Abstract

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A BOUNDARY VALUE CHARACTERIZATION OF WEIGHTED H^1

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ABSTRACT

We give a proof of the elementary result that for certain weight functions w, the Hardy space H_w^1 can be identified with the class of functions f such that f and all its Riesz transforms $R_j f$ belong to L_w^1. An important ingredient of the proof is that there exist positive constants c and μ, $0 < \mu < 1$, depending only on the dimension n such that if f belongs to L_w^1, then

$$N(f)(x) \leqslant c \left[M_\mu (f)(x) + \sum_{j=1}^{n} M_\mu (R_j f)(x) \right],$$

where $N(f)$ denotes the non-tangential maximal function of the Poisson (or any conjugate Poisson) integral of f, and M_μ denotes the Hardy-Littlewood maximal operator of order μ:

$$M_\mu (g)(x) = \left(\sup_{h>0} h^{-n} \int_{|y|<h} |g(x+y)|^\mu \, dy \right)^{1/\mu}.$$

§1. INTRODUCTION

Let $F(x, t) = \big(u(x, t), v_1(x, t), \ldots, v_n(x, t)\big)$, $x = (x_1, \ldots, x_n) \in R^n$, $t > 0$, satisfy the Cauchy-Riemann equations in the sense of Stein and Weiss [9]: i.e., $u, v_1, \ldots, v_n$ are harmonic in

$$R_+^{n+1} = \{(x, t) : x \in R^n, \; t > 0\}, \quad \frac{\partial u}{\partial t} + \sum_{j=1}^{n} \frac{\partial v_j}{\partial x_j} = 0$$

and

$$\frac{\partial v_j}{\partial x_i} = \frac{\partial v_i}{\partial x_j}, \quad \frac{\partial v_j}{\partial t} = \frac{\partial u}{\partial x_j}$$

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