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surely the object was to show the power of the method of geometrical algebra as much as to arrive at results" [23].

With the Babylonian accent on the algebraic form of geometry and the ensuing dependence of al-Khwārizmī upon this source, the latter's form of geometric algebra is fully expected. Thus, from the works of al-Khwārizmī and both Heron and Euclid, respectively representing the Babylonian and Greek forms of algebra, Abū Kāmil presented algebra on a unique level. This admits of theoretical explanation and demonstration, and provides the means of integrating Babylonian practice with Greek theory into a more virile approach [24].

C. Other examples of Abū Kāmil's methodology.

Abū Kāmil was the earliest algebraist to work out the solutions directly for the square of the unknown. In the problem quoted above he makes use of the following solutions:

$$x^2 = \frac{b^2}{2} - c - \sqrt{\left(\frac{c^2}{2}\right)^2 - b^2 c}$$

and for the second value

$$x^2 = \frac{b^2}{2} - c + \sqrt{\left(\frac{c^2}{2}\right)^2 - b^2 c}$$

The addition and substraction of [25] radicals was effected rhetorically by means of the relation now known as

$$\sqrt{a} \pm \sqrt{b} = \sqrt{a + b \pm 2\sqrt{a\,b}}$$

An example of this is given in $\sqrt{9} - \sqrt{4}$, whose solution is determined to be $\sqrt{9 + 4 - 2\sqrt{36}} = 1$ in the following [26]:

> " On subtraction of roots from each other.
>
> " When you wish to subtract (the root of) four from the root of nine so that the difference of the roots be another number, you add nine to four to give thirteen [27]. Then multiply nine by four to give thirty six [28]. Take two roots of it to give twelve. You subtract it from thirteen to get one. The root of one is the difference between the root of nine and

the root of four. It is one. I shall explain it to you by this figure:

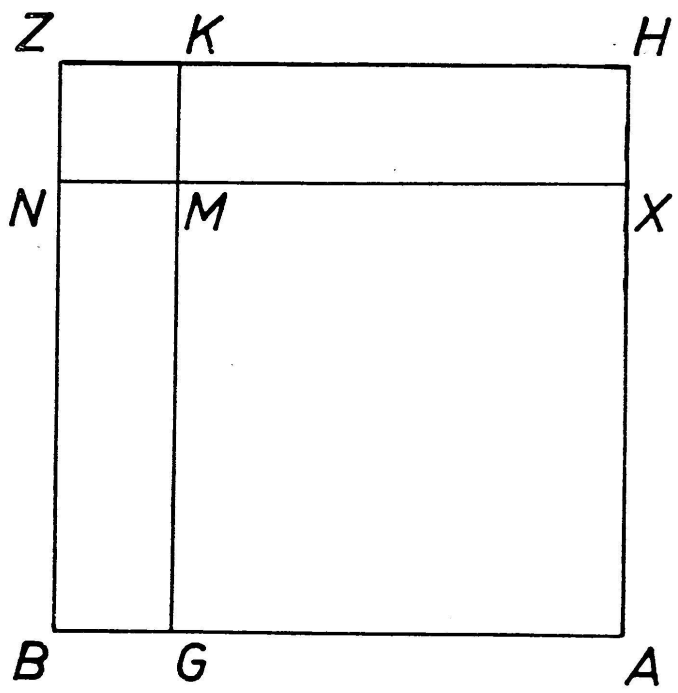

Fig.6

"We construct the line AB as the root of nine and the line AG as the root of four. When we subtract line AG from line AB, line GB remains. When we wish to know the value of line GB as a root, we construct on line AG [29], the square surface AM, which is four. You extend line XM to N and line GM to K. One knows that the square MZ is the product of GB by itself. Surface XK is two since the entire surface AK is six, or the product of line AG, which is the root of four, by AH which is the root of nine. Surface AM [30] is four and so surface XK remains as two. Also surface MB is two. Square MZ remains then as one and line MN is its root, or one. Line MN is equal to line GB and so it is demonstrated."

The use of this formula for the addition and subtraction of radicals is found in the later works of al-Karkhī [31] and Leonardo Fibonacci [32]. In Abū Kāmil, there can be no doubt that Book X of Euclid influenced him to introduce the irrational as a solution for some of his quadratic equations.

Abū Kāmil was the first Muslim algebraist to work with powers of the unknown higher than the square. In his algebra he uses the second, third, fourth, fifth, sixth and eighth powers of x. The names of these higher powers are based on the addition of exponents as we know them today. The development of this method of reckoning with exponents did not progress in a straight line. Hundred of years later symbols were still being used which were based on the systems of exponent multiplication [33].

D. Fusion of Babylonian and Greek Algebra.

From the passages of Abū Kāmil quoted above, it is evident that he was influenced by traditions which ultimately may be traced back to Babylonian and Greek Sources. On the one hand it is a further development of al-Khwārizmī's method, originally Babylonian, and on the other a utilization of the best algebraic innovations of the Greeks. It is possible that the latter were known, in part, to Abū Kāmil, through the works of Heron. The influence of Heron has already been established in the case of the great Hebrew geometer, Abraham Savasorda (12th century), as seen in his *Encyclopedia* [34]. It is interesting that Savasorda [35] who pioneered a new approach to geometry and Abū Kāmil who did the same for algebra should both have been influenced, directly or indirectly, by the great Alexandrian, Heron.

In turn, Abū Kāmil, as did Savasorda, exerted great influence upon al-Karkhī [36] and Leonardo Fibonacci, both of whom made use of many problems found in Abū Kāmil's algebra. In spite of the fact that Leonardo had squeezed Abū Kāmil's algebra dry of almost all his examples, nevertheless, enough material remained in the text so that Mordecai Finzi of the fifteenth century deemed it worthwhile to translate it into Hebrew and to insert further comments of his own.

In the painful growth of the integration of mathematical abstraction with its counterpart, the schematization and understanding of the practical basis, we have the seed of the forward development of mathematical science. With Abū Kāmil, mathematical abstraction attained recognition, not for its own