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implication is, quite likely, unsupported. Is it obvious? (Say so.) Will a counterexample be supplied later? (Promise it now.) Is it a standard but for present purposes irrelevant part of the literature? (Give a reference.) Or, horribile dictu, do you merely mean that you have tried to derive q from p, you failed, and you don't in fact know whether p implies q? (Confess immediately!) In any event: take the reader into your confidence.

There is nothing wrong with the often derided "obvious" and "easy to see", but there are certain minimal rules to their use. Surely when you wrote that something was obvious, you thought it was. When, a month, or two months, or six months later, you picked up the manuscript and re-read it, did you still think that that something was obvious? (A few months' ripening always improves manuscripts.) When you explained it to a friend, or to a seminar, was the something at issue accepted as obvious? (Or did someone question it and subside, muttering, when you reassured him? Did your assurance consist of demonstration or intimidation?) The obvious answers to these rhetorical questions are among the rules that should control the use of "obvious". There is another rule, the major one, and everybody knows it, the one whose violation is the most frequent source of mathematical error: make sure that the "obvious" is true.

It should go without saying that you are not setting out to hide facts from the reader; you are writing to uncover them. What I am saying now is that you should not hide the status of your statements and your attitude toward them either. Whenever you tell him something, tell him where it stands: this has been proved, that hasn't, this will be proved, that won't. Emphasize the important and minimize the trivial. There are many good reasons for making obvious statements every now and then; the reason for saying that they are obvious is to put them in proper perspective for the uninitiate. Even if your saying so makes an occasional reader angry at you, a good purpose is served by your telling him how you view the matter. But, of course, you must obey the rules. Don't let the reader down; he wants to believe in you. Pretentiousness, bluff, and concealment may not get caught out immediately, but most readers will soon sense that there is something wrong, and they will blame neither the facts nor themselves, but, quite properly, the author. Complete honesty makes for greatest clarity.

10. Down with the irrelevant and the trivial

Sometimes a proposition can be so obvious that it needn't even be called obvious and still the sentence that announces it is bad exposition, bad because it makes for confusion, misdirection, delay. I mean something like this: "If R is a commutative semisimple ring with unit and if x and y are in R, then $x^2 - y^2 = (x - y)(x + y)$." The alert reader will ask himself what semisimplicity and a unit have to do with what he had always thought was obvious. Irrelevant assumptions wantonly dragged in, incorrect emphasis, or even just the absence of correct emphasis can wreak havoc.

Just as distracting as an irrelevant assumption and the cause of just as much wasted time is an author's failure to gain the reader's confidence by explicitly mentioning trivial cases and excluding them if need be. Every complex number is the product of a non-negative number and a number of modulus 1. That is true, but the reader will feel cheated and insecure if soon after first being told that fact (or being reminded of it on some other occasion, perhaps preparatory to a generalization being sprung on him) he is not told that there is something fishy about 0 (the trivial case). The point is not that failure to treat the trivial cases separately may sometimes be a mathematical error; I am not just saying "do not make mistakes". The point is that insistence on legalistically correct but insufficiently explicit explanations ("The statement is correct as it stands—what else do you want ?") is misleading, bad exposition, bad psychology. It may also be almost bad mathematics. If, for instance, the author is preparing to discuss the theorem that, under suitable hypotheses, every linear transformation is the product of a dilatation and a rotation, then his ignoring of 0 in the 1-dimensional case leads to the reader's misunderstanding of the behavior of singular linear transformations in the general case.

This may be the right place to say a few words about the statements of theorems: there, more than anywhere else, irrelevancies must be avoided.

The first question is where the theorem should be stated, and my answer is: first. Don't ramble on in a leisurely way, not telling the reader where you are going, and then suddenly announce "Thus we have proved that ...". The reader can pay closer attention to the proof if he knows what you are proving, and he can see better where the hypotheses are used if he knows in advance what they are. (The rambling approach frequently leads to the "hanging" theorem, which I think is ugly. I mean something like: "Thus we have proved

THEOREM 2 ... ".

The indentation, which is after all a sort of invisible punctuation mark, makes a jarring separation in the sentence, and, after the reader has collected his wits and caught on to the trick that was played on him, it makes an undesirable separation between the statement of the theorem and its official label.)

This is not to say that the theorem is to appear with no introductory comments, preliminary definitions, and helpful motivations. All that comes first; the statement comes next; and the proof comes last. The statement of the theorem should consist of one sentence whenever possible: a simple implication, or, assuming that some universal hypotheses were stated before and are still in force, a simple declaration. Leave the chit-chat out: "Without loss of generality we may assume ... " and "Moreover it follows from Theorem 1 that ... " do not belong in the statement of a theorem.

Ideally the statement of a theorem is not only one sentence, but a short one at that. Theorems whose statement fills almost a whole page (or more!) are hard to absorb, harder than they should be; they indicate that the author did not think the material through and did not organize it as he should have done. A list of eight hypotheses (even if carefully so labelled) and a list of six conclusions do not a theorem make; they are a badly expounded theory. Are all the hypotheses needed for each conclusion ? If the answer is no, the badness of the statement is evident; if the answer is yes, then the hypotheses probably describe a general concept that deserves to be isolated, named, and studied.

11. Do and do not repeat

One important rule of good mathematical style calls for repetition and another calls for its avoidance.

By repetition in the first sense I do not mean the saying of the same thing several times in different words. What I do mean, in the exposition of a precise subject such as mathematics, is the word-for-word repetition of a phrase, or even many phrases, with the purpose of emphasizing a slight change in a neighboring phrase. If you have defined something, or stated something, or proved something in Chapter 1, and if in Chapter 2 you want to treat a parallel theory or a more general one, it is a big help to the reader if you use the same words in the same order for as long as possible, and then, with a proper roll of drums, emphasize the difference. The roll of drums is important. It is not enough to list six adjectives in one definition, and re-list five of them, with a diminished sixth, in the second. That's the thing to do, but what helps is to say, in addition: "Note that the