By hypothesis (1) $\sum | a_{\mu\mu} \alpha_\mu | < \infty$. Hence the second term of (3) is absolutely convergent if $\sum_{n=\mu}^{\infty} | A_{n\mu} | \leqq M$. Since $b_{n\mu}\uparrow$ for $\mu\uparrow$ we have

which is $0(1)$.

THEOREM 5. Let A, B be normal and absolutely regular with $A \dot{>} 0$, $B \dot{>} 0$, and $A' \dot{\leqq} 0$. Furthermore, assume

(4) $$a_{n\nu}\uparrow\,,\ \nu\uparrow (\nu \leqq n)$$

(5) $$b_{n\nu}\uparrow\,,\ \nu\uparrow (\nu \leqq n)$$

(6) $$\frac{a_{n\nu}}{b_{n\nu}}\downarrow,\ \nu\uparrow (\nu \leqq n)$$

(7) $$\frac{b_{n\nu}}{a_{n\nu}}(a_{kn} - a_{k\nu})\downarrow\,,\ \nu\uparrow \textit{ for all } n \leqq k$$

(8) $$a_{\nu+1,\nu+1} = 0(a_{\nu\nu})$$

(9) $$b_{\nu\nu} = 0(b_{\nu+1,\nu+1})$$

(10) $$p_{\nu+1} = 0(p_\nu)\,.$$

When these conditions are satisfied

$$\varepsilon_\nu(AP^k, c) = 0\left(\frac{a_{\nu\nu}}{b_{\nu\nu}}\left(\frac{p_\nu}{P_\nu}\right)^k\right)$$

implies

$$\varepsilon_\nu(AP^k, c) \in (| AP^k |, | B |)_r\,.$$

Proof. With $k = 0$ these condition imply $\varepsilon_\nu(A,c) \in (|A|, |B|)_r$ (see Theorem 3 in [2]. Hence, the theorem follows by induction from Theorem 3 and 4.

Theorem 5 extends Theorem 3 in [2] to include all Cesaro methods $A = (C, \alpha)$, $B = (C, \beta)$ where $\alpha \geqq 0$, and $0 \leqslant \beta \leqslant 1$.

BIBLIOGRAPHY

[1] IRWIN, R. L., Absolute Summability Factors I, *Tohoku Math. Journal, 1.*, 247-254 (1966).

[2] —— and A. PEYERIMHOFF, On Absolute Summability Factors, to appear in *L'Enseignement Mathématique.*

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