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PRIORITIES AND RESPONSABILITIES IN THE REFORM OF MATHEMATICAL EDUCATION: AN ESSAY IN EDUCATIONAL METATHEORY [1])

by Alexander WITTENBERG

One can ask two different kinds of questions concerning the teaching of mathematics—or, to be more precise, questions at two different levels of analysis. On the *second* level, we ask questions like these: Should the theory of groups be taught in high school ? Should we make use of the " language of sets " ? Should Euclid go ? Questions like the following also belong to the same level of inquiry: Should a group of teachers be commissioned here and now to write a grade 10 textbook ? Should one million dollars be ear-marked for summer courses for teachers ?— The questions at the *first* level are of a much more basic kind; these are questions like the following: What is it that we are trying to accomplish ? What are our criteria for deciding whether Euclid should go ? Generally speaking, what is the nature and structure of the second-level questions, and what are our criteria for dealing with them ?

If questions at the second level deal with concrete problems of educational theory and practice, the first-level questions deal, as it were, with educational *meta-theory.* They aim at the broad terms of reference which define the context within which alone the second-level questions can meaningfully be asked, investigated, and answered.

The most serious general criticism that must be levelled, I am afraid, against much of the work that has been done to reform mathematical education during the last decade or so (and against much of the earlier work as well), is that it has laid

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[1]) This paper was presented in slightly shortened form to the colloquium of the International Commission on Mathematical Instruction of the International Mathematical Union in Utrecht, Holland, December 19-22, 1964. The theme of the colloquium was " Modern Curricula in Secondary Mathematical Education ".

nearly exclusive emphasis on second-level questions, while almost completely neglecting serious study of the meta-questions. The result, all too often, has been the spectacle of a reform-movement surging ahead with great determination, fervour, and strength of conviction, yet utterly unable to give any rational account to itself or to others of *why* it was going where it was going, and how, once it arrived there, it would decide whether it had been a good idea trying to get there in the first place.

This situation is the more odd, since much of the mathematical inspiration behind the contemporary reform-movement derives from the impressive successes of the modern axiomatic method. And the first, meta-theoretical level which I was describing is quite analogous to the level of an axiomatic analysis: it is the level at which we try to gain analytical and critical clarity about our underlying assumptions and about what these do and do not logically imply. My criticism is, in effect, that while there has been a great deal of *talk about* the axiomatic method in the reform programs, there has been a sore lack of genuine axiomatic thinking in the work leading up to them. All too often, the place of that thinking has been taken by dogmatic, arbitrary assertions, occasionally even by obvious logical non-sequiturs; the latter occurred for instance when wide-ranging discourses concerning the role of mathematics in the modern world were used as motivations for specific curricular proposals without even an attempt to make a convincing case that the latter have anything to do with the former.

I believe that the most urgent requirement, if we are to attain responsibility and sanity in the reform of mathematical education, is to face up to the meta-questions, so as to establish at least a *framework of coherent and responsible discussion* of the many issues involved. This is the reason why I have chosen to give this talk at the meta-level, as it were. I am not going to discuss specific curricular matters in what follows; I shall not tell you how much mathematical libido I invest in Euclid and in the language of sets, respectively. Rather, I shall focus my remarks on basic principles of the reform of mathematics education. In order to bring the ideas I wish to present to you into even sharper focus, I shall formulate a series of clear-cut theses as I go along. A relatively elaborate attempt at detailed axiomatic analysis of the meta-problems will be found in a questionnaire in a forthcoming issue of DIALECTICA [1]).

I. The Teaching of Mathematics

1. The teaching. What a teacher actually does when teaching mathematics is determined by *two* parameters; I shall call them *content* and *form.*

(a) The *content* of his teaching is made up of the items included in the syllabus—e.g. equations, inequalities, functions, physical applications, etc. This also includes terminology (e.g. the " language of sets "), standard methods, etc.

