III. And what about " modernization " ?

while dissenting approaches, far from being fostered, are being ignored and starved. It is even more shocking to find organizations making a loyalty oath to H. M. Nicholas Bourbaki in effect a prerequisite for support. Bribery is no way to foster the free growth of ideas, in education no more than anywhere else.

Thesis 18.

Every effort should be made actively to maximize genuine creative diversity in the field of mathematical education, and to maximize at the same time the intellectual confrontation between the various approaches.

The way to accomplish this is indicated by the theses 16 and 17. Ideally, the chairs envisioned in thesis 16 should be held by men or women who each represent one distinguished and distinctive approach to the teaching of mathematics. One could well envision, for instance, one such chair being held in Paris by one of the intellectual children of Professor Nicholas Bourbaki, as a center of thought and experimentation along lines that have become familiar in theory, if not in practice, to everyone concerned with the teaching of mathematics.

The confrontation will arise in three ways. First, if those chairs live up to the requirements of thesis 16, then confrontation of varying approaches, in seminars, study of the literature, guest lectures, etc. will be one of their primary concerns. It would be as unthinkable for a distinguished professor of mathematical education to confine himself, and the horizon of his students, to only the approach that he himself favours, as it would be for a professor of physics or biology. Second, the journal envisioned in thesis 17 would of course operate as a constant, challenging medium of confrontation and discussion. Finally, once enough insight has matured to warrant it, time has come for international congresses.

III. And what about " modernization " ?

I do not seem to have spoken about our topic at all—the so-called " modernization " of secondary school mathematics,

that is, the introduction into secondary school teaching of certain specific types of terminology and content.

(In passing I would like to make this remark: That this type of reform has succeeded in appropriating for itself exclusively the term " modernization " is a veritable triumph of public relations. There would be a good case for arguing that the movement, far from being " modern ", is in fact exceedingly reactionary and backward-looking. It is intent on preserving in the teaching of mathematics the dry scholastic approach, the munching of non-understood, but sanctified words, the disregard of motivation and intrinsic interest, that are typical of a bygone age—although, to be sure, with a different content. In one perspective at least, that " modernization " consists in teaching *in the same way*, but something that is slightly different; true modernization would be to teach in a completely different way and with renewed aims—be it the traditional or a renovated content. This as an aside.)

In fact, I have been talking about *nothing but* the " modernization ". What I have been saying, in effect, is that there is no case for argument, so far—and for that reason, my mind is open.

I can only repeat what I said in " Bildung und Mathematik ": Those who believe that " modernization " is desirable should by all means try to establish it. That is, they should try to build a case.

Such a case cannot consist in mere exercises in science fiction, alleging without any shred of evidence that any child could learn anything at any age provided it is adequately taught. Nor can such a case consist in the listing of isolated desiderata —" it would be nice if group theory were taught in grade 10." Nor can such a case consist in isolated items of evidence reported out of any context—" a group of high school students in high school X allegedly learned something that Professor Y alleges resembles group theory—Professor Y has since left." Nor can such a case consist in " modern syllabi ", or " synopses " which are proclaimed and released to the world like Papal encyclicals, with a similar implicit assertion of self-justification and infallibility; synopses that state detailed and ambitious curricular proposals without any attempt at establishing their feasibility,

their intrinsic interest, or their relationship to the student's over-all education.

A case for " modernization " could only consist in a mature, coherent, integrated view of mathematics teaching as a whole —relating it to the contemporary state of mathematics and science on the one hand, to clearly-conceived aims of education on the other, and buttressing the case by a clear description of the standards by which the practicality of the case will be assessed. One main purpose of my book was to exhibit what such a case may look like. You may well disagree with the whole case I am making, and with every single one of my arguments. Still, I hope that I did show at least the necessary nature and range of such a case. If there is to be a case at all, we must be told, not only what is to be done, but also, in detail, why, how, for whom, by whom, and with what expected and verifiable results. And the description of what is to be done must itself include, not just a listing of isolated curricular items, but a broad and clearly patterned picture of the whole range of the mathematical education of the child, in its integration into the whole of that child's education.

