8. Automorphic functions and Beltrami differentials

Although this aspect has not been emphasized it should be clear that the author is trying to develop a theory which is immediately applicable to the study of discrete subgroups of G. All the definitions have been chosen with this in mind, and the relevant theorems for subgroups follow effortlessly.

Let G^0 be a discrete subgroup of G. A vector-valued function f is *automorphic* with respect to G^0 if $A^* f = f$, or more explicitly $A'(x)^{-1} f(Ax) = f(x)$ for all $A \in G^0$. Similarly, an SM_n-valued function v will be called a *Beltrami differential* for G^0 if $A^* v = v$, or $A'(x)^{-1} v(Ax) A'(x) = v(x)$, for all $A \in G^0$. If v is a Beltrami differential, then $A^*(\rho\, v\, dx) = \rho\, v\, dx$ for all $A \in G^0$, and $\rho\, v\, dx$ is called an nth order differential. The terminology is borrowed from the corresponding notions for $n = 2$.

If v is Beltrami and in L^∞, then it is also in $L^p(B)$ for all p, and Theorems 2-5 are applicable. They gain added significance from the fact that Iv is automatically automorphic with respect to G^0 (it is easy to show that $A^* Iv = IA^* v$ for all v and $A \in G$). As a consequence SIv is Beltrami, and by Theorem 2 the same is true of Γv. It follows that Theorems 2-5 may be interpreted as referring to the quotient space $G^0 \backslash B$, provided that we start from the hypothesis $v \in L^\infty$. In the conclusion we know, for instance, that

$$\int_B \| SI\gamma \|^p dx = \int_{G^0 \backslash B} \| SI\, v \|^p \rho_0 dx < \infty$$

where, by a theorem of Godement,

$$\rho_0(x) = \sum_{A \in G^0} | A'(x) |^n$$

is known to converge.

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