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infinitely many integer solutions can always be found if V is linear, i.e. is a subspace. For the non-linear case we have neither a good generalization of Dirichlet's Theorem nor anything like Roth's Theorem.

Suppose now that V is a hypersurface containing no integer point $\mathbf{x} \neq \mathbf{0}$ and defined by the equation $F(\mathbf{x}) = 0$ where F is a form of degree d with rational integer coefficients. For every integer point $\mathbf{x} \neq \mathbf{0}$ we have $| F(\mathbf{x}) | \geqq 1$, and since $\left| \frac{\partial}{\partial x_i} F(\mathbf{x}) \right| \leqq c_1 | \mathbf{x} |^{d-1}$ $(i=1, ..., n)$, the distance from $\mathbf{x}$ to V is $\geqq c_2 | \mathbf{x} |^{1-d}$, which in turn implies that

$$\psi(V, \mathbf{x}) \geqq c_3 | \mathbf{x} |^{-d},$$

where the constants depend only on V. This inequality may be interpreted as a generalization of Liouville's Theorem. Any improvement of this inequality, even though perhaps it may apply only to special classes of non-linear hypersurfaces, would be of great interest and would shed light on certain diophantine equations different from the equations with norm forms discussed in §10.

REFERENCES

ADAMS, W. W. (1967). Simultaneous asymptotic diophantine approximations. *Mathematika 14*, 173-180.

—— (1969a). Simultaneous asymptotic diophantine approximations to a basis of a real number field. *J. Number Theory 1*, 179-194.

—— (1969b). Simultaneous diophantine approximations and cubic irrationals. *Pacific J. Math. 30*, 1-14.

—— (To appear). Simultaneous Diophantine Approximations to a Basis of a Real Number Field. *Nagoya Math. J. 42*.

BAKER, A. (1962). Continued fractions of transcendental numbers. *Mathematika 9*, 1-8.

—— (1964a). Rational approximations to certain algebraic numbers. *Proc. London Math. Soc. (3) 14*, 385-398.

—— (1964b). Rational approximations to $\sqrt[3]{2}$ and other algebraic numbers. *Quart. J. Math. Oxford Ser.* (2) *15*, 375-383.

—— (1965). On some Diophantine inequalities involving the exponential function. *Can. J. Math. 17*, 616-626.

—— (1966). Linear forms in the logarithms of algebraic numbers. *Mathematika 13*, 204-216.

—— (1967a). Simultaneous approximations to certain algebraic numbers. *Proc. Camb. Phil. Soc. 63*, 693-702.

—— (1967b). Linear forms in the logarithms of algebraic numbers (II). *Mathematika 14*, 102-107.

BAKER, A. (1967c). Linear forms etc. (III). *Mathematika 14*, 220-224.

—— (1968a). Linear forms etc. (IV). *Mathematika 15*, 204-216.

—— (1968b). Contributions to the theory of diophantine equations (I). On the representation of integers by binary forms. *Phil. Trans. Royal Soc. London A 263*, 173-191.

—— (1968c). Contributions etc. (II). The diophantine equation $y^2 = x^3 + k$. *Ibid.*, 193-208.

—— (1968d). The diophantine equation $y^2 = ax^3 + bx^2 + cx + d$. *J. London Math. Soc. 43*, 1-9.

—— (1969). Bounds for the solutions of the hyperelliptic equation. *Proc. Camb. Phil. Soc. 65*, 439-444.

—— (1971). Effective methods in diophantine problems. *Proc. of Symp. in Pure Math. XX* (1969 Number Theory Institute), 195-205.

—— (to appear). A sharpening of the bounds for linear forms in logarithms. *Acta Arith.*

BAKER, A. and J. COATES (1970). Integer points on curves of genus 1. *Proc. Camb. Phil. Soc. 67*, 595-602.

BAKER, A. and H. M. STARK (1971). On a fundamental inequality in number theory. *Annals of Math. 94*, 190-199.

BOREVICH, Z. I. and I. R. SHAFAREVICH (1966). Number theory. (Translated from the Russian (1964) ed. *Academic Press*: New York and London.

