Zeitschrift: L'Enseignement Mathématique

Band: 22 (1976)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: HOW TO USE RUNGE'S THEOREM

Bibliographie

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DOI: https://doi.org/10.5169/seals-48184

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where G is some open superset in D of the set E. Now to construct f_2, we observe that the complement in D of G is contained in $\bigcup_{k=1}^{\infty} S_k$, where each S_k is a closed annular sector of the form

$$S_k = \{z = re^{i\theta}: s_k \leqslant r \leqslant t_k, \delta_k \leqslant \theta \leqslant 2\pi - \delta_k\}$$

and $s_k \uparrow 1$, $t_k \uparrow 1$ and $t_k < s_{k+1}$ for $k = 1, 2, 3, \ldots$.

Define g_1 in D by $g_1(z) = 2$. Having defined $g_1, \ldots, g_n$, consider a closed disc U^n with center at 0 that contains $S_1, \ldots, S_n$ and a slightly larger open disc D^n that excludes S_{n+1}. Let S'_{n+1} be an open superset of S_{n+1} that does not intersect D^n, and define φ_n in $D^n \cup S'_{n+1}$ by $\varphi_n(z) = n + 2 - \sum_{i=1}^{n} g_1(z)$ for $z \in S'_{n+1}$ and $\varphi_n(z) = 0$ in D^n. By Runge's theorem, there is a polynomial g_n such that

$$|g_n(z) - \varphi_n(z)| < 2^{-n-2}$$

for $z \in U^n \cup S_{n+1}$. Let $f_2 = \sum_{j=1}^{\infty} g_j$. It is easily verified that the series converges uniformly on compact subsets of D to a function f_2 that is analytic on D. On S_n,

$$f_2(z) = g_n(z) + \sum_{i=1}^{n-1} g_i(z) + \sum_{i=n+1}^{\infty} g_i(z),$$

so that in S_n

$$|f_2(z)| \geqslant n + 1 - \sum_{i=n+1}^{\infty} 2^{-i-2} \geqslant n .$$

Hence

$$(**) \qquad \liminf_{r \to 1} \{|f_2(z)|: z \in \bigcup_{n=1}^{\infty} S_n, |z| \geqslant r\} = \infty$$

Since $G \cup \left(\bigcup_{n=1}^{\infty} S_n\right) = D$, we have the desired result on putting (*) and (**) together.

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(Reçu le 10 février 1976)

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