(b) The *form* of the teaching can be analyzed into two different, although closely related, elements:

b 1) *Inner organization of the content.* The content can be presented in a coherent or an incoherent way, with an atomistic or a holistic structure, as a collection of many isolated items (" units ") or as an exploration of a few major connected topics (in the sense of the " Themenkreis-methode " which I discuss in my book " Bildung und Mathematik ".[2]))

b 2) *Type of classroom work.* The major emphasis in the actual teaching process can lie on the communication of ready-made mathematical information by the teacher to the student (dogmatic, magisterial teaching); in that case, the underlying structure of the teaching situation is: active output by the teacher, passive reception by the student. (The student may still by very *busy* in such a situation —but his busyness is embedded into a basic passivity; it consists in busily carrying out what he has been told to do, in the way which has been prescribed to him). The role of the teacher essentially is that of a mechanical purveyor of information and checker of routine problems. Not surprisingly, many will think that this operation can

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1) Dialectica, forthcoming.
2) Ernst Klett Verlag, Stuttgart, 1963.

be mechanized altogether, and the teacher's job done more cheaply and more efficiently by a teaching machine.

Alternatively, the major emphasis can lie on the generation by the teacher of broadly directed, yet largely autonomous intellectual activity on the part of the students: genetic teaching, rediscovery method, méthodes actives, Arbeitsunterricht.[1]) The students think by themselves, and are ultimately guided by their own personal insight. The primary function of the teacher in this case is to question, to guide, to stimulate, to catalyze, to help synthesize and to assess within a broad predetermined framework of teaching aims: The teacher makes the students think creatively. In this case the details of the teaching process cannot be completely predetermined; they are to some extent essentially unpredictable. A fortiori it is impossible to *program* this kind of teaching. Between teacher and students, a true interpersonal relationship develops as a support for, and integral part of, this educational process—a genuine, continuing dialogue, whose educational value extends far beyond the mere acquisition of mathematical knowledge by the students.

Thesis 1.

In the teaching of mathematics, the form *of the teaching is more important than the* content. *In other words:* how *the student learns is more important than* what *he learns.*

Thesis 2.

The teaching of mathematics should maximize the inner organization of the content; it should present the content as a meaningful, coherent, interconnected exploration of a few clearly significant topics. At the same time, the teaching should maximize autonomous intellectual activity by the student—insight, understanding,

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1) See G. Polya, How to Solve It, Anchor paperback, and Mathematical Discovery, Vol. I and II, Wiley. M. Wagenschein, Exemplarisches Lehren im Mathematikunterricht, Der Mathematikunterricht 8, 1962, part 4. A. Wittenberg, *op. cit.*, and A. Wittenberg, Sr. Ste Jeanne de France, Fernand Lemay, Redécouvrir les Mathématiques, Neuchâtel, 1963. Further literature is cited in these books.

creative problem-solving, active intellectual analysis and synthesis of genuinely interesting mathematical situations.

Thesis 3.

Reform of the form *of teaching mathematics should take clear precedence over reform of the* content. *The latter should be subordinated to the former; that is, a minimum requirement for proposed changes in content is that they should be consistent with, and a fortiori should not jeopardize, desirable changes in form.*

To exemplify the impact of the three theses in a nutshell, they mean that it is more desirable, for the mathematical *education* of a student, to acquire a thorough, versatile, resourceful understanding of elementary algebra, and no more, than it is to acquire insightless skill in the performance of integrations and differentiations.

The three theses also imply that reforms that consist in a mere increase, or change, in content can be highly misleading, creating a façade of improvement over what may actually be a deterioration of the teaching. This will occur, in particular, if the changes in content force a change in form away from what is desirable.—One of the most typical examples for this is the set-theoretical approach to geometry, in which the student is required to take for granted that a line, or a plane, is a set of infinitely many points; an assertion that raises a number of very old puzzles, and one that a student can only be made to accept unhesitatingly if he is in effect prevented from thinking by himself.[1]

As to the content of the teaching, an essential minimum requirement is that it should not be the result of an arbitrary choice. The motivations for a given choice of content can be of several kinds—intrinsic interest, practical necessity, professional preparation of the student, etc. Whatever they are in any given case, they should be bona fide motivations, they should be explicitly spelt out, and the connection between these motivations and the actual curricular proposals should be explicitly

[1] See A. Wittenberg, Formation et Information Scientifiques dans l'Enseignement secondaire, Revue Internationale de Pédagogie, Vol. 9, No. 4, 1963-64, pp. 407-417.

analyzed.[1]) It goes without saying that the analysis must also extend to the compatibility of the proposals with thesis 3:

Thesis 4.

The content of the teaching must be an effectively stated function of effectively stated motivations; the implications of any given content for both aspects of the form of the teaching must be explicitly analyzed.

