For any ξ_1 and ξ_2 satisfying

$$\xi = \eta + t\delta\,, \qquad (A\xi, \xi) = 0$$

and corresponding t_1 and t_2 of (2. 2) for which $|t_1| \cdot |t_2| = |t'|^2$ [see 3 III], there is a point ξ called the harmonic cojugate of η with respect to ξ_1 and ξ_2, so that the parameter t corresponding to this ξ satisfies the harmonic relation

$$\frac{2}{|t|} = \frac{1}{|t_1|} + \frac{1}{|t_2|}\,.$$

This implies that the image of t in the complex plane is the polar of the origin with respect to the image of t_0, defining the intersection of the line $\xi = \eta + t_0\,\delta$ and $(A\xi, \xi) = 0$. Thus

$$t = -\,\frac{(A\eta, \eta)}{(A\eta, \delta)}\,, \qquad (A\eta, \delta) \neq 0\,.$$

If $(A\eta, \delta) = 0$, then the polar is at infinity. Substituting this t in (9. 1) we get

$$\overline{(A\eta, \delta)}\, \xi = \overline{(A\eta, \delta)}\, \eta - (A\eta, \eta)\, \delta\,.$$

From the inner product of both sides with $A\eta$ we get

$$\overline{(A\eta, \delta)}\, (A\eta, \xi) = 0\,,$$

and since $(A\eta, \delta) \neq 0$ we get $(A\eta, \xi) = 0$, the equation of a plane called the polar of η with respect to the quadric.

Note that if η is on the quadric, then this plane becomes the tangent plane.

In this paper we have discussed only the properties of a quadric in any location. The problem of transformation to the most convenient position is well known, and we made use of it in section 7.

REFERENCES

1. A. R. Amir-Moéz, A. L. Fass, Quadrics in R_n. *Amer. Math. Monthly*, vol. 67, nº 7, pp. 632-636 (1960).
2. A. R. Amir-Moéz, Some Equalities in a Unitary Space Leading to Equalities Concerning Singular Values of Sets of Matrices. *Mathematische Annalen*, 135 Band, 5 (Schluss) Heft, 1958, S. 388-390.

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