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### **Haftungsausschluss**

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$$\begin{aligned} \tilde{F} - F &= \sum_{\alpha; \beta_1, \dots, \beta_\alpha} p_{\beta_1, \dots, \beta_\alpha}(x_1, \dots, x_n) \\ &\times \varphi_{\beta_1, \dots, \beta_\alpha}(q_{\beta_1, \dots, \beta_\alpha, 1}(x_1, \dots, x_n), \dots, q_{\beta_1, \dots, \beta_\alpha, k}(x_1, \dots, x_n)) + R(x_1, \dots, x_n), \end{aligned}$$

where  $|R(x_1, \dots, x_n)| \leq \gamma(\varepsilon)\varepsilon$ ,  $\gamma(\varepsilon) \rightarrow 0$  as  $\varepsilon \rightarrow 0$ , and

$$\begin{aligned} \varepsilon &= \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sup_t |\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)| \\ &\leq \lambda \sup_{x \in G_n} |\tilde{F}(x_1, \dots, x_n) - F(x_1, \dots, x_n)|. \end{aligned}$$

That  $\gamma(\varepsilon) \rightarrow 0$  as  $\varepsilon \rightarrow 0$  follows from the fact that as  $\varepsilon \rightarrow 0$  the quantity

$$\varepsilon' = \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sum_{i=1}^k \sup \left| \frac{\partial \varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)}{\partial t_i} \right| \rightarrow 0,$$

provided only that the modulus of continuity of the partial derivatives of the functions  $\{\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)\}$  is fixed. By 5.1.10 it follows that  $r(A, F) \leq k$  in some subregion  $G_n \subset D_n$ . So we have obtained a contradiction to the assumption that  $r(A, F) > k$  in any subregion  $G_n \subset D_n$  and this proves the theorem.

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