2. THE TEACHERS. No teacher can teach in the way required by thesis 2 without thorough personal mastery of the subject-matter taught. Without such mastery, he will be at the mercy of every unexpected remark by a student, of every original suggestion, of every surprising question. Lack of grasp of the subject-matter compels the teacher to minimize personal and original thinking on the part of the students; this becomes for him a matter of sheer self-preservation. In addition, it is the most basic fact of life in education that no teacher can convey an understanding of, nor a taste for, that which he does not understand himself.

Thesis 5.

No teacher should teach mathematical subject-matter unless he has thorough personal understanding of that subject-matter in its broader context.

For instance, no teacher should teach an axiomatic or quasi-axiomatic approach to geometry, who has no clear insight, both into the meaning of such an approach, and into the nature of the axiomatic method. A teacher should only teach sets if he knows " naive set theory " reasonably well, and has personal knowledge of at least some significant examples of important applications of the set-theoretical approach; etc.

An immediate consequence of thesis 5 is the following:

Thesis 6.

The requirement of thesis 5 should be the primary, overriding factor governing the choice of content. In particular, introduction

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[1]) Cf. the Dialectica-questionnaire, footnote 2.

of new content should be subordinated to the availability of teachers satisfying the requirements of thesis 5.

This means, for instance, that if in a given situation a choice must be made between having traditional elementary algebra taught by teachers who have or can acquire a thorough understanding of it, or having some brand of " modern " algebra taught by teachers without such understanding, the decision *must* be in favour of the former possibility, irrespective of any other elements influencing the choice.

Rethinking the teaching of mathematics in the sense of thesis 2 is a difficult and sophisticated intellectual exercise; so is the mastery of new and unfamiliar mathematical material. To do both things at once is nearly impossible for many teachers. In the light of the priority of form over content, that fact entails the following:

Thesis 7.

When educating or re-educating teachers, priority should be given to the rethinking of the teaching of familiar material.

Some critics of my book " Bildung und Mathematik " have overlooked the fact that it is this thesis which, quite explicitly, underlies the choice of the material discussed in the book (see pp. 70-71).

3. The results. No rational discussion of the teaching of mathematics (or, for that matter, of the teaching of any other discipline) is possible without clear criteria of success. If we do not know what it is that we are trying to do, we cannot evaluate whether we succeed in doing it or not, nor even whether we are *in fact* attempting to do what we *believe* we are attempting to do. For instance, we may believe that we are trying to teach an understanding for the mathematical idea of structure; while in fact, conceivably we are not even making a beginning in that direction. In the teaching of mathematics as elsewhere, wishful thinking and self-delusion are no substitute for careful, critical, in effect axiomatic, analysis.

Thesis 8.

Any proposal for the teaching of mathematics, and in particular any reform undertaking, must state beforehand *the terms in which it defines its success, and the means by which it proposes to check whether it succeeds or not. Its proposed criteria must be demonstrably consistent with, and a check upon, its stated aims.*

The most straightforward (although not necessarily the only) check on the actual aims of our teaching is provided by the actual content of our examinations. Essentially, we are really teaching that which we are prepared to examine in our students. It is a sorry and a ridiculous sight when reform undertakings define themselves in the most ambitious and uninhibited terms, claiming to teach everything from the axiomatic method to the structural approach to mathematics, and then end up setting final examinations in which the students are required to do nothing more subtle than, say, find the cardinality of the union of two sets of given cardinalities.

Thesis 9.

Examinations must be framed in terms of the stated aims of the teaching. The first test of the seriousness of the commitment of those responsible for the teaching to their stated aims lies in their willingness to devise examinations that are explicitly geared to these aims.

But candid and searching examinations are of no avail if the examination *results* are manipulated in such a way as to effectively destroy the value of the examination as a control on the success of the teaching. In particular, it is a rather crude statistical fallacy (and one that is particularly surprising when it comes from leaders in the movement to introduce the teaching of statistics in the schools !) to adapt the results of examinations on new curricula to standard statistical norms, and still expect to get from these examinations useful information concerning the value of the curricula.[1])

[1]) See Cooperative Mathematics Texts, A Progress Report; Mathematical Education Notes, Amer. Math. Monthly, 69, 3, March 1962.

Thesis 10. (Corollary to theses 8 and 9).

