1. Introduction

QUADRATIC FORMS IN AN ADELIC SETTING [1])

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1. INTRODUCTION

The connection between Siegel's main theorem in the analytical theory of quadratic forms and the determination of the Tamagawa number of the orthogonal group has been discussed in expository articles by Kneser [1] and Tamagawa [5]. Both of these papers, however, consider only a special case of Siegel's general theorem, namely the number of representations of a quadratic form by itself. In the present paper we consider the problem of representing a positive definite form by another positive definite form. Siegel's main theorem is derived from an adelic integral formula, Ono's "mean value theorem", which is the analogue for the adelized orthogonal group of Siegel's mean value theorem in the geometry of numbers.

2. THE MEAN VALUE FORMULA

The adelic mean value formula generalizes Siegel's mean value theorem in the geometry of numbers [3]. We first describe Siegel's theorem as reformulated by Weil [8].

Let Φ be a continuous function of $\mathbf{R}^n$ $(n \geqslant 2)$ with compact support. Then

$$\Phi(g) = \sum_{X \in Z^n - \{0\}} \Phi(gx)$$

defines a function on $SL_n(\mathbf{R})$, right invariant by $SL_n(\mathbf{Z})$. According to Siegel's theorem, $SL_n(\mathbf{R}) / SL_n(\mathbf{Z})$ has finite measure, Φ is integrable on this space, and

$$\frac{\int\limits_{SL_n(\mathbf{R})/SL_n(\mathbf{Z})} \Phi(g)\, dg}{\int\limits_{SL_n(\mathbf{R})/SL_n(\mathbf{Z})} dg} = \int\limits_{\mathbf{R}^n} \Phi(x)\, dx \,.$$

In Siegel's original formulation, Φ is taken to be the characteristic function

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