REFERENCES

- 1 Affentranger F.: Generalization of a formula of C. Buchta about the convex hull of random points. *Elem. Math.* 43, 39–45 (1988).
- 2 Buchta, C.: Über die konvexe Hülle von Zufallspunkten in Eibereichen. *Elem. Math.* 38, 153–156 (1983).