proof which possibly goes more to the heart of the matter emerges from the point of view of Atiyah and Hirzebruch. They define the groups:

$$KU^i(X) = \pi[E^{-i}X;\ Z \times B_U] \qquad i \leqq 0$$

where $\pi[A, B]$ denotes homotopy classes of maps. In this terminology the periodicity formula: $\Omega^2(Z \times B_U) = Z \times B_U$ is expressed by:

$$KU^i(X) = KU^{i+2}(X) \qquad i \leqq -2 .$$

Now Atiyah and Hirzebruch use this recurrence to define $KU^i(X)$ for all integers i, and then show that the resulting functor $X \rightarrow \{KU^i(X)\}$ satisfies all the axioms of a cohomology theory —except for the dimension axiom. Further they observe that the uniqueness theorem of Eilenberg-Steenrod can be generalized to yield a spectral sequence relating $E_2 = H^*(X;\ KU^*(p))$ to $KU^*(X)$. (Here $KU^i(p)$—the KU—theory of a point—is Z if i is even and 0 otherwise.) This sequence immediately implies the proposition. (See [8].)

To return to our bundles ξ^m on P_n. By the proposition just discussed the restriction of ξ^m to P_{n-k} will be trivial if $m \geqq n - k$. By trivializing this element on P_{n-k} we obtain bundles ξ^m on $P_{n,k}$ which under the projection π: $P_n \rightarrow P_{n,k}$ go over into ξ^m, $m \geqq n - k$. In particular, $\pi^* ch(\xi^m) = (e^x - 1)^m$. Thus in any case we obtain these criteria the S-reducibility of $P_{n,k}$:

$P_{n,k}$ is S-*reducible only if the coefficient of* x^{n-1} *in* $(e^x - 1)^m$, $m \geqq n - k$, *is an integer.*

This is the number theoretical condition from which Atiyah and Todd deduce theorem III. Their result is the best possible one obtainable from the test-space B_U, because one can show quite easily, with the spectral sequence alluded to earlier, that the elements 1, ξ^m; $n - 1 \geqq m \geqq n - k$; form a base of $KU(P_{n,k})$.

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