1. Euclid Book II, proposition 5.

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Bernheimer, *La Bibliofilia*, XXVI, pp. 300-25, 1924/25). Through the kindness of Prof. S. Gandz, use has also been made of his autograph copy of a copy made by Dr. Joseph Weinberg who made a German translation, " Die Algebra des Abū Kāmil Šoğa' ben Aṣlam " (München, 1935).

B. The classical equation $x^2 + 21 = 10x$.

1. *Euclid Book II, proposition 5.*

From Euclid, we have the geometric solution of the equation $x^2 + b = ax$. According to the *Commentary of Proclus* (ed. Friedlein, p. 44), this is an ancient proposition and a discovery of the Muse of the Pythagoreans.

> " If a straight lines be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half."

> " For let a straight line AB be cut into equal segments at C and into unequal segments at D; I say that the rectangle contained by AD, DB together with the square on CD is equal to the square on CB." [6]

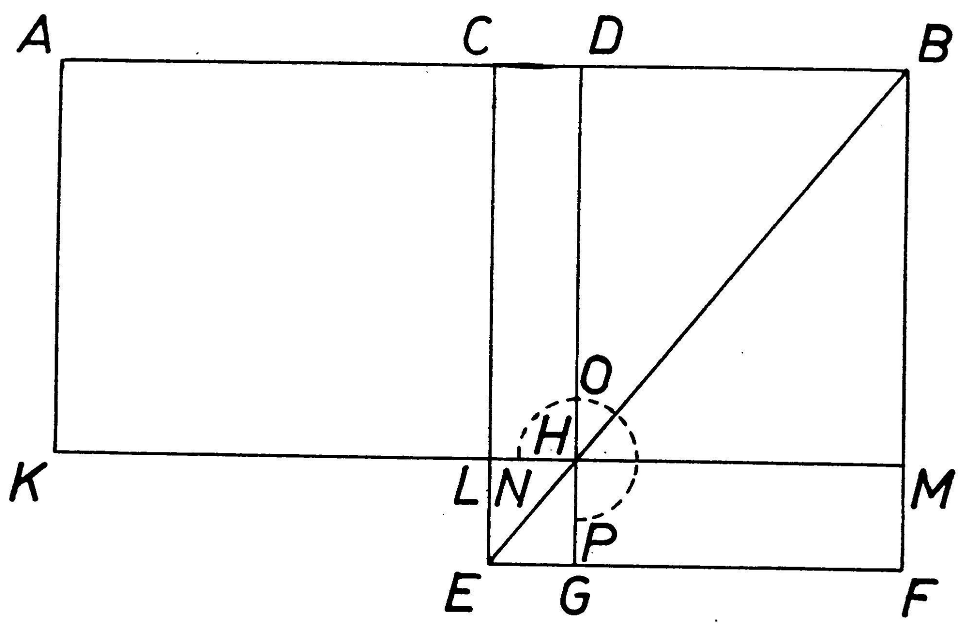

Fig. 1

In the geometric algebra of Euclid, addition and subtraction of simple numbers are, of course, performed by increasing and decreasing the lengths of lines. Multiplication is effected by construction of a rectangle using factors equivalent to the adjacent sides.

2. *Heron's solution.*

Heron proved many of the propositions of Book II by the algebraic method with the use of one line as a figure. The following excerpt is from a later Arabic commentary [1].

> " Then if we wish to demonstrate Heron's proof of this proposition, and the reasoning, we must show that the area outlined by the two parts AD and DB together with the square on line GD is equal to the square on line GB. We take two lines; one of them AD, is divided by point G, and the other line, DB, is not divided. In the proof of proposition 1 of (Book) II, the area that is outlined by the two lines AD and DB is equal to the sum of the two areas, each outlined by line BD with the two divisions AG and GD respectively. Since AG equals GB, then the sum of the two areas, bounded respectively by the two lines GB and BD, and the two lines GD and DB, are equal to the area outlined by the two lines AD and DB. Thus, there remains to us the square on GD. We distribute it as to partners (add it to both sides equally). Then the sum of the two areas bounded by the lines GB and BD, and the lines GD and DB respectively, together with the square on GD is equal to the area outlined by the two lines AD and DB plus the square on GD. But the area that it outlined by the two lines GD and DB plus the square on GD is equal to the area outlined by the two lines BG and GD, from proposition 3 of (Book) II [8]. The sum of the two areas, one outlined by lines BG and GD, and the other by the two lines GB and BD is equal to the area outlined by the two lines AD and DB plus the square on GD. But the demonstration of proposition 2 of (Book) II, the sum of the two areas, outlined respectively by the two lines GB and BD, and the two lines BG and GD, is equal to the square on line GB. The square