Zeitschrift: L'Enseignement Mathématique

Band: 5 (1959)

Heft: 2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE SOLUTION OF SIMULTANEOUS IMPLICIT EQUATIONS

Bibliographie

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DOI: https://doi.org/10.5169/seals-35481

independent of x. In this case the functions $Y_i(x)$ become constants Y_i. The following theorem corresponds to Theorems 1 and 2.

Theorem 6. Given the functions $g_i(y_1, \ldots, y_p) \equiv g_i(y)$ continuous on the closed region $N_1 \subset E^p$ determined by the relations $|y_i - b_i| \leqq \beta_{i1}$, where the β_{i1} are positive constants, let there exist a non-singular matrix of constants (C_{ij}) and a matrix of constants (D_{ij}) with $\sum_j D_{ij} < 1$, and such that, for $y \in N_1$,

$$\left| \delta_{ij} \Delta y_j + \sum_k C_{ik} \Delta_j f_k \right| \leqq D_{ij} \left| \Delta y_j \right| .$$

Then there exist p positive constants $\beta_i \leqq \beta_{i1}$ such that $\beta_i - \sum_j D_{ij} \beta_j > 0$. If furthermore the quantities $g_k(b) = g_k(b_1, \ldots, b_p)$ satisfy

$$\left| \sum_k C_{ik} g_k(b) \right| < \beta_i - \sum_j D_{ij} \beta_j ,$$

then the system of simultaneous equations $g_i(y) = 0$ has a unique solution $y_i = Y_i$ in the closed region $N \subset N_1$ determined by $|y_i - b_i| \leqq \beta_i$.

Moreover, if for $y \in N_1$ we define $G_i(y) = y_i + \sum_k C_{ik} g_k(y)$, and if $Y_i(0)$ is any constant satisfying $|Y_i(0) - b_i| \leqq \beta_i$, then for $m \geqq 0$ the constants $Y_i(m+1) = G_i[Y(m)]$ are well defined, and $Y = \lim_{m \to \infty} Y_i(m)$.

The appraisals of the remainder error given in Theorems 3 and 4 remain valid.

REFERENCES

1. S. Abian and A. B. Brown, On the Solution of an Implicit Equation. *Illinois Journal of Mathematics.* (Accepted for publication.)
2. T. H. Hilderbrandt and L. M. Graves, Implicit Functions and their Differentials in General Analysis. *Trans. Amer. Math. Soc.*, Vol. 29 (1927), pp. 127-153.