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## 8. AUTOMORPHIC FUNCTIONS AND BELTRAMI DIFFERENTIALS

Although this aspect has not been emphasized it should be clear that the author is trying to develop a theory which is immediately applicable to the study of discrete subgroups of  $G$ . All the definitions have been chosen with this in mind, and the relevant theorems for subgroups follow effortlessly.

Let  $G^0$  be a discrete subgroup of  $G$ . A vector-valued function  $f$  is *automorphic* with respect to  $G^0$  if  $A^* f = f$ , or more explicitly  $A'(x)^{-1} f(Ax) = f(x)$  for all  $A \in G^0$ . Similarly, an  $SM_n$ -valued function  $v$  will be called a *Beltrami differential* for  $G^0$  if  $A^* v = v$ , or  $A'(x)^{-1} v(Ax) A'(x) = v(x)$ , for all  $A \in G^0$ . If  $v$  is a Beltrami differential, then  $A^*(\rho v dx) = \rho v dx$  for all  $A \in G^0$ , and  $\rho v dx$  is called an  $n$ th order differential. The terminology is borrowed from the corresponding notions for  $n = 2$ .

If  $v$  is Beltrami and in  $L^\infty$ , then it is also in  $L^p(B)$  for all  $p$ , and Theorems 2-5 are applicable. They gain added significance from the fact that  $Iv$  is automatically automorphic with respect to  $G^0$  (it is easy to show that  $A^* Iv = IA^* v$  for all  $v$  and  $A \in G$ ). As a consequence  $SIv$  is Beltrami, and by Theorem 2 the same is true of  $\Gamma v$ . It follows that Theorems 2-5 may be interpreted as referring to the quotient space  $G^0 \backslash B$ , provided that we start from the hypothesis  $v \in L^\infty$ . In the conclusion we know, for instance, that

$$\int_B \|SI \gamma\|^p dx = \int_{G^0 \backslash B} \|SI v\|^p \rho_0 dx < \infty$$

where, by a theorem of Godement,

$$\rho_0(x) = \sum_{A \in G^0} |A'(x)|^n$$

is known to converge.

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