

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 9 (1963)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: MATRICES OF LINEAR OPERATORS
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Bibliographie
DOI: <https://doi.org/10.5169/seals-38783>

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Since T_j is of the form $T_j = a_j I + K_j$, it follows from (25) that (37) is equivalent to

$$[P(\lambda_1)I - \sum_{j=1}^m \lambda_1^{m-j} K_j] u_0 = 0. \quad (38)$$

Since $P(\lambda_1) \neq 0$ by hypothesis, this is a generalized Fredholm equation of the second kind. The number λ_1 is an eigenvalue of \mathbf{T} if and only if (38) has a non-zero solution u_0 , in which case (36) gives a corresponding eigenvector \vec{u}_0 .

A special case of a composite recursion relation was studied by D. Greenspan and the author in [4]. Asymptotic results were obtained there which go beyond those given above.

We conclude this paper with a generalization of Theorem 1. Let \mathfrak{A} be an algebra with unit I over the complex field. Let \mathcal{I} be an ideal in \mathfrak{A} . Let \mathfrak{A}_m denote the algebra of all $m \times m$ matrices $\mathbf{T} = [T_{ij}]$ with $T_{ij} \in \mathfrak{A}$. Then the set

$$\mathcal{I}_m = \{\mathbf{K} = [K_{ij}] : K_{ij} \in \mathcal{I}\} \quad (39)$$

is an ideal in \mathfrak{A}_m .

Theorem 2. Let $\mathbf{T} = [a_{ij}I + K_{ij}]$, where $K_{ij} \in \mathcal{I}$. Let $P(\lambda)$ be the characteristic polynomial of the scalar matrix $[a_{ij}]$. Then $P(\mathbf{T}) \in \mathcal{I}_m$.

Since the proof is essentially the same as that for Theorem 1, it is omitted.

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