BRYUNO, A. D. (1964). The expansion of algebraic numbers in continued fractions (Russian). *Zh. Vychisl. Mat. i Mat. Fiz. 4*, 211-221.

CASSELS, J. W. S. (1955). Simultaneous diophantine approximation. *J. London Math. Soc. 30*, 119-121.

—— (1957). An introduction to diophantine approximation. *Cambridge Tracts 45*, Cambridge University Press.

—— (1959). An introduction to the geometry of numbers. Grundlehren 99. *Springer Verlag*: Berlin-Göttingen-Heidelberg.

CASSELS, J. W. S. and H. P. F. SWINNERTON-DYER (1955). On the product of three homogeneous linear forms and indefinite ternary quadratic forms. *Philos. Trans. Roy. Soc. London Ser. A 248*, 73-96.

CHABAUTY, C. (1938). Sur les équations diophantinnes liées aux unités d'un corps de nombres algébriques fini. *Ann. Mat. Pura Appl. 17*, 127-168.

COATES, J. (1969). An effective p-adic analogue of a Theorem of Thue. *Acta Arith. 15*, 279-305.

—— (1970a). An effective etc. (II). The greatest prime factor of a binary form. *Acta Arith. 16*, 399-412.

—— (1970b). An effective etc. (III). The diophantine equation $y^2 = x^3 + k$. *Acta Arith. 16*, 425-435.

CUGIANI, M. (1959). Sulla approssimabilità dei numeri algebrici mediante numeri razionali. *Ann. Mat. Pura Appl. (4) 48*, 135-145.

DAVENPORT, H. (1937). Note on a result of Siegel. *Acta Arith. 2*, 262-265.

—— (1968). A note on Thue's Theorem. *Mathematika 15*, 76-87.

DAVENPORT, H. and K. F. ROTH (1955). Rational approximations to algebraic numbers. *Mathematika 2*, 160-167.

DAVENPORT, H. and W. M. SCHMIDT (1967). Approximation to real numbers by quadratic irrationals. *Acta Arith. 13*, 169-176.

—— (1969). Approximation to real numbers by algebraic integers. *Acta Arith. 15*, 393-416.

DIRICHLET, L. G. P. (1842). Verallgemeinerung eines Satzes aus der Lehre von den Kettenbrüchen nebst einigen Anwendungen auf die Theorie der Zahlen. *S. B. Preuss Akad. Wiss.* 93-95.

DYSON, F. J. (1947). The approximation to algebraic numbers by rationals. *Acta Math. 79*, 225-240.

FELDMAN, N. I. (1968a). Estimate for a linear form of logarithms of algebraic numbers (Russian). *Mat. Sbornik 76 (118)*, 304-319. *English Transl. Math. USSR Sbornik 5*, 291-307.

—— (1968b). An improvement of the estimate of a linear form in the logarithms of algebraic numbers (Russian). *Mat. Sbornik 77 (119)*, 423-436. *English Transl. Math. USSR Sbornik 6*, 393-406.

—— (1969). A certain inequality for a linear form in the logarithms of algebraic numbers (Russian). *Mat. Zametki 5*, 681-689.

—— (1970a). Bounds for linear forms of certain algebraic numbers (Russian). *Mat. Zametki 7*, 569-580. English transl. *Math. Notes 7* (1970) , 343-349.

—— (1970b). Effective bounds for the size of the solutions of certain diophantine equations (Russian). *Mat. Zametki 8*, 361-371.

FELDMAN, N. I. and A. B. SHIDLOVSKII (1967). The development and the present state of the theory of transcendental numbers. *Russian Math. Surveys 22*, 1-79.

FRAENKEL, A. S. (1962). On a theorem of Ridout in the theory of diophantine approximations. *Trans. Am. Math. Soc. 105*, 84-101.

GELFOND, A. O. (1952). Transcendental and algebraic numbers (Russian). (English transl. (1960), *Dover Publications*: New York.)

GYÖRY, K. (1968). Sur une classe des équations diophantinnes. *Publ. Math. Debrecen 15*, 165-179.

—— (1969). Représentation des nombres par des formes décomposables. I. *Publ. Math. Debrecen 16*, 253-263.

