Let the function F be defined on $a \leqq x \leqq b$ by $F(x) = f(x) - kx$. Then [2, p. 191]

$$D^- F(b) = D^- f(b) - k > 0 \text{ and } D_+ F(a) = D_+ f(a) - k < 0.$$

Hence there is a point ξ, $a < \xi < b$, such that F has a minimum at ξ. We note that $D^+ F(\xi) D^- F(\xi) \leqq 0$. If $D^+ F(\xi) = 0$, we choose $p = 1$ and $q = 0$. If $D^+ F(\xi) \neq 0$, we choose

$$p = \frac{D^- F(\xi)}{D^- F(\xi) - D^+ F(\xi)} \text{ and } q = \frac{D^+ F(\xi)}{D^+ F(\xi) - D^- F(\xi)}.$$

In either case $p \geqq 0$, $q \geqq 0$, $p + q = 1$, and $p\, D^+F(\xi) + q\, D^-F(\xi) = 0$. Hence

$$p\left[D^+ f(\xi) - k\right] + q\left[D^- f(\xi) - k\right] = 0.$$

It follows that

$$p D^+ f(\xi) + q D^- f(\xi) = k$$

which was to be shown.

The reader will observe that we have not required the derivates at a and b to be finite. In fact, we need not require that all derivates be finite at each point of the open interval $a < x < b$. However, since these cases are easily resolved, they will be left for the consideration of the reader.

A closely related theorem may be obtained by substituting $D^+ f(a) > k > D_- f(b)$ for $D_+ f(a) < k < D^- f(b)$ in the statement of our theorem. A proof of the new theorem can be obtained by marking minor modifications in the above proof.

REFERENCES

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2. McShane, E. J., *Integration*. Princeton, Princeton University Press, 1944.

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