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# REMARK ON A FÉJER'S INEQUALITY WHICH IS USED IN THE WEIERSTRASS FACTORIZATION THEOREM

by Michael SHASHKEVICH

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In E. Hille's *Analytic Function Theory*, Vol. 1 (1959), page 227, the following theorem is given:

Let

$$E_p(z) = (1-z) \exp \left\{ z + \frac{z^2}{2} + \dots + \frac{z^p}{p} \right\}, \quad p = 1, 2, \dots;$$

for  $|z| \leq 1$  we have

$$|E_p(z) - 1| \leq |z|^{p+1}.$$

There is also the remark: « This proof was communicated to me some forty years ago by my teacher Marcel Riesz. If I remember correctly, he ascribed it to Fejer. The proof does not seem to have been published. »

The proof is based on the fact that the coefficients of the development of

$$E_p(z) = 1 + \sum_{k=1}^{\infty} A_{k,p} z^k$$

have the property

$$A_{1,p} = A_{2,p} = \dots = A_{p,p} = 0, \text{ and } A_{k,p} < 0 \text{ for } k > p,$$

which is a consequence of

$$E'_p(z) = -z^p \exp \left\{ z + \frac{z^2}{2} + \dots + \frac{z^p}{p} \right\}. \quad (1)$$

But starting from (1) the proof can be obtained immediately as follows. Indeed, if  $|z| \leq 1$  and  $0 \leq t \leq 1$  it follows that

$$|E'_p(tz)| \leq -|z|^p E'_p(t),$$

and hence

$$|E_p(z) - 1| = \left| z \int_0^1 E'_p(tz) dt \right| \leq -|z|^{p+1} \int_0^1 E'_p(t) dt = |z|^{p+1}.$$

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