6. FUNCTIONS OF FIXED CENTER

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Proof: At a vertex point the projection of the direction of quadric curvature on the tangent plane is zero. Thus

$$-\left(\frac{\partial f}{\partial \xi}\right) Q^{-1} (I-P) = 0 .$$

This implies that

$$Q^{-1} P Q = P .$$

In all cases this implies

$$PQ = QP .$$

A vertex point in particular may become a spherical point, i.e. a point where

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = \lambda \delta_{ij} , \lambda$$

is a constant.
A vertex point will be called a cylindrical point when

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = \lambda \delta_{ij} , i , j \leqq k ,$$

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = 0 , i , j > k .$$

6. Functions of fixed center

An interesting fact about these functions is that they are not necessarily quadrics.

The equation.

$$\xi Q = -\left(\frac{\partial f}{\partial \xi}\right) \tag{6.1}$$

where $\xi = (c_1 - x_1, \ldots, c_n - x_n)$, and $(c_1, \ldots, c_n)$ is the fixed center gives f. To produce a counter example we let the origin be the center and the dimension of the space be two. Then in the real case (6.1) becomes

$$\left.\begin{aligned} x\frac{\partial^2 f}{\partial x^2} + y\frac{\partial^2 f}{\partial x \partial y} &= \frac{\partial f}{\partial x} \\ x\frac{\partial^2 f}{\partial x \partial y} + y\frac{\partial^2 f}{\partial y^2} &= \frac{\partial f}{\partial y}\,. \end{aligned}\right\} \tag{6.2}$$

We can easily find a solution of (6.2) which is not a quadric. For example

$$f = \frac{x^2}{2}\log\left(\frac{\sqrt{x^2+y^2}+y}{x}\right) + \frac{y}{2}\sqrt{x^2+y^2}\,.$$

REFERENCES

[1] A. R. Amir-Moéz, Quadrics in a Unitary space, *L'Enseignement Mathématique*, tome VII (1961), pp. 252-257.

[2] Penrose, A generalized inverse of matrices, *Proc. Cambridge Philos. Soc.*, vol. 51 (1953), pp. 406-413.

[3] Ali R. Amir-Moéz, Les sommets d'une surface, *L'Enseignement Mathématique* 10 (1964) p. 255

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