Proposals for the teaching of mathematics must include proposals for typical examinations, together with a statement of the kinds of examination results that will be considered acceptable *in terms of the success or failure of the proposals.*

I may add here that, if the aims include the mastery of *ideas* by the student (for instance an understanding for the idea of mathematical structure), then the examinations must include tests for mastery of ideas, that is essay-type questions; e.g. " Explain and discuss the idea of mathematical structure ". In a system of education like the French one, such questions might build valuable bridges between the teaching of mathematics and that of philosophy, particularly philosophy of science.

II. The Process of Reform

So far, I have been discussing basic principles that, in my opinion, ought to govern the process of elaborating, implementing, and assessing proposals for shaping the teaching of mathematics.

The most basic principle of all, however, has not been stated so far. This simply is that *proper care* must be exercised in shaping the teaching of mathematics. Everything else will be to no avail if such care is absent, if we allow sloppiness and irresponsibility to prevail; exactly as progress in medical science will be of no avail if medical practitioners fail to wash their hands and to learn about proper dosages of antibiotics. In the past, unfortunately, proper care has not always been exercised, and we have seen in the field of mathematical education major reform undertakings that were open to serious criticisms not on recondite grounds of educational or mathematical philosophy, but on the down-to-earth grounds of sheer mathematical and educational competence or conscientiousness.

Educational reform has this in common with medical research that it deals with the lives of human beings. Untold damage can be wrought if it deals with them carelessly and callously.

It is therefore appropriate to spell out explicitly what the exercise of proper care does mean in the field of mathematical education. To this I turn now.

First of all, there can be no excuse whatever for finding out from experiment with children what could have been found out by the mere exercise of careful judgment. Educational experimentation should not be a substitute for thought, nor a means for dispensing with the need for thought.

Thesis 11.

No major proposals should be implemented in actual practice —not even on an experimental scale—unless they have been elaborated with maximum care, and have had the benefit of careful and informed scrutiny and discussion. There can be no excuse for the use in actual teaching of hastily written, shoddy, obviously imperfect teaching materials.

Probably the worst example, considering the scale of the operation and the way in which it was advertised, is provided by some of the sample textbooks produced by the American School Mathematics Study Group—textbooks that were launched into large-scale use in actual teaching without even having had the benefit of responsible editing by responsible editors willing to sign their names.[1])

This example points up a further problem. The School Mathematics Study Group widely, and correctly, advertised the fact that respected professional mathematicians had been involved in its work. There have been several disturbing instances, in countries on both sides of the Atlantic Ocean, in which respected, and sometimes even very distinguished, mathematicians allowed themselves to be associated with, or even explicitly endorsed, novel textbooks which, quite obviously, they had not read carefully; textbooks which contained purely mathematical nonsense that any competent mathematician could detect.[2])

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1) See for instance my paper: Sampling a Mathematical Sample Text, Amer. Math. Monthly, Vol. 70, No. 4, April 1963, pp. 452-459.

2) G. Polya, Mathematical Discovery. Vol. II, p. 134, 14.16, makes the same remark in the form of a lovely anecdote.

In my view, this is exceedingly serious. The public at large—including, very often, the educational public—is nowhere more helpless and more dependent on reliable advice than when it comes to judging the mathematical correctness of proposed texts and programs. No non-mathematician, and very few practicing teachers, can challenge the word of a university mathematician who tells them that this or that proposal is "in the mainstream of contemporary mathematics". There arises therefore a heavy responsibility for the professional mathematician not, in effect, to endorse mathematical quackery. Indeed, he, and he alone, can expose it—he is at least responsible not to condone it.

Thesis 12.

It should be a matter of professional ethics for any mathematician not to endorse or lend his name to any mathematical textbook or method, unless he has satisfied himself of at least that text's or method's mathematical correctness and quality, or unless he explicitly specifies on the record whatever reservations he has to make.

Thesis 13.

New mathematical textbooks, particularly novel ones, should be carefully reviewed at least for mathematical content in the major mathematical journals.

I may add in passing that the amount of self-advertising allowed to the various reform undertakings might well be drastically cut down. It is inconsistent with well-established standards of editorial propriety when reputable journals, on the one hand do not review novel textbooks, yet on the other hand give the authors of these texts editorial space for advertising the alleged merits of their work. More generally, I believe that there are altogether too many publications that merely rehash the claims of the various reform undertakings uncritically, while there are far too few publications attempting to give a serious critical evaluation of them.