When such a case will be before us, then time will have come for argument. So far, to my knowledge, nobody has found the time to elaborate such a case for " modernization ". The overriding slogan has been *Act Now—Think Later*—write reports that state with monotonous regularity in their foreword that there was no time to consider the major problems and issues (although it would be desirable if someone, somewhere would consider them); write textbooks that are sometimes so much rushed off the press that nobody even finds time to proofread them adequately; write curricula that have to meet deadlines... I cannot help wondering at times whether we should really grant in every case that the authors *could* deal with all those problems adequately if only they took more time. Sometimes at least, I am afraid, lack of time has been both a ready excuse and an alibi. However that may be—I propose a little moratorium on action. Time has come for *thought.*

Let me add one or two specific comments on the proposals for " modernization ":

Even in terms of their own stated aims and frames of reference, these proposals, as I see them, fall into two broad categories, categories that are really quite distinct and should be kept carefully separate. We might call them " pseudo-modernization " and " genuine modernization ". Pseudo-modernization is modernization in the most external trappings of mathematics only—terminology, some uncalled-for concepts that perform no useful function within actual teaching, some isolated and disconnected semblances of " rigorous " proofs of theorems like this one: A line segment has only one middle point. Probably the most popular example of this pseudo-modernization is the introduction, for its own sake, of the " language of sets " from kindergarten onwards. This kind of approach leads very easily into what I have called " pseudo-sophistication ". Its intrinsic significance is negligible, its practical significance is enormous because it provides a means for acquiring the semblance of modernization without the substance, effectively destroying mathematical education in the process.

The second category comprises those proposals that aim at carrying into the schools some genuine mathematical theories or approaches, of the type that we are wont to call " modern ". These may be proposals to do some genuine axiomatic geometry, for instance, at an appropriate level of care and sophistication; or proposals to teach some genuine group theory, comprising not only the definition of a group and some disconnected examples, but a fair amount of substantial theory with applications. Only these proposals are at all debatable, I believe; only for them is a case at all conceivable. I discussed the case for axiomatic geometry explicitly in my book—a fact that seems to have been overlooked by those critics who accused me of disregarding axiomatics altogether.

The problems that arise for this latter class of proposals are circumscribed by my theses 1 to 10. The two primary problems revolve around the questions of *justification* and of *feasibility*—the questions, that is, *why it should* be done, and *whether it can* be done. Under the first heading, one essential consideration is quite simply that of intrinsic interest. In the teaching of mathematics (as in research), there must be a reason-

able ratio between effort and reward, between the intrinsic interest or importance of the insights gained, and the amount of work that is necessary in order to gain them. A sound education cannot consist in shooting with cannons at pigeons. Yet, if you look even at the exceedingly ambitious syllabus in group theory contained in the OECD-synopses, you will find, I believe, that the syllabus for the first two years hardly contains one result of genuine and obvious intrinsic interest.[1]) As to the question of feasibility, it resolves itself into two questions: Are any given proposals at all feasible, under realistic criteria of success ? But also: if they are intrinsically feasible, what is the price that must be paid—in particular, the price in terms of teaching time, and in terms of segregation of students by ability and vocational interests ? Is this a price that we are willing to pay ?

IV. Two concluding remarks

A large share of the responsibility for the soundness of the reform-movement in mathematical education rests upon the mathematical community; this includes both the concern for the intrinsic mathematical quality of any proposals that are being advanced, and the willingness to pursue a constructive dialogue with those outside the mathematical community who share a legitimate interest in the shaping of mathematical education.

In order that this heavy responsibility be adequately discharged, I would like to suggest the following.

Thesis 19.

The International Mathematical Union should consider preparing, after wide debate, a statement of guidelines and basic principles for the process of reforming mathematical education. The statement should not deal with the details of any possible reforms, but with such matters as: procedures for the elaboration of proposals, standards for publication, standards for evaluation,

1) A notable example which does not leave room for this criticism was presented at the colloquium by Professor H. G. Steiner, Muenster-Westfalen.