Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 22 (1976)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: BOUNDARY VALUE CHARACTERIZATION OF WEIGHTED H^1

Autor: Wheeden, Richard L.

Kurzfassung: Abstract

DOI: https://doi.org/10.5169/seals-48178

A BOUNDARY VALUE CHARACTERIZATION OF WEIGHTED H^1

by Richard L. WHEEDEN [1])

ABSTRACT

We give a proof of the elementary result that for certain weight functions w, the Hardy space H_w^1 can be identified with the class of functions f such that f and all its Riesz transforms $R_j f$ belong to L_w^1. An important ingredient of the proof is that there exist positive constants c and μ, $0 < \mu < 1$, depending only on the dimension n such that if f belongs to L_w^1, then

$$N(f)(x) \leqslant c\left[M_\mu(f)(x) + \sum_{j=1}^{n} M_\mu(R_j f)(x)\right],$$

where $N(f)$ denotes the non-tangential maximal function of the Poisson (or any conjugate Poisson) integral of f, and M_μ denotes the Hardy-Littlewood maximal operator of order μ:

$$M_\mu(g)(x) = \left(\sup_{h>0} h^{-n} \int_{|y|<h} |g(x+y)|^\mu \, dy\right)^{1/\mu}.$$

§1. INTRODUCTION

Let $F(x, t) = \big(u(x, t), v_1(x, t), \ldots, v_n(x, t)\big)$, $x = (x_1, \ldots, x_n) \in R^n$, $t > 0$, satisfy the Cauchy-Riemann equations in the sense of Stein and Weiss [9]: i.e., $u, v_1, \ldots, v_n$ are harmonic in

$$R_+^{n+1} = \{(x, t) : x \in R^n, \ t > 0\}, \frac{\partial u}{\partial t} + \sum_{j=1}^{n} \frac{\partial v_j}{\partial x_j} = 0$$

and

$$\frac{\partial v_j}{\partial x_i} = \frac{\partial v_i}{\partial x_j}, \frac{\partial v_j}{\partial t} = \frac{\partial u}{\partial x_j}$$

[1] Supported in part by NSF-MPS75-07596.