Once texts or curricular proposals are as sound as competent and knowledgeable thinking can make them, time has come to try them out in the class-room. I say *try them out*! That is, even at that stage it would be utterly irresponsible to introduce them for large-scale use. The next stage must rather be one of careful small-scale experimentation, using the texts in a carefully selected sample of actual classrooms. The selection must be made in such a way as to accurately reflect the kinds of classroom situations in which the texts are to be used, if found to be successful; e.g. classes of gifted children or of average children; of highly motivated or of poorly motivated children; classrooms with highly qualified teachers or classrooms with poorly qualified teachers, etc. It is important that this experimentation be as little glamourized as possible; otherwise, the Hawthorne-effect will be maximized, and strong drives will be created, whether conscious or unconscious ones, to ignore or hide possible adverse results.

The use of a new approach only constitutes bona fide experimentation if there is careful evaluation of the resutls. This is elementary. Unfortunately, it is necessary to state this. There is a veritable mountain of publications offering, proposing, developing " new thinking in school mathematics " (to borrow the title of a well-known OECD publication) and proclaiming its miraculous therapeutic virtues. There are tragically, scandalously few publications giving a careful and candid account of observations made with the use of those new approaches. In fact, I can only think of one—a very interesting and informative paper by F. M. Hall, Group Theory in the Sixth Form, Math. Gazette XLV, 353, October 1961. This is probably not the only one; but it does belong to a pitifully small group. Can it possibly be that so few results are being reported because there are so few encouraging results to report ?

However that may be, I think that the publication of candid accounts of experiments is essential. Indeed, an " experiment " has only truly become that once its results are on the public record.

Thesis 14.

Careful, small-scale experimentation followed by candid, independent, competent evaluation in terms of aims stated beforehand *should precede any large-scale introduction of reforms. The results of these evaluations should be published in every case.*

I may add two remarks here. First, the willingness to have one's results evaluated and the evaluation published really is the crucial test for the seriousness of a reform undertaking. No financial support should therefore be given to any organization or project that does not include *in its plans* satisfactory plans for the evaluation of results.

In many cases, particularly in the case of wholesale approaches to the teaching of mathematics, the evaluation might well include an appraisal by non-mathematicians involved in education, for instance concerning possible side-effects. Mathematics teaching is not the whole of education ! A proposal might for instance succeed on its own terms, but require an inordinate amount of time or home-work; or destroy the chances of establishing meaningful connections between mathematics and physics; or require groupings of children that are inherently undesirable, whatever their specific usefulness might be. These aspects should not be disregarded.

Contrarily to, say, history or literature, mathematics is a fairly international discipline, and the teaching of mathematics is a fairly international problem with solutions that vary relatively little from one country to another. If it is possible and makes sense to teach certain kinds of students in a certain way in one place, there is a strong likelihood that this will also be possible and make sense in other places, at least if these have the same basic aims of education. There is therefore a strong case to be made for an international pooling of resources and experience in the reform of mathematics teaching. By the same token, there is a strong case to be made *against* the proliferation of little local reform undertakings ignoring each other and everything that went on before them, duplicating each other, refraining from talking to each other, each one proceeding with similar lack of sensible evaluation. The reform of mathematics teaching must not become a form of international *featherbedding* for school and university teachers in need of prestige, summer earnings, and openings with textbook publishers.

Among the many dangers in that senseless, sometimes ridiculously nationalistic, proliferation, two are particularly conspicuous. On the one hand, genuinely valuable and significant work may simply be buried under the accumulation of worthless and monotonously similar work; or it may be shouted down in the contest for publicity by undertakings of less intrinsic merit, but greater financial strength and Madison-Avenue-type acumen. Where too much is going on, not as a coherent and focussed effort but as a disjointed collection of isolated ventures, it is very much as if nothing were going on: everybody talks and nobody listens.

