General Theory

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **14 (1968)**

Heft 1: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **10.08.2024**

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.
Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.
Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Ein Dienst der *ETH-Bibliothek*
ETH Zürich, Rämistrasse 101, 8092 Zürich, Schweiz, www.library.ethz.ch

http://www.e-periodica.ch

tion because $\overline{U^*_{\iota_0\ldots\iota_\lambda}} \subset \overline{U^*_{\iota_0}} \cap \ldots \cap \overline{U^*_{\iota_\lambda}} \subset U_{\iota_0\ldots\iota_\lambda}$. In the same way we can prove that condition 3) is satisfied if ρ_2 is sufficiently small and the theorem is clear.

General Theory

Let G be an analytic manifold. We put $\hat{G} = G \times E^n(\rho_1)$ where ρ_1 is an n-tuple of positive numbers. Let $\pi: \hat{G} \to E^n(\rho_1)$ and $\mathfrak{P}: \hat{G} \to G$ be the projection maps. $\hat{G}^* \subset \hat{G}$ denotes an open subset and $G^* = \hat{G}^* \cap G \times \{0\}$.

The set G^* can be identified with an open subset of G. We denote by $\alpha: G^* \times E^n(\rho_1) \to \hat{G}^*$ a biholomorphic fiber preserving map, i.e. $\pi \circ \alpha = \pi^*$ where $\pi^*: G^* \times E^n(\rho_1) \to E^n(\rho_1)$ is the natural projection. Let $\rho \leqslant \rho_2 = \gamma\rho_1 < \rho_1$ where $0 < \gamma < 1$ is a fixed number. We put $\hat{G}(\rho) = G \times E^n(\rho)$. If f is a holomorphic function on $\hat{G}(\rho)$ we write $f = \sum a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. We define the norm $\|f\|_\rho$ of f by $\|f\|_\rho = \sup_\nu \{\sup |a_\nu(G)|\}$. If $f \in I(\hat{G}(\rho))$ we see that $f \circ \alpha$ is a well defined function on $G^* \times E^n(\rho)$ because α is fiber preserving. We define $\|f \circ \alpha\|_\rho$ using G^* instead of G as above. We have the proposition:

Proposition 1. There exists a constant K such that $\|f \circ \alpha\|_\rho \leqslant K\|f\|_\rho$ where $K = K(\rho_2)$ is independent of $\rho \leqslant \rho_2$.

Proof. We write $f = \sum_{|\nu|=0}^{\infty} a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. Now we get $f \circ \alpha = \sum (a_\nu \circ \mathfrak{P} \circ \alpha)(t/\rho)^\nu$ because α is fiber preserving. Since $\mathfrak{P}(\hat{G}^*) \subset G$ we get $|a_\nu \circ \mathfrak{P}(\hat{G}^*)| \leqslant |a_\nu(G)| \leqslant \|f\|_\rho$. Now $a_\nu \circ \mathfrak{P} \circ \alpha$ admits a Taylor series: $a_\nu \circ \mathfrak{P} \circ \alpha = \sum C_{\nu\lambda}(t/\rho)^\lambda$ with $C_{\nu\lambda} \in I(G^*)$. Since $|\sum C_{\nu\lambda}(t/\rho)^\lambda| \leqslant \|f\|_\rho$ in $G^* \times E^n(\rho_1)$ and $\rho \leqslant \rho_2 = \gamma\rho_1$ Cauchy's inequalities give us $|C_{\nu\lambda}(G^*)| \leqslant \|f\|_\rho \gamma^{|\lambda|}$. Let us put $b_\mu = \sum_{\nu+\lambda=\mu} C_{\nu\lambda}$. We get $|b_\mu(G^*)| \leqslant \|f\|_\rho \sum \gamma^{|\lambda|} = \|f\|_\rho (1-\gamma)^{-n} = K\|f\|_\rho$. Now we can write $f \circ \alpha = \sum_\nu a_\nu \circ \mathfrak{P} \circ \alpha (t/\rho)^\nu = \sum_{\lambda,\nu} C_{\nu\lambda}(t/\rho)^\lambda (t/\rho)^\nu = \sum_\mu b_\mu (t/\rho)^\mu$. By definition we have $\|f \circ \alpha\|_\rho = \sup_\mu |b_\mu(G^*)| \leqslant K\|f\|_\rho$.

Let us now consider $\mathbf{h} = (h_{\nu\mu})$ which is a $q \times q$ matrix with $h_{\nu\mu} \in I(\hat{G})$. The $h_{\nu\mu}$ are also assumed to be bounded on $\hat{G}$.

Proposition 2. Let $\mathbf{f} = (f_1 \dots f_q) \in qI(\hat{G}(\rho))$. Then $\| \mathbf{h}(\mathbf{f}) \|_\rho \leqslant K \| \mathbf{f} \|_\rho$. As before $\rho \leqslant \rho_2 = \gamma \rho_1 < \rho_1$ and K only depends on ρ_2.

