Theorem 6. Suppose that assumptions A_1, A_2, A_3 and A_4 hold and in addition that $a(t) > 0$ and $a'(t) \geqq 0$ for $t \geqq T$; then all solutions of (3.3) are bounded.

Proof. Integrate (3.3) in the following manner:

$$G\big(u'(t)\big) - G\big(u'(t_0)\big) + a(t)F\big(u(t)\big) - a(t_0)F\big(u(t_0)\big)$$
$$= \int_{t_0}^{t} a'(s)F\big(u(s)\big)ds + \int_{t_0}^{t} \frac{h(t,u,u')\,u'(s)\,ds}{g(u')} \tag{3.4}$$

where $G(v) = \int_0^{\gamma} \frac{s\,ds}{g(s)}$ and $F(u) = \int_0^{u} f(s)\,ds$. Taking absolute values and noting that $G(v) \geqq 0$ and $F(u) \geqq 0$, we obtain

$$a(t)F\big(u(t)\big) \leqq c_0 + c_1 + \int_{t_0}^{t} a'(s)F\big(u(s)\big)ds \tag{3.5}$$

where $c_0 = G(u'(t_0)) + a(t_0)F(u(t_0\,\mathrm{O})$ and $c_1 = \int_{t_0}^{\infty} \gamma(s)\,ds$ are non-negative constants. From (3.5) and A_4 it is now clear that every solution of (3.3) are bounded (cf. [1]).

Corollary. In addition to the hypothesis of Theorem 6, suppose that assumption A_5 also holds and that $\lim_{t\to\infty} a(t) = k > 0$; then all solutions of (3.3) and their derivatives are bounded.

We note that by setting $h(t, u, u') \equiv 0$, the above result again reduces to Theorem 1 and its corollary. Other comparison theorems may be formulated in a similar way as Theorem 6 by extending the corresponding result for the homogeneous equation. Since the procedure is clear, the statements and proofs of these results will be omitted.

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