HASSE, H. (1939). Simultane Approximation algebraischer Zahlen durch algebraische Zahlen. *Monatsh. Math. 48*, 205-225.

HOOLEY, C. (1967). On binary cubic forms. *Journal f. Math. 226*, 30-87.

HURWITZ, A. (1891). Über die angenäherte Darstellung der Irrationalzahlen durch rationale Brüche. *Math. Ann. 39*, 279-284.

HYYRÖ, S. (1964). Über die Gleichung $ax^n - by^n = z$ und das Catalansche Problem. *Ann. Acad. Fennicae*, Ser. AI 355.

KEATES, M. (1969). On the greatest prime factor of a polynomial. *Proc. Edinb. Math. Soc. (2) 16*, 301-303.

KHINTCHINE, A. (1925). Zwei Bemerkungen zu einer Arbeit des Herrn Perron. *Math. Zeitschr. 22*, 274-284.

—— (1926a). Über eine Klasse linearer Diophantischer Approximationen. *Rend. Circ. Mat. Palermo 50*, 170-195.

—— (1926b). Zur metrischen Theorie der diophantischen Approximationen. *Math. Z. 24*, 706-714.

KOKSMA, J. F. (1936). Diophantische Approximationen. *Ergebnisse d. Math. u. Grenzgeb. 4*. Springer Verlag: Berlin.

—— (1939). Über die Mahlersche Klasseneinteilung der transzendenten Zahlen und die Approximation komplexer Zahlen durch algebraische Zahlen. *Mh. Math. Phys. 48*, 176-189.

LANG, S. (1962). Diophantine Geometry. Interscience tracts in pure and applied math. 11. J. Wiley & Sons: New York — London.

—— (1965a). Report on diophantine approximations. *Bull. de la Soc. Math. de France 93*, 117-192.

—— (1965b). Asymptotic approximation to quadratic irrationalities. *Am. J. Math. 87*, 481-487.

—— (1965c). Asymptotic approximation etc (II). *Ibid.*, 488-496.

LANG, S. (1966a). Asymptotic diophantine approximation. *Proc. of the Nat. Acad. of Sci. 55*, 31-34.

—— (1966b). Introduction to diophantine approximations. Addison-Wesley Publ. Co.: Reading, Mass.

—— (1971). Transcendental numbers and diophantine approximations. *Bull. Am. Math. Soc. 77*, 635-677.

LEKKERKERKER, C. G. (1969). Geometry of Numbers. Wolters-Noordhoff Publishing: Groningen.

LE VEQUE, W. J. (1955). Topics in number theory. Addison-Wesley Publ. Co.: Reading, Mass.

LIOUVILLE, J. (1844). Sur des classes très étendues de quantités dont la valeur n^e est ni algébrique, ni même réductible à des irrationnelles algébriques. *C. R. Acad. Sci. Paris 18*, 883-885 and 910-911.

LUTZ, E. (1955). Sur les approximations diophantinnes linéaires P-adiques. *Actualités scient. et ind. N° 1224*, Paris.

MAHLER, K. (1932). Zur Approximation der Exponentialfunktion und des Logarithmus I. *J. reine ang. Math. 166*, 118-136.

—— (1933a). Zur Approximation algebraischer Zahlen (I). Über den grössten Primteiler binärer Formen. *Math. Ann. 107*, 691-730.

—— (1933b). Zur Approximation etc. (II). Über die Anzahl der Darstellungen ganzer Zahlen durch Binärformen. *Math. Ann. 108*, 37-55.

—— (1933c). Zur Approximation etc. (III). Über die mittlere Anzahl der Darstellungen grosser Zahlen durch binäre Formen. *Acta Math. 62*, 91-166.

—— (1936). Ein Analogon zu einem Schneiderschen Satz. *Nederl. Akad. Wetensch. Proc. 39*, 633-640 and 729-737.

—— (1939). Ein Übertragungsprinzip für lineare Ungleichungen. *Cas. Pest Mat. 68*, 85-92.

—— (1953). On the approximation of π. *Proc. Akad. Wetensch. Ser. A 56*, 30-42.