In addition, when many ventures arise not because genuinely creative educators have an important contribution to make, but because there is a bandwagon which some local people would not like to miss, the result is the spread of a stifling and unreasoning orthodoxy, and beyond this, the development of an attitude of mind which is clearly incompatible with educational progress of any kind. I feel that this to some extent has been the fate of the movement for " modernization ". Whatever the intrinsic merits of the substantial proposals that have been made for teaching " modern " topics, there is no merit whatsoever in the spread of numerous " reforms " which are " modern " to the extent that they use the " language of sets ", and no more—indeed, occasionally use it in a way which makes clear that the authors of the reforms have no inkling of what it is that they are talking about.[1])

Thesis 15.

Appropriate steps should be taken for an international pooling of resources in the field of mathematical education: the pooling should extend over the whole range of the process of reform, from the first elaboration of new approaches to the final evaluation of their feasibility and merit. It should particularly also include the creation of new textbooks.

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1) A wide-spread general instance of this is the introduction of the " language of sets " by authors who are not aware of the basic difference between the *extensional* and the *intensional* idea of a set. These authors will then give a family, a class, a football team as examples of sets, unaware of the fact that from the intensional standpoint of ordinary speech these remain *the same* family, class, or team after the addition of a new member.

How can this be done in practice ? Not primarily, I believe, through organizational means. The impact of Parkinson's Law on the reform movement has been much too heavy as it is already. In fact, there is no need to devise new means. The means for establishing international cooperation in research are wellknown; they apply in the field of mathematical education as they do elsewhere. They all revolve around two foci: professorial chairs as centers of broad-based scholarship, research, and teaching; international journals as organs for the publication, review, confrontation, and discussion of work done.

Thesis 16.

To ensure continuous work in the field of mathematical education, at an appropriate level of intellectual distinction, the creation of a limited number of university chairs for mathematical education should be encouraged. The requirements for the holders of these chairs—both in terms of their qualifications, and in terms of the duties to be discharged by them—should be comparable in every way to the requirements for holders of chairs in other fields of learning. Necessary financial support for these chairs should include the provision of appropriate scholarships to enable students from all over the world to come and study under the holders of these chairs.

Thesis 17.

There is an urgent need for a genuinely international journal —international in its editing and in its diffusion—to deal with all aspects of the reform of the teaching of mathematics, at an appropriate level of editorial sophistication.

If we are to make enlightened choices concerning desirable ways of teaching mathematics, the first requirement is a wide range of thought and experience concerning conceivable alternatives. The development of a wide and diversified range of approaches of suitable quality should therefore be actively encouraged. It is shocking to find support being given to dozens of enterprises that all do essentially the same thing, while dissenting approaches, far from being fostered, are being ignored and starved. It is even more shocking to find organizations making a loyalty oath to H. M. Nicholas Bourbaki in effect a prerequisite for support. Bribery is no way to foster the free growth of ideas, in education no more than anywhere else.

Thesis 18.

Every effort should be made actively to maximize genuine creative diversity in the field of mathematical education, and to maximize at the same time the intellectual confrontation between the various approaches.

The way to accomplish this is indicated by the theses 16 and 17. Ideally, the chairs envisioned in thesis 16 should be held by men or women who each represent one distinguished and distinctive approach to the teaching of mathematics. One could well envision, for instance, one such chair being held in Paris by one of the intellectual children of Professor Nicholas Bourbaki, as a center of thought and experimentation along lines that have become familiar in theory, if not in practice, to everyone concerned with the teaching of mathematics.

The confrontation will arise in three ways. First, if those chairs live up to the requirements of thesis 16, then confrontation of varying approaches, in seminars, study of the literature, guest lectures, etc. will be one of their primary concerns. It would be as unthinkable for a distinguished professor of mathematical education to confine himself, and the horizon of his students, to only the approach that he himself favours, as it would be for a professor of physics or biology. Second, the journal envisioned in thesis 17 would of course operate as a constant, challenging medium of confrontation and discussion. Finally, once enough insight has matured to warrant it, time has come for international congresses.

III. And what about “ modernization ” ?

I do not seem to have spoken about our topic at all—the so-called “ modernization ” of secondary school mathematics, that is, the introduction into secondary school teaching of certain specific types of terminology and content.

(In passing I would like to make this remark: That this type of reform has succeeded in appropriating for itself exclusively the term " modernization " is a veritable triumph of public relations. There would be a good case for arguing that the movement, far from being " modern ", is in fact exceedingly reactionary and backward-looking. It is intent on preserving in the teaching of mathematics the dry scholastic approach, the munching of non-understood, but sanctified words, the disregard of motivation and intrinsic interest, that are typical of a bygone age—although, to be sure, with a different content. In one perspective at least, that " modernization " consists in teaching *in the same way*, but something that is slightly different; true modernization would be to teach in a completely different way and with renewed aims—be it the traditional or a renovated content. This as an aside.)

