

# Existence of admissible refinements of measure coverings

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$= \{ \hat{U}_i(\rho) \}_1^{i^*}$  is a Stein covering of  $X(\rho)$ . We say then that  $\hat{\mathfrak{U}}(\rho)$  is a measure covering of  $X(\rho)$ .

*Admissible refinements of measure coverings.* Let  $\hat{\mathfrak{U}}(\rho)$  and  $\hat{\mathfrak{U}}^*(\rho)$  be two measure coverings of  $X(\rho)$ . We say that  $\hat{\mathfrak{U}}^*(\rho)$  is an admissible refinement of  $\hat{\mathfrak{U}}(\rho)$  if the following conditions hold:

- 1)  $U_i^* \subset \subset U_i$  for each  $i$ .
- 2) If  $U_{i_0 \dots i_\lambda}^* = U_{i_0}^* \cap \dots \cap U_{i_\lambda}^*$  we put  $(U_{i_0 \dots i_\lambda}^*)_v = \Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho))$  for each  $v \in \{i_0 \dots i_\lambda\}$ . It is now required that  $(U_{i_0 \dots i_\lambda}^*)_v \subset (U_{i_0 \dots i_\lambda}^*)_\mu$  for all  $v, \mu \in \{i_0 \dots i_\lambda\}$ .
- 3)  $\hat{U}_{i_0 \dots i_\lambda}^* = \hat{U}_{i_0}^* \cap \dots \cap \hat{U}_{i_\lambda}^* \subset (U_{i_0 \dots i_\lambda}^*)_\mu$  for each  $\mu \in \{i_0 \dots i_\lambda\}$ .

#### EXISTENCE OF ADMISSIBLE REFINEMENTS OF MEASURE COVERINGS

*Existence Theorem.* For every fixed integer  $s$  we can find, for some  $\rho > 0$ , a sequence  $\mathfrak{U}_s \ll \mathfrak{U}_{s-1} \ll \dots \ll \mathfrak{U}_1 \ll \mathfrak{U}_0$  of finer measure coverings of  $X(\rho)$  each of which is an admissible refinement of the following.

*Proof.* We first construct a measure covering of  $X(\rho)$  for some  $\rho < \min \rho_i$ . Let  $\mathfrak{U}_0 = \{ \mathfrak{U}_i \}_1^{i^*}$  be a Stein covering of  $X_0$  such that  $U_i \subset \subset W_i$  for  $i \in \{1, \dots, i^*\}$ . Choose a fixed  $\rho_0 < \min \rho_i$ . Now the open sets  $\Phi_i^{-1}(U_i \times E^n(\rho_0))$  cover  $X_0$  and hence they also cover  $X(\rho)$  for some sufficiently small  $\rho$ . Hence  $\mathfrak{U}_0$  defines a measure covering of  $X(\rho)$ . It is also clear that  $\mathfrak{U}_0$  defines a measure covering of  $X(\rho')$  for each  $\rho' \leq \rho$ . Let us now construct  $\mathfrak{U}_1$ . We let  $\mathfrak{U}^* = \{ U_i^* \}_1^{i^*}$  be a Stein covering such that  $U_i^* \subset \subset U_i$  always holds. Now we can find  $\rho_1 \leq \rho$  such that  $\{ \hat{U}_i^*(\rho_1) = \Phi_i^{-1}(U_i^* \times E^n(\rho_1)) \}_1^{i^*}$  cover  $X(\rho_1)$ . Hence  $\hat{\mathfrak{U}}^*(\rho_1)$  and  $\hat{\mathfrak{U}}(\rho_1)$  are measure coverings of  $X(\rho_1)$ . But we do not yet know if  $\hat{\mathfrak{U}}^*(\rho_1) \ll \hat{\mathfrak{U}}(\rho_1)$ . We claim that if  $\rho_2 \leq \rho_1$  is sufficiently small then  $\hat{\mathfrak{U}}^*(\rho_2) \ll \hat{\mathfrak{U}}(\rho_2)$ . For suppose this is false. Say that 2) fails for  $\hat{\mathfrak{U}}^*(\rho_2)$  and  $\hat{\mathfrak{U}}(\rho_2)$  when  $0 < \rho_2 \leq \rho_1$ . Hence  $\Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho_2)) - \Phi_\mu^{-1}(U_{i_0 \dots i_\lambda} \times E^n(\rho_2))$  are non empty for suitable indices while  $\rho_2 \rightarrow 0$ . Choose a point  $x_t$  from each of these sets. Because  $x_t \in X(\rho_1)$  which is relatively compact we may assume that  $x_t \rightarrow x_0$ . Obviously we get  $x_0 \in \overline{U_{i_0 \dots i_\lambda}^*} - U_{i_0 \dots i_\lambda}$ , a contradic-

tion because  $\overline{U_{\iota_0 \dots \iota_\lambda}^*} \subset \overline{U_{\iota_0}^*} \cap \dots \cap \overline{U_{\iota_\lambda}^*} \subset U_{\iota_0 \dots \iota_\lambda}$ . In the same way we can prove that condition 3) is satisfied if  $\rho_2$  is sufficiently small and the theorem is clear.