—— (1955). On compound convex bodies (I). *Proc. London Math. Soc. (3) 5*, 358-379.

—— (1961). Lectures on diophantine approximation. Notre Dame University.

—— (1963). On the approximation of algebraic numbers by algebraic integers. *J. Austral. Math. Soc. 3*, 408-434.

MINKOWSKI, H. (1907). Diophantische Approximationen. Teubner: Leipzig u. Berlin.

—— (1896) und (1910). Geometrie der Zahlen. Teubner: Leipzig u. Berlin. (The 1910 ed. prepared posthumously by Hilbert and Speiser).

NEUMANN, J. VON and B. TUCKERMAN (1955). Continued fraction expansion of $2^{1/3}$. *Math. Tables Aids Comp. 9*, 23-24.

NIVEN, I. (1963). Diophantine Approximations. Interscience tracts in pure and applied math. 14. J. Wiley & Sons: New York — London.

OSGOOD, C. F. (1970). The simultaneous approximation of certain *k-th* roots. *Proc. Camb. Phil. Soc. 67*, 75-86.

PARRY, C. J. (1940). The p-adic generalization of the Thue-Siegel theorem. *J. London Math. Soc. 15*, 293-305.

—— (1950). The $\mathfrak{p}$-adic generalization of the Thue-Siegel theorem. *Acta Math. 83*, 1-100.

PECK, G. (1961). Simultaneous rational approximations to algebraic numbers. *Bull. Am. Math. Soc. 67*, 197-201.

PERRON, O. (1921). Über diophantische Approximationen. *Math. Ann. 83*, 77-84.

—— (1932). Über mehrfach transzendente Erweiterungen des natürlichen Rationalitätsbereiches. Sitzungsber. Bayer. Akad. Wiss. H 2, 79-86.

—— (1954). Die Lehre von den Kettenbrüchen. 3. Aufl. B. G. Teubner: Stuttgart.

POPKEN, J. (1929). Zur Transzendenz von e. *Math. Z. 29*, 525-541.

RAMACHANDRA, K. (1966). Approximation of algebraic numbers. Nachrichten d. Akad. d. Wiss. in Göttingen, *Math.-Phys. Kl*, 45-52.

—— (1969). A lattice point problem for norm forms in several variables. *J. Number Theory 1*, 534-555.

RICHTMYER, R. D., M. DEVANY and N. METROPOLIS (1962). Continued fraction expansions of algebraic numbers. *Numerische Math. 4*, 68-84.

RIDOUT, D. (1957). Rational approximations to algebraic numbers. *Mathematika 4*, 125-131.

—— (1958). The p-adic generalization of the Thue-Siegel-Roth Theorem. *Mathematika 5*, 40-48.

ROTH, K. F. (1955a). Rational approximations to algebraic numbers. *Mathematika 2*, 1-20.

—— (1955b). Corrigendum. *Ibid.*, 168.

SCHINZEL, A. (1967). Review of a paper by Hyyrö. *Zentralblatt Math. 137*, 257-258.

—— (1968). An improvement of Runge's Theorem on diophantine equations. *Commentarii Pontif. Acad. Soc. 2*, No. 20.

SCHMIDT, W. M. (1962). Simultaneous approximation and algebraic independence of numbers. *Bull. Am. Math. Soc. 68*, 475-478.

—— (1965). Über simultane Approximation algebraischer Zahlen durch rationale. *Acta Math. 114*, 159-206.

—— (1966). Simultaneous approximation to a basis of a real number field. *Amer. J. Math. 88*, 517-527.

—— (1967a). On simultaneous approximation of two algebraic numbers by rationals. *Acta Math. 119*, 27-50.

—— (1967b). Some diophantine equations in three variables with only finitely many solutions. *Mathematika 14*, 113-120.

—— (1970). Simultaneous approximation to algebraic numbers by rationals. *Acta Math. 125*, 189-201.

—— (1971a). Linear forms with algebraic coefficients. I. *J. of Number Theory 3*, 253-277.

—— (1971b). Linearformen mit algebraischen Koeffizienten. II. *Math. Ann. 191*, 1-20.

—— (in preparation). Norm form equations.