Proof. We have $\mathbf{h}(\mathbf{f}) = (g_1 \dots g_q)$ with $g_\nu = \sum\limits_\mu h_{\nu\mu} f_\mu$. Let us write $h_{\nu\mu} = \sum\limits_\lambda a_{\nu\mu\lambda} (t/\rho)^\lambda$. By assumption $| h_{\nu\mu}(\hat{G}) | \leqslant M$ for some constant M and hence we have, by Cauchy's inequalities, $| a_{\nu\mu\lambda}(G) | \leqslant M\gamma^{|\lambda|}$. Let us also write $f_\mu = \sum\limits_\lambda b_{\mu\lambda} (t/\rho)^\lambda$. By definition $\sup\limits_{\mu,\lambda} | b_{\mu\lambda}(G) | = \| \mathbf{f} \|_\rho$. Now we get $g_\nu = \sum\limits_\mu \sum\limits_{\lambda_1, \lambda_2} a_{\nu\mu\lambda_1} b_{\mu\lambda_2} (t/\rho)^{\lambda_1 + \lambda_2} = \sum C_{\nu\lambda} (t/\rho)^\lambda$ where $C_{\nu\lambda} = \sum\limits_\mu \sum\limits_{\lambda_1 + \lambda_2 = \lambda} a_{\nu\mu\lambda_1} b_{\mu\lambda_2}$. We get easily $| C_{\nu\lambda}(G) | \leqslant qM \| \mathbf{f} \|_\rho (1 - \gamma)^{-n} = K \| \mathbf{f} \|_\rho$. Hence $\| \mathbf{h}(\mathbf{f}) \|_\rho = \sup\limits_\nu \| g_\nu \|_\rho = \sup\limits_{\nu,\lambda} | C_{\nu\lambda}(G) | \leqslant K \| \mathbf{f} \|_\rho$.

We shall now apply these two propositions to our situation. Let $G^* \subset G \subset W_{\iota_0 \dots \iota_\lambda} \subset X_0$. Here G^* and G are open sets and $W_{\iota_0 \dots \iota_\lambda}$ comes from the measure atlas $\mathscr{W}$. As before $\rho \leqslant \rho_2 < \rho_* = \min \rho_\iota$. We are given ι and ι' from $\{ \iota_0, \dots, \iota_\lambda \}$ and the following inclusions are assumed: $(G^*)_{\iota'}(\rho_1) \subset (G)_\iota (\rho_1)$, $(G^*)_{\iota'}(\rho_1) \subset\subset \hat{W}_{\iota'}$, $(G)_\iota (\rho_1) \subset\subset \hat{W}_\iota$.

The following theorem is very important.

Theorem I. Let $S \in \Gamma((G)_\iota(\rho), \mathbf{F})$. Then $\| S | (G^*)_{\iota'}(\rho) \|_{\iota'} \leqslant K \| S \|_\iota$. K depends only on ρ_2.

Proof. We have the following diagram:

$$\begin{array}{ccc} (G)_\iota(\rho_1) & \xrightarrow{\Phi_\iota} & G \times E^n(\rho_1) \\ \text{injection} \uparrow & & \uparrow \alpha \\ (G^*)_{\iota'}(\rho_1) & \xrightarrow{\Phi_{\iota'}} & G^* \times E^n(\rho_1) \end{array}$$

α being a fiber preserving holomorphic map. We identify $S | (G^*)_{\iota'}(\rho)$ with an element of $qI(G^* \times E^n(\rho))$ using the trivialization of F in the chart $\mathscr{W}_{\iota'}$. Call this element S^*. Also S itself is considered as an element of $qI(G \times E^n(\rho))$ using the trivialization in the chart $\mathscr{W}_\iota$. Now we have $S^* = \mathbf{h}(S \circ \alpha)$ where $\mathbf{h}$ is a $q \times q$ matrix. The elements of $\mathbf{h}$ are holomorphic functions defined on $\Phi_{\iota'}(\hat{W}_{\iota\iota'}) \supset\supset G^* \times E^n(\rho_1)$. Hence the elements of $\mathbf{h}$ are bounded on $G^* \times E^n(\rho_1)$. It is now obvious how we can use 1) and 2) to finish the proof.

We shall need one more general result. Let G be an analytic manifold. G is assumed to be Stein and $R^* = \{ U_1, \dots, U_{\iota^*} \}$ a Stein covering of G.

The set $G^* \subset G$ is open and $R^{**} = \{ V_1, ..., V_{\iota^*} \}$ an open covering of G^* such that $V_\iota \subset\subset U_\iota$ for $\iota \in \{ 1, ..., \iota^* \}$. We have:

Cartan's Theorem. There exists a constant K such that if $\xi \in Z^l (R^*, q\mathcal{O})$ then $\xi \mid R^{**} = \delta\eta$ where $\eta \in C^{l-1} (R^{**}, q\mathcal{O})$ and $\| \eta \| \leqslant K \| \xi \|$ for $l \geqslant 1$.

