Theorem 1.4, for $d = 3$ and special measures, may be found in the book of Blaschke [5] (p. 152, 154-155). The general case is due to A. D. Aleksandrov [1] (§8) (though in the form of a seemingly more special geometric theorem); compare also Petty [16] (p. 1545-1546). The case $d = 3$ (and φ specialized) of Theorem 1.5 is due to Funk [9]; another proof (of special cases in geometric formulation) has been given by Kubota [11]. The common generalization of both uniqueness theorems, which is given above, may be found in [18]. To this paper we refer also for references to the known geometric consequences of Theorems 1.4 and 1.5, as well as for some new applications thereof.

The question leading to Theorem 1.5 can be generalized in the following way: Let $D \subset \Omega_d$ be any domain, and let us say for the moment that D is *non-special* if and only if every measure φ on Ω_d for which $\varphi(D') = 0$ for each domain D' (properly) congruent to D, must vanish identically; otherwise D is called *special.* If D is a spherical cap of radius $\alpha \in (O, \pi)$, it has been shown that D is special if and only if α is contained in a certain set of values which is denumerable and dense in (O, π) (Ungar [21], more general in [18]). Ungar [21] has given an example of a non-circular special domain on Ω_3. Now Theorem 2.1 (if generalized to piecewise continuous functions) allows, at least theoretically, to decide whether a given domain $D \subset \Omega_d$ is non-special: For this to be the case it is necessary and sufficient that

$$\int_D Y_{ni}\, d\omega \neq 0$$

for each $n \in \{0, 1, 2, \ldots\}$ and some $i \in \{1, \ldots, N_{d,n}\}$. Thus the answer depends on the computation of denumerably many definite integrals.

Finally we mention that a special case of the 2-dimensional analogue of Theorem 2.1 was used by Görtler [10] in characterizing those pairs of plane convex domains whose mixed area is invariant under arbitrary (non-simultaneous) motions of the domains.

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