SCHNEIDER, Th. (1936). Über die Approximation algebraischer Zahlen. *J. reine angew. Math. 175*, 182-192.

—— (1957). Einführung in die transzendenten Zahlen. Grundlehren 81. Springer Verlag: Berlin-Göttingen-Heidelberg.

SIEGEL, C. L. (1921a). Approximation algebraischer Zahlen. *Math. Zeitschr. 10*, 173-213.

—— (1921b). Über Näherungswerte algebraischer Zahlen. *Math. Ann. 84*, 80-99.

—— (1929). Über einige Anwendungen diophantischer Approximationen. Abh. d. Preuss. Akad. d. Wiss., *Math. Phys. Kl.*, Nr. 1.

—— (1937). Die Gleichung $ax^n - by^n = c$. *Math. Ann. 114*, 57-88.

—— (1970). Einige Erläuterungen zu Thues Untersuchungen über Annäherungswerte algebraischer Zahlen und diophantische Gleichungen. Nachr. Akad. d. Wiss. Göttingen, *Math. Phys. Kl.*, Nr. 8.

SKOLEM, Th. (1935). Einige Sätze über p-adische Potenzreihen mit Anwendung auf gewisse exponentielle Gleichungen. *Math. Ann. 111*, 399-424.

—— (1937). Anwendung exponentieller Kongruenzen zum Beweis der Unlösbarkeit gewisser diophantischer Gleichungen. *Vid. Akad. Avh. Oslo I*, Nr. 12.

—— (1938). Diophantische Gleichungen. Ergebnisse d. Math. 5. Springer Verlag: Berlin.

SPRINDZUK, V. G. (1969). Effective estimates in " ternary " exponential diophantine equations (Russian). *Dokl. Akad. Nauk Belorusskoj SSR 13, No. 9*, 777-780.

—— (1970a). A new application of p-adic analysis on the representation of integers by binary forms (Russian). *Istvestia Akad. Nauk SSR, ser. math. 34, No. 5*, 1038-1063.

SPRINDZUK, V. G. (1970b). An effective estimate of rational approximations to algebraic numbers (Russian). *Dokl. Akad. Nauk Belorusskoj SSR 14, No. 8*, 681-684.

—— (1971a). An improvement of the estimate of rational approximations to algebraic numbers (Russian). *Dokl. Akad. Nauk Belorusskoj SSR 15, No. 2*, 101-104.

—— (1971b). On the greatest prime divisor of a binary form (Russian). *Ibid.*, No. 5, 389-391.

—— (1971c). Rational approximations to algebraic numbers (Russian). *Istvestia Akad. Nauk SSR 5.*

STEPANOW, S. A. (1967). The approximation of an algebraic number by algebraic numbers of a special form (Russian). Vestnik Moskov. Univ., Ser. I, *Math. Meh. 22, No. 6*, 78-86.

THUE, A. (1908). Bemerkungen über gewisse Näherungsbrüche algebraischer Zahlen. Über rationale Annäherungswerte der reellen Wurzel der ganzen Funktion dritten Grades $x^3 - ax - b$. On en general i store hele tal uløsbar ligning. Skrifter udgivne of Videnskabs-Selskabet i Christiania.

—— (1909). Über Annäherungswerte algebraischer Zahlen. *Journal f. Math. 135*, 284-305.

TIJDEMAN, R. (1971). On the algebraic independence of certain numbers. *Indag. Math. 33*, 146-162.

VINOGRADOV, A. I. and V. G. SPRINDZUK (1968). The representation of numbers by binary forms (Russian). *Mat. Zametki 3*, 369-376.

WALLISER, R. (1969). Zur Approximation algebraischer Zahlen durch arithmetisch charakterisierte algebraische Zahlen. *Arch. Math. (Basel) 20*, 384-391.

WIRSING, E. (1961). Approximation mit algebraischen Zahlen beschränkten Grades. *J. reine ang. Math. 206*, 67-77.

—— (1971). On approximations of algebraic numbers by algebraic numbers of bounded degree. *Proc. of Symp. in Pure Math. XX.* (1969 Number Theory Institute) 213-247.

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