

# 5. THINK ABOUT THE ALPHABET

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you may for instance decide to leave probability out, but put Haar measure in.

There is a sense in which the preparation of an outline can take years, or, at the very least, many weeks. For me there is usually a long time between the first joyful moment when I conceive the idea of writing a book and the first painful moment when I sit down and begin to do so. In the interim, while I continue my daily bread and butter work, I daydream about the new project, and, as ideas occur to me about it, I jot them down on loose slips of paper and put them helter-skelter in a folder. An “idea” in this sense may be a field of mathematics I feel should be included, or it may be an item of notation; it may be a proof, it may be an aptly descriptive word, or it may be a witticism that, I hope, will not fall flat but will enliven, emphasize, and exemplify what I want to say. When the painful moment finally arrives, I have the folder at least; playing solitaire with slips of paper can be a big help in preparing the outline.

In the organization of a piece of writing, the question of what to put in is hardly more important than what to leave out; too much detail can be as discouraging as none. The last dotting of the last *i*, in the manner of the old-fashioned Cours d'Analyse in general and Bourbaki in particular, gives satisfaction to the author who understands it anyway and to the helplessly weak student who never will; for most serious-minded readers it is worse than useless. The heart of mathematics consists of concrete examples and concrete problems. Big general theories are usually afterthoughts based on small but profound insights; the insights themselves come from concrete special cases. The moral is that it's best to organize your work around the central, crucial examples and counterexamples. The observation that a proof proves something a little more general than it was invented for can frequently be left to the reader. Where the reader needs experienced guidance is in the discovery of the things the proof does not prove; what are the appropriate counterexamples and where do we go from here?

## 5. THINK ABOUT THE ALPHABET

Once you have some kind of plan of organization, an outline, which may not be a fine one but is the best you can do, you are almost ready to start writing. The only other thing I would recommend that you do first is to invest an hour or two of thought in the alphabet; you'll find it saves many headaches later.

The letters that are used to denote the concepts you'll discuss are worthy of thought and careful design. A good, consistent notation can be a tremendous help, and I urge (to the writers of articles too, but especially to the writers of books) that it be designed at the beginning. I make huge tables with many alphabets, with many fonts, for both upper and lower case, and I try to anticipate all the spaces, groups, vectors, functions, points, surfaces, measures, and whatever that will sooner or later need to be baptized. Bad notation can make good exposition bad and bad exposition worse; ad hoc decisions about notation, made mid-sentence in the heat of composition, are almost certain to result in bad notation.

Good notation has a kind of alphabetical harmony and avoids dissonance. Example: either  $ax + by$  or  $a_1x_1 + a_2x_2$  is preferable to  $ax_1 + bx_2$ . Or: if you must use  $\Sigma$  for an index set, make sure you don't run into  $\sum_{\sigma \in \Sigma} a_\sigma$ . Along the same lines: perhaps most readers wouldn't notice that you used  $|z| < \varepsilon$  at the top of the page and  $z \in U$  at the bottom, but that's the sort of near dissonance that causes a vague non-localized feeling of malaise. The remedy is easy and is getting more and more nearly universally accepted:  $\in$  is reserved for membership and  $\varepsilon$  for ad hoc use.

Mathematics has access to a potentially infinite alphabet (e.g.,  $x, x', x'', x''', \dots$ ), but, in practice, only a small finite fragment of it is usable. One reason is that a human being's ability to distinguish between symbols is very much more limited than his ability to conceive of new ones; another reason is the bad habit of freezing letters. Some old-fashioned analysts would speak of "xyz-space", meaning, I think, 3-dimensional Euclidean space, plus the convention that a point of that space shall always be denoted by " $(x,y,z)$ ". This is bad: it "freezes"  $x$ , and  $y$ , and  $z$ , i.e., prohibits their use in another context, and, at the same time, it makes it impossible (or, in any case, inconsistent) to use, say, " $(a,b,c)$ " when " $(x,y,z)$ " has been temporarily exhausted. Modern versions of the custom exist, and are no better. Example: matrices with "property L"—a frozen and unsuggestive designation.

There are other awkward and unhelpful ways to use letters: "CW complexes" and "CCR groups" are examples. A related curiosity that is probably the upper bound of using letters in an unusable way occurs in Lefschetz [6]. There  $x_i^p$  is a chain of dimension  $p$  (the subscript is just an index), whereas  $x_p^i$  is a co-chain of dimension  $p$  (and the superscript is an index). Question: what is  $x_3^2$ ?

As history progresses, more and more symbols get frozen. The standard examples are  $e, i$ , and  $\pi$ , and, of course,  $0, 1, 2, 3, \dots$  (Who would dare

write “Let 6 be a group.”?) A few other letters are almost frozen: many readers would feel offended if “ $n$ ” were used for a complex number, “ $\varepsilon$ ” for a positive integer, and “ $z$ ” for a topological space. (A mathematician’s nightmare is a sequence  $n_\varepsilon$  that tends to 0 as  $\varepsilon$  becomes infinite.)

Moral: do not increase the rigid frigidity. Think about the alphabet. It’s a nuisance, but it’s worth it. To save time and trouble later, think about the alphabet for an hour now; then start writing.

## 6. WRITE IN SPIRALS

The best way to start writing, perhaps the only way, is to write on the spiral plan. According to the spiral plan the chapters get written and re-written in the order 1, 2, 1, 2, 3, 1, 2, 3, 4, etc. You think you know how to write Chapter 1, but after you’ve done it and gone on to Chapter 2, you’ll realize that you could have done a better job on Chapter 2 if you had done Chapter 1 differently. There is no help for it but to go back, do Chapter 1 differently, do a better job on Chapter 2, and then dive into Chapter 3. And, of course, you know what will happen: Chapter 3 will show up the weaknesses of Chapters 1 and 2, and there is no help for it ... etc., etc., etc. It’s an obvious idea, and frequently an unavoidable one, but it may help a future author to know in advance what he’ll run into, and it may help him to know that the same phenomenon will occur not only for chapters, but for sections, for paragraphs, for sentences, and even for words.

The first step in the process of writing, rewriting, and re-rewriting, is writing. Given the subject, the audience, and the outline (and, don’t forget, the alphabet), start writing, and let nothing stop you. There is no better incentive for writing a good book than a bad book. Once you have a first draft in hand, spiral-written, based on a subject, aimed at an audience, and backed by as detailed an outline as you could scrape together, then your book is more than half done.

The spiral plan accounts for most of the rewriting and re-rewriting that a book involves (most, but not all). In the first draft of each chapter I recommend that you spill your heart, write quickly, violate all rules, write with hate or with pride, be snide, be confused, be “funny” if you must, be unclear, be ungrammatical—just keep on writing. When you come to rewrite, however, and however often that may be necessary, do not edit but rewrite. It is tempting to use a red pencil to indicate insertions, deletions, and permutations, but in my experience it leads to catastrophic blunders. Against human impatience, and against the all too human partiality everyone