In fact, I have been talking about *nothing but* the " modernization ". What I have been saying, in effect, is that there is no case for argument, so far—and for that reason, my mind is open.

I can only repeat what I said in " Bildung und Mathematik ": Those who believe that " modernization " is desirable should by all means try to establish it. That is, they should try to build a case.

Such a case cannot consist in mere exercises in science fiction, alleging without any shred of evidence that any child could learn anything at any age provided it is adequately taught. Nor can such a case consist in the listing of isolated desiderata —" it would be nice if group theory were taught in grade 10." Nor can such a case consist in isolated items of evidence reported out of any context—" a group of high school students in high school X allegedly learned something that Professor Y alleges resembles group theory—Professor Y has since left." Nor can such a case consist in " modern syllabi ", or " synopses " which are proclaimed and released to the world like Papal encyclicals, with a similar implicit assertion of self-justification and infallibility; synopses that state detailed and ambitious curricular proposals without any attempt at establishing their feasibility, their intrinsic interest, or their relationship to the student's over-all education.

A case for " modernization " could only consist in a mature, coherent, integrated view of mathematics teaching as a whole —relating it to the contemporary state of mathematics and science on the one hand, to clearly-conceived aims of education on the other, and buttressing the case by a clear description of the standards by which the practicality of the case will be assessed. One main purpose of my book was to exhibit what such a case may look like. You may well disagree with the whole case I am making, and with every single one of my arguments. Still, I hope that I did show at least the necessary nature and range of such a case. If there is to be a case at all, we must be told, not only what is to be done, but also, in detail, why, how, for whom, by whom, and with what expected and verifiable results. And the description of what is to be done must itself include, not just a listing of isolated curricular items, but a broad and clearly patterned picture of the whole range of the mathematical education of the child, in its integration into the whole of that child's education.

When such a case will be before us, then time will have come for argument. So far, to my knowledge, nobody has found the time to elaborate such a case for " modernization ". The overriding slogan has been *Act Now—Think Later*—write reports that state with monotonous regularity in their foreword that there was no time to consider the major problems and issues (although it would be desirable if someone, somewhere would consider them); write textbooks that are sometimes so much rushed off the press that nobody even finds time to proofread them adequately; write curricula that have to meet deadlines... I cannot help wondering at times whether we should really grant in every case that the authors *could* deal with all those problems adequately if only they took more time. Sometimes at least, I am afraid, lack of time has been both a ready excuse and an alibi. However that may be—I propose a little moratorium on action. Time has come for *thought.*

Let me add one or two specific comments on the proposals for " modernization ":

Even in terms of their own stated aims and frames of reference, these proposals, as I see them, fall into two broad categories, categories that are really quite distinct and should be kept carefully separate. We might call them " pseudo-modernization " and " genuine modernization ". Pseudo-modernization is modernization in the most external trappings of mathematics only—terminology, some uncalled-for concepts that perform no useful function within actual teaching, some isolated and disconnected semblances of " rigorous " proofs of theorems like this one: A line segment has only one middle point. Probably the most popular example of this pseudo-modernization is the introduction, for its own sake, of the " language of sets " from kindergarten onwards. This kind of approach leads very easily into what I have called " pseudo-sophistication ". Its intrinsic significance is negligible, its practical significance is enormous because it provides a means for acquiring the semblance of modernization without the substance, effectively destroying mathematical education in the process.

The second category comprises those proposals that aim at carrying into the schools some genuine mathematical theories or approaches, of the type that we are wont to call " modern ". These may be proposals to do some genuine axiomatic geometry, for instance, at an appropriate level of care and sophistication; or proposals to teach some genuine group theory, comprising not only the definition of a group and some disconnected examples, but a fair amount of substantial theory with applications. Only these proposals are at all debatable, I believe; only for them is a case at all conceivable. I discussed the case for axiomatic geometry explicitly in my book—a fact that seems to have been overlooked by those critics who accused me of disregarding axiomatics altogether.

