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Artikel: VERTEX POINTS OF FUNCTIONS
Kapitel: 5. Vertex points
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II. Let the rank of Q be k, and centers exist. Then these centers are solutions of

$$\xi_k = \xi E = -\left(\frac{\partial f}{\partial \xi}\right) EQ^{-1}, \tag{3.2}$$

where Q^{-1} is the reciprocal of Q, see [2]. That is, if E is the projection on the range of Q, then

$$Q^{-1}Q = QQ^{-1} = E.$$

Here we choose the center of quadric curvatures at a point of (3.2) so that, it is at the shortest distance from γ.

III. When the rank of Q is k and the quadric does not have centers, then we say that f does not have a center of quadric curvature.

4. Direction of quadric curvature

In part I and II of section 3 we respectively call the vectors ξ and ξ_k the directions of quadric curvature of f at $(c_1, \ldots, c_n)$. In III of section 3, we define the direction of quadric curvature to be a vector δ which satisfies

$$\delta = \delta E = -\left(\frac{\partial f}{\partial \xi}\right) EQ^{-1},$$

where E is the projection described in section 3.

5. Vertex points

Let at the point $\gamma = (c_1, \ldots, c_n)$ of f the direction of quadric curvature be the same as the normal to $f = 0$. Then γ is called a vertex point of the function f.

Theorem: A necessary and sufficient condition for a point to be a vertex point of the function f is that at that point

$$PQ = QP,$$

where P and Q are the matrices described in section 3.

Proof: At a vertex point the projection of the direction of quadric curvature on the tangent plane is zero. Thus

$$-\left(\frac{\partial f}{\partial \xi}\right) Q^{-1} (I-P) = 0 .$$

This implies that

$$Q^{-1} PQ = P .$$

In all cases this implies

$$PQ = QP .$$

A vertex point in particular may become a spherical point, i.e. a point where

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = \lambda \delta_{ij} , \lambda$$

is a constant.

A vertex point will be called a cylindrical point when

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = \lambda \delta_{ij} , i , j \leqq k ,$$

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = 0 , i , j > k .$$

6. Functions of fixed center

An interesting fact about these functions is that they are not necessarily quadrics.

The equation.

$$\xi Q = -\left(\frac{\partial f}{\partial \xi}\right) \tag{6.1}$$

where $\xi = (c_1 - x_1, \ldots, c_n - x_n)$, and $(c_1, \ldots, c_n)$ is the fixed center gives f. To produce a counter example we let the origin be the center and the dimension of the space be two. Then in the real case (6.1) becomes