1. Introduction

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BOUNDEDNESS THEOREMS FOR SOLUTIONS OF $u''(t) + a(t)f(u)g(u') = 0$ (IV)

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1. Introduction

In our previous work [1-3], we have presented rather fragmentary results concerning the boundedness of solutions to certain second order non-linear differential equations of the following form:

$$u''(t) + a(t)f(u)g(u') = 0 \tag{1.1}$$

where $a(t), f(u)$ and $g(u')$ satisfy certain assumptions to be described below. The purpose of the present paper is to further extend these results and establish comparison theorems. Some of our results presented here may be considered as generalizations to the results of Zhang [4], where a special case of equation (1.1):

$$u''(t) + a(t)f(u) = 0 \tag{1.2}$$

was treated [1]).

Throughout the discussion of this paper, we will need the following assumptions:

(A_1) $g(u')$ is a positive continuous function of u',

(A_2) $f(u)$ is a continuous function of u satisfying $uf(u) > 0$, if $u \neq 0$,

(A_3) $a(t)$ is continuous in t,

$$(A_4) \quad \lim_{|u| \to \infty} \int_0^u f(s)\, ds = \infty,$$

$$(A_5) \quad \lim_{|v| \to \infty} \int_0^v \frac{g(s)}{s\, ds} = \infty.$$

We also list in the following a brief résumé of our previous results on boundedness.

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[1]) Eor other boundedness result concerning (1.2), see [5]-[7], [13], [14].

Theorem (I). Suppose that assumptions A_1, A_2, A_3, and A_4 hold and in addition that $a(t) > 0$ and $a'(t) \geqq 0$ for $t \geqq T$. Then all solutions of (1.1) are bounded.

Corollary. In addition to the hypothesis of Theorem (I), suppose that assumption A_5 also holds and that $\lim_{t\to\infty} a(t) = k > 0$; then all solutions of (1.1) and their derivatives are bounded.

Theorem (II). Suppose that assumptions A_1, A_2, A_3 and A_4 hold and in addition that $a'(t) \leqq 0$ for $t \geqq T$. Then all solutions of (1.1) are bounded.

Corollary. In addition to the hypothesis of Theorem (II), suppose that assumption A_5 also holds and $\lim_{t\to\infty} a(t) = k > 0$; then all solutions of (1.1) and their derivatives are bounded.

Theorem (III). Suppose that assumptions A_1, A_2, A_3, and A_4 hold and in addition that $a(t) \geqq a_0 > 0$ for $t \geqq T$, and $\int^{\infty} | a'(t) | \, dt < \infty$. Then all solutions of (1.1) are bounded.

Corollary. In addition to the hypothesis of Theorem (III), suppose that assumption A_5 also holds; then all solutions of (1.1) and their derivatives are bounded.

The method of proof for the above results is based essentially on the well-known lemma of Gronwall [10], which is also known as the Bellman's lemma. In this paper, we use in addition to this fundamental lemma, its generalizations [11], [12], and techniques borrowed from Lyapunov's stability theory.

It might be of interest to note that quite a few results in [4] are incorrect; in particular Theorems 5 and 6. Also, Theorems 3 and 4 are stated incorrectly.

2. Boundedness Theorems I

Theorem 1. Suppose that assumptions A_1, A_2, A_3 and A_4 hold and that $a(t) > 0$ for $t \geqq T$ and there exists a non-negative function $\alpha(t)$ such that $-a'(t) \leqq \alpha(t) \, a(t)$ with $\int^{\infty} \alpha(s) \, ds < \infty$. Then all solutions of (1.1) are bounded.