This is a simple consequence of Theorem B and Banach's open mapping theorem.

Now we apply Cartan's theorem. We keep the notations as above. Let $\hat{G} = G \times E^n(\rho)$ and put $\hat{R}^* = \{ U_\iota \times E^n(\rho) \}$. Now $\hat{R}^*$ is a Stein covering of $\hat{G}$. Let $\hat{G}^* = G^* \times E^n(\rho)$ and $\hat{R}^{**} = \{ V_\iota \times E^n(\rho) \}$. Let $\hat{\xi} \in Z^l (\hat{R}^*, q\mathcal{O})$ and write $\hat{\xi} = \sum \xi_{(\nu)} (t/\rho)^\nu$ with $\xi_{(\nu)} \in Z^l (R^*, q\mathcal{O})$. We assume $\| \hat{\xi} \|_\rho = \sup_\nu \| \xi_{(\nu)} \| < \infty$. Now Cartan's theorem gives $\xi_{(\nu)} \mid R^{**} = \delta\eta_\nu$ with $\eta_\nu \in C^{l-1}(R^{**}, q\mathcal{O})$ and $\| \eta_\nu \| \leqslant K \| \xi_{(\nu)} \| < \infty$. It follows that $\hat{\eta} = \sum \eta_\nu (t/\rho)^\nu$ is well defined in $C^{l-1} (\hat{R}^{**}, q\mathcal{O})$ and by definition we have $\| \hat{\eta} \|_\rho \leqslant K \| \hat{\xi} \|_\rho$.

Smoothing

We are given a sequence of admissible refinements of measure coverings in $X(\rho_1)$. Here $\rho_1 < \rho_0 = \min \rho_\iota$ as usual. Let l be a fixed integer $\geqslant 1$. We are given $\mathfrak{B}^* \ll \mathfrak{B}' = \mathfrak{B}_{3l} \ll \mathfrak{B}_{3l-1} \ll ... \ll \mathfrak{B}_1 \ll \mathfrak{B}_0 \ll \mathfrak{B} \ll \mathfrak{U}^* \ll \mathfrak{U} = \mathfrak{U}_{3l} \ll ... \ll \mathfrak{U}_0 \ll \mathfrak{U}'$. Here it is also required that $(\mathfrak{B}_{\nu+1}, \mathfrak{U}_{\nu+1}) \ll (\mathfrak{B}_\nu, \mathfrak{U}_\nu)$; $(\mathfrak{B}^*, \mathfrak{U}^*) \ll (\mathfrak{B}', \mathfrak{U})$ and $(\mathfrak{B}_0, \mathfrak{U}_0) \ll (\mathfrak{B}, \mathfrak{U}')$. These extra conditions mean: 1) $\hat{U}^{(\nu+1)}_{i_0 \dots i_\kappa} \cap \hat{V}^{(\nu+1)}_{\iota_0 \dots \iota_l} \subset (U^{(\nu)}_{i_0 \dots i_\kappa} \cap V^{(\nu)}_{\iota_0 \dots \iota_l})_i$ for each $i \in \{ i_0, ..., i_\kappa \}$ and 2) $(U^{(\nu+1)}_{i_0 \dots i_\kappa} \cap V^{(\nu+1)}_{\iota_0 \dots \iota_l})_j \subset (U^{(\nu)}_{i_0 \dots i_\kappa} \cap V^{(\nu)}_{\iota_0 \dots \iota_l})_i$ for all $i, j \in \{ i_0, \dots i_\kappa, \iota_0, \dots \iota_l \}$. Recall that all operations are done with respect to ρ_1. Let us put $\hat{R}^{(\nu)}_{i_0 \dots i_k \iota_0 \dots \iota_\kappa} = \hat{U}^{(\nu)}_{i_0 \dots i_k} \cap \hat{V}^{(\nu)}_{\iota_0 \dots \iota_\kappa}$. We consider elements $\xi_{i_0 \dots i_k \iota_0 \dots \iota_\kappa} \in \hat{\Gamma}(\hat{R}^{(\nu)}_{i_0 \dots i_k \iota_0 \dots \iota_\kappa}, \mathbf{F})$. Now we take a full collection $\hat{\xi} = \{ \hat{\xi}_{i_0 \dots i_k \iota_0 \dots \iota_\kappa} \}$ of such elements which is anticommutative in $\{ i_0, \dots i_k \}$ and $\{ \iota_0, ..., \iota_\kappa \}$. In this way we get a double complex $C_\nu^{k,\kappa}$. Here $\delta : C_\nu^{k,\kappa} \to C_\nu^{k+1,\kappa}$ and $\partial : C_\nu^{k,\kappa} \to C_\nu^{k,\kappa+1}$ are the usual coboundary operators.

Norm in $C_\nu^{k,\kappa}$: Let $\hat{\xi} \in C_\nu^{k,\kappa}$; we put