The problems that arise for this latter class of proposals are circumscribed by my theses 1 to 10. The two primary problems revolve around the questions of *justification* and of *feasibility*—the questions, that is, *why it should* be done, and *whether it can* be done. Under the first heading, one essential consideration is quite simply that of intrinsic interest. In the teaching of mathematics (as in research), there must be a reasonable ratio between effort and reward, between the intrinsic interest or importance of the insights gained, and the amount of work that is necessary in order to gain them. A sound education cannot consist in shooting with cannons at pigeons. Yet, if you look even at the exceedingly ambitious syllabus in group theory contained in the OECD-synopses, you will find, I believe, that the syllabus for the first two years hardly contains one result of genuine and obvious intrinsic interest.[1]) As to the question of feasibility, it resolves itself into two questions: Are any given proposals at all feasible, under realistic criteria of success ? But also: if they are intrinsically feasible, what is the price that must be paid—in particular, the price in terms of teaching time, and in terms of segregation of students by ability and vocational interests ? Is this a price that we are willing to pay ?

IV. Two concluding remarks

A large share of the responsibility for the soundness of the reform-movement in mathematical education rests upon the mathematical community; this includes both the concern for the intrinsic mathematical quality of any proposals that are being advanced, and the willingness to pursue a constructive dialogue with those outside the mathematical community who share a legitimate interest in the shaping of mathematical education.

In order that this heavy responsibility be adequately discharged, I would like to suggest the following.

Thesis 19.

The International Mathematical Union should consider preparing, after wide debate, a statement of guidelines and basic principles for the process of reforming mathematical education. The statement should not deal with the details of any possible reforms, but with such matters as: procedures for the elaboration of proposals, standards for publication, standards for evaluation,

[1]) A notable example which does not leave room for this criticism was presented at the colloquium by Professor H. G. Steiner, Muenster-Westfalen.

ways and means for ensuring the widest international exchange of ideas and experiences, both within the mathematical community and with those outside.

The most crucial single factor for sound teaching of mathematics is and remains the *teacher.* The key alternative before us, transcending by far in importance the alternatives between various programs or approaches, it this one: Whether the teaching community at large consists of highly and broadly educated, very competent men and women, with independent, mature judgment, a broad and informed awareness of the issues involved in their teaching, and a keen desire to learn and to experiment; or whether it consists of narrowly and superficially educated, intellectually timid, men and women of limited intelligence, with little grasp on the issues underlying their teaching, little ability and zest for the exercise of independent judgment, little autonomy in the face of the claims of authority—and little genuine intellectual fire to communicate to their students.

Nothing could be more surely destructive of mathematical education than the disappearance of first-rate teachers.

The most disquieting trend to-day is the tendency to accept that first-rate minds will no longer become teachers. It is significant, for instance, to read that in England the recruitment of first-class mathematics graduates into the schools is practically ceasing, and to read at the same time that in that country the most creative young people tend to go into Arts, not into Science.

A major concern of everyone concerned with mathematical education should be this one, therefore: How to ensure a continuing supply of truly distinguished teachers to the schools. Partly, this is a matter of external factors, which may not be disregarded: salary, teaching loads, status, independence. But partly, it depends on the nature of the teaching process itself, and on that of the process of reform. The more formal and predetermined the teaching, the more narrowly vocational and technical, the less open towards wider vistas of the mind and the more confined to the introductory chapters of university mathematics—the smaller the likelihood that a truly intellectually distinguished person will choose that teaching as a lifelong commitment. Similarly, the more prestructured any reforms, the more narrowly directive and the more obviously predicated on the teacher's ignorance and lack of ability—the greater the likelihood that a potential teacher worth his salt will not submit to the requirement of carrying out these reforms on such terms, even if this means leaving the teaching profession altogether.

It is fitting to devote my last thesis to the teacher:

Thesis 20.

No single issue is more important, in the teaching of mathematics as in education generally, than that teachers should be men and women of genuine intellectual distinction. An over-riding concern, in the shaping of the teaching of mathematics, must therefore be whether any proposals made will tend to attract or to repel, to challenge or to stifle, to stimulate or to discourage, to reward or to punish truly distinguished and creative teachers. Every one of the parameters which influence the recruitment of such teachers —including selection, training, salary scales, teaching loads, class sizes, independence...—must be a legitimate concern of those who care for excellence in the teaching of mathematics.

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