8. Applications

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and

$$\| x^*x \| = \| x \|^2 \quad \textit{for all normal} \quad x \in A .$$

Then A is a B^*-algebra.

In a later paper [48] Sebestyén claimed to prove that continuity of the involution can be dropped from Theorem 2 above. However, G. A. Elliott has pointed out an error in [48]; indeed, on line four of page 212 the series displayed, although convergent, is not shown to converge to the quasi-inverse of $\lambda^{-1}x$. The paper does reduce the problem to the commutative case; but in this case it remains an interesting open question.

8. Applications

Numerous applications of the Gelfand-Naimark theorems appear in the literature. Indeed, utilizing the representation theorem for commutative algebras important theorems in abstract harmonic analysis can be established. For example both the Plancherel theorem and the Pontryagin duality theorem are proved in [33] via the commutative theorem. Further applications to harmonic analysis can be found in [15], [30], [33] and [37]. The representation theorem for commutative algebras can be used to establish important results on compactifications of topological spaces and locally compact abelian groups (see [15], [30] and [33]); it also provides the most elegant method of proof of the spectral theorem for normal operators on a Hilbert space ([15], [30], [33]).

Applications to group representations and von Neumann algebras can be found in [13] and [37]. For applications to numerical ranges of operators see [7], [9], [10], [11] and [34].

In recent years the theory of C*-algebras has entered into the study of statistical mechanics and quantum theory. The basic principle of the algebraic approach is to avoid starting with a specific Hilbert space scheme and rather to emphasize that the primary objects of the theory are the fields (or observables) considered as purely algebraic quantities, together with their linear combinations, products, and limits in an appropriate topology. The representations of these algebraic objects as operators acting on a suitable Hilbert space can then be obtained in a way that depends essentially only on the states of the physical system under investigation. The principal tool needed to build the required Hilbert space and associated representation is the Gelfand-Naimark-Segal construction discussed earlier in this article.

A substantial literature has now emerged from this new algebraic point of view and a recent book by G. Emch [18] has been written with the express purpose of offering a systematic introduction to the ideas and techniques of the C*-algebra approach to physical problems. The authors recommend this book to the reader who would like to pursue this subject further. The book contains a bibliography of more than four hundred items which should aid the interested reader in his study of this new and interesting application of operator algebras.

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