### GENERAL THEORY

Let  $G$  be an analytic manifold. We put  $\hat{G} = G \times E^n(\rho_1)$  where  $\rho_1$  is an  $n$ -tuple of positive numbers. Let  $\pi: \hat{G} \rightarrow E^n(\rho_1)$  and  $\mathfrak{P}: \hat{G} \rightarrow G$  be the projection maps.  $\hat{G}^* \subset \hat{G}$  denotes an open subset and  $G^* = \hat{G}^* \cap G \times \{0\}$ .

The set  $G^*$  can be identified with an open subset of  $G$ . We denote by  $\alpha: G^* \times E^n(\rho_1) \rightarrow \hat{G}^*$  a biholomorphic fiber preserving map, i.e.  $\pi \circ \alpha = \pi^*$  where  $\pi^*: G^* \times E^n(\rho_1) \rightarrow E^n(\rho_1)$  is the natural projection. Let  $\rho \leq \rho_2 = \gamma \rho_1 < \rho_1$  where  $0 < \gamma < 1$  is a fixed number. We put  $\hat{G}(\rho) = G \times E^n(\rho)$ . If  $f$  is a holomorphic function on  $\hat{G}(\rho)$  we write  $f = \sum a_\nu (t/\rho)^\nu$  with  $a_\nu \in I(G)$ . We define the norm  $\|f\|_\rho$  of  $f$  by  $\|f\|_\rho = \sup_\nu \{ \sup |a_\nu(G)| \}$ .

If  $f \in I(\hat{G}(\rho))$  we see that  $f \circ \alpha$  is a well defined function on  $G^* \times E^n(\rho)$  because  $\alpha$  is fiber preserving. We define  $\|f \circ \alpha\|_\rho$  using  $G^*$  instead of  $G$  as above. We have the proposition:

*Proposition 1.* There exists a constant  $K$  such that  $\|f \circ \alpha\|_\rho \leq K \|f\|_\rho$  where  $K = K(\rho_2)$  is independent of  $\rho \leq \rho_2$ .

*Proof.* We write  $f = \sum_{|\nu|=0}^{\infty} a_\nu (t/\rho)^\nu$  with  $a_\nu \in I(G)$ . Now we get  $f \circ \alpha = \sum (a_\nu \circ \mathfrak{P} \circ \alpha) (t/\rho)^\nu$  because  $\alpha$  is fiber preserving. Since  $\mathfrak{P}(\hat{G}^*) \subset G$  we get  $|a_\nu \circ \mathfrak{P}(\hat{G}^*)| \leq |a_\nu(G)| \leq \|f\|_\rho$ . Now  $a_\nu \circ \mathfrak{P} \circ \alpha$  admits a Taylor series:  $a_\nu \circ \mathfrak{P} \circ \alpha = \sum C_{\nu\lambda} (t/\rho)^\lambda$  with  $C_{\nu\lambda} \in I(G^*)$ . Since  $|\sum C_{\nu\lambda} (t/\rho)^\lambda| \leq \|f\|_\rho$  in  $G^* \times E^n(\rho_1)$  and  $\rho \leq \rho_2 = \gamma \rho_1$  Cauchy's inequalities give us  $|C_{\nu\lambda}(G^*)| \leq \|f\|_\rho \gamma^{|\lambda|}$ . Let us put  $b_\mu = \sum_{\nu+\lambda=\mu} C_{\nu\lambda}$ . We get  $|b_\mu(G^*)| \leq \|f\|_\rho \sum \gamma^{|\lambda|} = \|f\|_\rho (1-\gamma)^{-n} = K \|f\|_\rho$ . Now we can write  $f \circ \alpha = \sum_\nu a_\nu \circ \mathfrak{P} \circ \alpha (t/\rho)^\nu = \sum_{\lambda,\nu} C_{\nu\lambda} (t/\rho)^\lambda (t/\rho)^\nu = \sum_\mu b_\mu (t/\rho)^\mu$ . By definition we have  $\|f \circ \alpha\|_\rho = \sup_\mu |b_\mu(G^*)| \leq K \|f\|_\rho$ .

Let us now consider  $\mathbf{h} = (h_{\nu\mu})$  which is a  $q \times q$  matrix with  $h_{\nu\mu} \in I(\hat{G})$ . The  $h_{\nu\mu}$  are also assumed to be bounded on  $\hat{G}$ .